Theoretical perspectives on, and methodologies for, researching teacher decision making

Andreas Ebbelind[1]; Ralf Erens[2]; Tracy Helliwell[3]

[1] Linnaeus University, Sweden
[2] University of Education, Freiburg, Germany
[3] University of Bristol, United Kingdom

Abstract: In this paper, we reflect on the role of our respective research directions to gain knowledge about ourselves as mathematics teachers, teacher educators and researchers through presenting a range of perspectives from the field of affect with a particular focus on the phenomenon of teacher decision making. The basis of this reflection is our different backgrounds in beliefs, narrative identity and enactivist research perspectives. In this paper, we follow a research tradition based on what is known as duoethnography, where the discussions amongst us are the research site, and the outcome of these discussions is the data on which we base our results. In this paper, a shared understanding of one another's perspectives is crucial. For example, a shared understanding is needed when conducting scientific work with others, reviewing papers outside your perspective or participating fully in conferences.

Keywords: beliefs, identity, learning, duoethnography, teacher decision making

Correspondence: tracy.helliwell@bristol.ac.uk

1 The process of research

In 1995, the first MAVI conference was held in Duisburg, Germany. Ten papers were presented, mostly relating to the concept of beliefs. Nearly 30 years later, the MAVI community is preparing for yet another MAVI conference, MAVI 30, in Freiburg. Nowadays, the MAVI community is more diverse, covering the broad area of affect in mathematics education, which entails attitudes, beliefs, emotions, identity, meaning-making, motivation, values, and the like. It is at the heart of the MAVI spirit that each participant is an integrated part of the whole MAVI process. That means writing a paper proposal in line with the call for papers, writing well-informed, in-depth reviews, and being well-prepared for the conference. The local organising team arranging the conference is obliged to give each contributor enough time to engage in in-depth discussions to both develop the work presented and enrich the general discussion brought to attention at the conference. As frequent MAVI participants, we recognise the need to maintain this engagement given the diversity of work now being presented at each conference. To enable this, it is essential to look beyond one's own perspective to engage in the various joint discussions. In light of this need, our paper aims

to elaborate on our diverse research backgrounds concerning the process of research to increase our awareness of one another's research perspectives for ourselves and for other MAVI participants. We do this by adopting a methodological approach called duoethnography (which we discuss in section 2).

Eisenhart (1991) and Lester (2005) both highlight the role of three fundamental elements of the research process: the focus of interest; the theoretical perspective and the methodological principles. In this paper, our phenomenon of interest (something that each of us have researched) is mathematics teacher (and mathematics teacher educator) decision making, hereon referred to as teacher decision making. The theoretical perspective provides a guiding framework for the way in which the research process is comprehended and advanced, comprising predefined concepts and assumptions. Though we share a common research focus, in our research we have drawn on different theoretical perspectives. In this paper, we make our different perspectives visible, exploring the similarities, differences and connections between them. It follows that each framework establishes a set of methodological principles, which in turn determines the form of the outcomes. A research methodology entails methodological choices, including the process of reducing empirical material into meaningful data. We discuss some of the methodological implications of our chosen perspectives in relation to researching teacher decision making.

2 Duoethnography

Duoethnography, exemplified by works such as Sawyer and Norris (2015), is a qualitative research methodology in which researchers collaboratively craft a dialogic narrative in their distinctive voices. The process of duoethnography is a collaborative approach to research where two or more researchers re-engage in their (research) life stories to illuminate various facets of a social phenomenon. By delving into their personal narratives, they offer readers multiple viewpoints, forcing them to engage in the discourse. Through this dialectical process, duoethnography sheds light on social phenomena and provides opportunities for researchers to work on their identity, work that might transform the researchers themselves. The narrative dialogical structure of duoethnography is pivotal as it signifies how personal experiences can be cast as interconnected "stories" culminating in a coherent dialogic text. Duoethnography sets out to present varied perspectives on a phenomenon but also captures its evolution over time. Through duoethnography, we are encouraged as researchers to introspect and re-evaluate our past experiences and viewpoints. This willingness to self-reflect

places us in a vulnerable position (Zazkis & Koichu, 2015) yet concurrently provides an arena for sharing our research journeys and insights.

Few mathematics education researchers have employed duoethnography as a methodology. Lloyd et al. (2021) used the approach to carry out and report on a collaborative self-study of mathematics teacher educators. Zazkis & Koichu (2015) adapted the approach by combining it with a fictional technique of data analysis known as virtual monologue (e.g., Ejersbo & Leron, 2005) which they call virtual duoethnography where "researchers produce a text of a dialogic format in the voices of fictional characters" (Zazkis & Koichu, 2015, p. 163).

The narrative we present in this paper interlaces our recollections of personal histories, all tied to the phenomenon of teacher decision making whilst offering diverse perspectives. We, as duoethnographers ourselves, serve as the site of research, not merely its subject matter. Our experiences facilitate a deeper comprehension of the phenomenon under scrutiny, in this case, our different research perspectives and their consequences when doing research.

3 Teacher decision making

This section consists of our duoethnographic dialogue which begins from the point at which a common phenomenon of interest was identified by the three authors, namely mathematics teacher (and mathematics teacher educator) decision making. The duoethnographic text presented below is not a script of an actual dialogue, rather, it is the result of "purposeful juxtaposition and intertwining of stories" (Apostolidou & Daskalakithat, 2021) which were previously told separately, through email communications and recorded conversations. This material constitutes the raw data from which the final dialogue has been produced through a process of "aesthetic structuring" (Winter, 1988, p. 233), a method of theorising, or meaning making, "rather than merely a format in which theory (created by some other process) can be presented" (p. 233).

The dialogue is organised under two main headings: theoretical perspectives on teacher decision making; and methodologies for researching teacher decision making. Throughout the dialogue, the three authors assume various positions, specifically: Ralf talks from his background in beliefs research; Andreas from his background in identity research; and Tracy from her background in researching learning from an enactivist perspective. Given the focus of the MAVI community on affective aspects of mathematics teaching and learning, we explore how the affective aspects of teacher

decision making are dealt with from our different perspectives, to better understand the convergences, divergences and connections across our different worlds of research.

3.1 Theoretical perspectives on teacher decision making

Ralf: Having worked as a teacher and teacher educator, I have always been curious about what guides me and other teachers in their decision making in mathematics classrooms and what their underlying goals and beliefs are.

Tracy: The question of what guides me in my decision making as a teacher of mathematics, or as a mathematics teacher educator, is such a fundamental one, both at an ontological level and an epistemological one. Are my actions as a teacher or teacher educator *guided* by my decision making? Or is there a more suitable metaphor? Is there something *underlying* my decision making? Maybe some would refer to this as my identity or set of values. Or is my decision making *triggered by* the various external stimuli that I encounter when working with students or prospective teachers? Varela (1999) suggested we are "always operating in some kind of immediacy of a given situation" (p. 9, emphasis original), a mode which he refers to as "immediate coping" (p. 9), akin to being in a state of flow without the need (or even capacity) for rational deliberation. When operating purely in this mode, I doubt whether I would have access to my decision making. I am simply responding to the situation at hand in a seemingly automated way. Imagine if we needed to rationalise every decision, we would never cope with the complex nature of classrooms or teacher education situations (or any situation, come to think of it). One reading of Varela's immediate coping could be that we act based on our intuitions, but what are our intuitions made up from and how do we develop them or access them in a meaningful way? Claxton (2000) wrote that intuition "provides the 'glue' that holds together our conscious intellect and our intelligent action" (p. 36) and that it is "the bedrock on which all other ways of knowing are constructed" (p. 48). So basically, we act first, and later construct meaning based on those actions.

Andreas: Previously, I have approached decision making as a reciprocal relationship between language and social practice, as Wetherell (2001) and Halliday (1978) have both highlighted. Halliday describes this reciprocal relationship as context shaping language and vice versa. This echoes the idea that our linguistic expressions are triggered by and contribute to the broader social context. Moreover, Holland and Eisenhart (1990) emphasise how culture mediates our interpretation of the world,

suggesting that cultural meanings also inform our language and shape our perception of reality. This interplay between language, context, and culture has significant implications for me in the discussion of my decision-making. However, I now find myself scrutinising this view of reciprocity and tend to view the interaction as mutual rather than reciprocal (Skott, 2022). I act immediately, in the moment, that is, I mirror myself with "the other" (Blumer, 1969) when making decisions. This move will have consequences for me as a researcher in the future. From a theoretical perspective, social practices' temporal and spatial structuring contribute to the process by which individuals draw upon past and present social practices and cultural meanings in their immediate interactions, mainly through language use (Giddens, 1984; Holland et al., 1998). Identity then reflects the immediate social interaction since individuals develop their understanding of themselves and their environment through participation in diverse social practices, that is, act differently in different social situations. When trying to conceptualise various ways of participating, we utilise language as a primary medium, both self-communication and communication with others.

Ralf: A comprehensive worldview that is focused on the learning, doing, and teaching of mathematics can be fundamental when looking at individual structure with regard to mathematics. The basic components of the structure of beliefs concerning the nature of mathematics have been conceptualised. Different notions more or less share a common ground which can be distinguished in two overarching perspectives: a static view which is characterised by formalist or schema-related views, and a dynamic view which consists of either a process-oriented or application-oriented perspective (e.g. Törner et al, 2014). Yet, teachers' decision-making and instructional choices in planning their teaching may be affected by a range of factors including professional development interventions (e.g., teacher training), curricular trends propagated by education authorities and mathematics educators. The common ground in research has been given agreement, that such world views and beliefs are "used to designate individual, subjectively true, value-laden mental constructs that are the relatively stable results of substantial social experiences and that have significant impact on one's interpretations of and contributions to classroom practice" (Skott, 2015).

Tracy: You talk about the *structure of beliefs* here, Ralf. Enactivists would not tend to use that terminology, but a fundamental concept in the enactivist theory of cognition is that of *structure*. As living systems, humans each have their own complex structure that determines the way in which they respond (i.e., our decision making) in a given situation, a notion that is known as structural determinism (Maturana & Varela,

1998) and which has profound implications on the way we conceptualise learning. If the way we each respond is determined by our own unique structures, then no stimulus (from, say, a teacher, or a teacher educator) can directly determine what happens (i.e., what is learned). This is part of a fundamental argument that rejects a transmission model of learning, and at the same time, could be read in terms of a learning paradox (that is, how can we learn anything new if the way we respond to a stimulus is determined by our existing structures?) Enactivism deals with this potential paradox using the concept of *emergence* claiming that learning is "emergent action" (Proulx & Simmt, 2016, p. 102). Since individuals are understood as parts of a series of increasingly complex systems, novelty can arise out of the interactions of these different existing components. Thus decision making can never really be assigned to an individual, rather it happens in interaction with that individual's environment (which, in a teaching situation, includes other individuals).

Andreas: I agree that there are things that affectively relate to how we act in immediate social interaction. I would not term them beliefs since I try to avoid reifying how we act and know. I talk about identities, not in the singular, nor as static and residing within, but as something emerging in situ. If we look at how the most prominent researchers define identity, they have two words in common: context and practice (Gee, 2011; Holland et al., 1998). From an interactionist perspective (Blumer, 1969), human beings engage with things based on the significance these things hold for them. When doing this, meanings are attributed to things that emerge from individuals' social interactions with others. In this process, individuals continuously interpret and adjust the meanings of things through an ongoing interaction process with the objects they encounter. Aligning with Skott (2022), I want to utilise dynamic and participatory approaches to affect.

Ralf: I wonder how beliefs become beliefs; is it through individual, life-long learning? In a framework by Hannula (2012), beliefs consist of cognitive, motivational and affective aspects. Teaching experience and professional enculturation and education are connected to an evolutionary process of developing beliefs about mathematics and its teaching. Existing research (e.g., Oliveira & Hannula, 2008; Liljedahl et al, 2012) indicates crucial phases in which such an evolutionary process of developing beliefs takes place. Although beliefs have often been referred to as a messy construct, there has been a consensus that they can be considered as personal philosophies about the teaching and learning of mathematics. Influenced by experience in (one's own) learning and teaching, beliefs can be formed to a certain extent on the basis of evidence.

As Skott (2015) suggests, beliefs are considered to be stable reifications as a result of engagement in relevant social practices. Thus, it can be argued that beliefs may or may not change during the process of social inculturation. This could be seen as a connecting issue relating to identity.

Tracy: Our different perspectives seem to converge around the social, cultural, and historical nature of teacher decision making. Where they diverge is perhaps the extent to which decision making is seen as a property of the individual, from individual belief systems, to the enactment of multiple identities influenced by the social and cultural settings in which they are enacted, to co-emerging in the relations between the individual teachers and their changing environments.

3.1.1 Shaping teachers' future decision making

Tracy: Given our roles as mathematics teacher educators, we are responsible for shaping the learning of the mathematics teachers with whom we work. In what ways do these different theoretical perspectives help us to conceptualise how we might influence or change our own future decision making or the future decision making of others? Returning to Varela's (1999) idea that our predominant mode of operating is immediate coping, then for us to respond to a stimulus in a way that is different to our habitual way of responding requires a change to an individual's structure (since this is what determines a response). This is difficult, since our structures are established over our lifetimes and are based on the history of all of our interactions. Change is hard when most of our responses happen without rational deliberation. Mason (2002) writes convincingly about the difficulty of personal change, he writes about being awake in the moment of decision making to have available a range of new and different responses given the situation at hand. According to both Mason (2002) and Varela (1999), this kind of change requires "some form of sustained, disciplined practice" (Varela, 1999, p. 75).

Andreas: According to identity research, how one responds might differ in different situations. What I strive to understand is professional development from the perspectives of prospective and in-service teachers of mathematics. I agree with Battey and Franke's (2008) conclusion that "there remains a large and often undocumented variability in how teachers make use of ideas learned" (p. 127), that is, how professional development can inform their decision-making. One way of describing this variability is through identity studies, so that is why I ended up there. We, as a community, already know a great deal about identity and in many different ways. However,

we often fail to integrate research on identity development into professional development and teacher education contexts. Here the notion of *identity work* is particularly valuable, although there is not yet enough research on how identity work could be operationalised in terms of preparing teachers for sustainable and long-lasting carriers. Tracy and I began contributing to this discussion by developing a methodology for teacher educators' *identity work* (Helliwell & Ebbelind, 2024).

Ralf: After more than a decade of experience in classrooms and in-service teacher education, my PhD project on teachers' beliefs allowed me to change my perspective from being a teacher to being a researcher. I asked myself; do teachers stick to educational (curriculum) trends or other teaching objectives? The role attributed to teachers' beliefs as part of mathematics-related affect has been regarded as a significant, explanatory determinant in the transformation process from the given curriculum to different practices in the classroom (Fives & Buehl, 2011; Pajares, 1992; Erens & Eichler, 2019). As mathematics education research regarding beliefs has recognised a potential in prevailing approaches in which teachers position themselves as experts of their teaching and in findings about their professional understanding in the classroom, it seems expedient to regard beliefs as a key concern in order "to understand teaching from the teachers' perspectives" and "to understand the beliefs with which they define their work" (Nespor, 1987). As central and core beliefs are commonly said to be rather stable, teacher identity is often seen as a dynamic construct, i.e., changing over time and, with a general consensus on its contextuality (Liljedahl et al., 2012). The variability mentioned by Andreas seems an interesting perspective from the perspective of a beliefs researcher. It is often claimed that knowledge and beliefs (about teaching & about learners) are major determinants of what happens in classrooms. As an explanatory principle, what links are there between teachers' beliefs, identity and the role attributed to these constructs in relation to praxeology in schools?

Andreas: It seems to me that whether we consider it to be our beliefs, identities, or structures that determine our actions and decision making as teachers and teacher educators (regardless of whether or not they are fixed or fluid, conscious or habitual), we agree that change in practice is challenging and we do not yet understand fully how teachers realise ideas from professional development contexts in their own teaching.

3.2 Methodologies for researching teacher decision making

Andreas: I consistently ask myself whether the techniques employed by a researcher correspond with the phenomenon under investigation and how they harmonise with

the theoretical framework. When we report on our research, it is essential that we transparently demonstrate how results emerge from data through methodological choices and when viewed through the lens of the theoretical framework.

Tracy: My reading based on a limited amount of beliefs research is that a belief system, which Ralf mentioned consists of a set of basic components, can be *accessed* by researchers (and thus conceptualised) using sophisticated questionnaire tools. Presumably, the conceptualisation of the relationship between these beliefs and what teachers and teacher educators then do in their teaching (i.e., their decision making) impacts on how this relationship can be researched. What does an enactivist perspective mean in terms of how we might be able to research teacher decision making? The enactivist non-separability of knowing and doing means we can *observe* decision making in our interactions with the environment and with others. It is unlikely that a methodology informed by enactivism would aim to uncover something behind these actions but enactivist researchers do sometimes employ narrative methods as a way of re-entering moments of teaching, to "reconstruct the intelligent awareness that justifies the action" (Varela, 1999, p. 31). It is the relationship between actions and rational deliberation on those actions which enactivists are more concerned with in relation to teacher and teacher educator learning.

Ralf: A crucial question in research on beliefs and their enactment is what guides teachers in their decisions concerning their beliefs and identity? Some research has tried to explain potential cognitive conflicts and tensions in decision-making with situated conceptual change or sociocultural factors (Liljedahl et al., 2012). Getting to the core of decision-making, one might consider pursuing multiple goals with varying degrees of commitment. In terms of beliefs, goals, and identity, a psychological perspective can help to explain such processes with a model of action phases (see "Rubicon Model" by Heckhausen and Gollwitzer (cf. Achtziger & Gollwitzer, 2018)), which addresses questions relating to the selection of goals, the planning of execution of goals, and the way goals are accomplished in these different action phases. Andreas said that identity can be conceptualised as being multiple and situated. A discursive lens may help to find a methodological basis for both constructs (beliefs and identity) as subjective truths can be uncovered in, e.g. semi-structured interviews. The complexity and contradictions between subjective beliefs and self-report narratives are sometimes counted as arguments for non-accountability. However, such a discourse may serve to uncover the complexity and situatedness of both beliefs and identities.

Andreas: As I mentioned earlier, there remains a large and often undocumented variability in how teachers make use of ideas learned. For me, our methodological choices need to capture this variability from particular aspects. For me, one level of analysis is about reconstructing patterns in a teacher's involvement in individual classroom events. That is, for teachers to access the intelligent awareness that led to action, in Tracy's terms, and to articulate how they recognise the situation retrospectively. Simultaneously, at another level, I engage in longitudinal studies to identify overarching trends and developments in how teachers interact with students and content (Skott, 2022), i.e., how they teach and how this might change over time.

Tracy: One methodological issue that we have not yet considered here relates to our choice of presentation. It remains relatively unquestioned in mathematics education that the predominant mode of communicating our research is through written text, mostly formal-analytical in form, yet how does this well-established practice align with our different theoretical perspectives? Andreas and I have recently written about the role of the research text (see Ebbelind & Helliwell, 2023). For me, our discussion is linked to the bigger question of the purpose of research. Perhaps, if we would like teachers and other teacher educators to use the research texts that we produce as a source for ongoing personal growth and development (that may even lead to changes in decision making), then we might think about producing more research texts that are moving, that evoke empathy and a critical examination of one's own practices. How can our research texts do this? What literary techniques can be employed that remain consistent with our underlying perspectives? I think our mode of presentation is an often-overlooked aspect of methodology. Who has not been moved, emotionally, by a novel, autobiography or even film?

Ralf: To sum up, our different theoretical perspectives determine how it is possible for us to research teacher decision making (i.e., our methods and tools), as well as how we communicate our findings. How we conceptualise the relationship between what we believe/know and what we do justifies where, when and how we look at social phenomena and raises the question of whether we can say we are even talking about the same things. What is possible to *access* or *observe* or *collect* and the distance between that data and the phenomena itself varies greatly across our different perspectives, even though we may use the same words. It appears, for example, that self-reports/narratives are valid forms of data from each of our perspectives, yet their ontological and epistemic status are quite different.

Conclusions

Tracy: To grow as teachers, teacher educators and researchers, it is essential to engage with the perspectives of others, otherwise, as mathematic education researchers, we have the potential to constrain ourselves. As Krammer and Mangiardi (2016) claim, our "dialogues about these experiences—and specifically the dialectical interplay between and among them—promote researcher change and reflexivity" (p. 41). The process of duoethnography is itself a process of growth and development, a way to engage in and with the perspectives of others. Sawyer and Norris (2015) suggest that the process of duoethnography can help us to examine "the formation of our beliefs, values and ways of knowing" (p. 1), and for that reason we might look to support mathematics teachers in engaging in a similar process so that they themselves can question their own assumptions. Some differences in perspectives can be linguistically subtle (a noun instead of a verb, a different choice of metaphor, an emphasis in a different place). Nonetheless, these subtle distinctions often imply the basis of profoundly different assumptions. To hear these subtleties requires an act of deep listening to one another, understanding the nuances and more profound divergences, and having conversations focused on the boundaries and overlaps of one another's perspectives. If we can do this, then it will only enrich what is possible to learn from one another's research, at places like MAVI conferences.

Ralf: I agree. As researchers, we have our particular stories and projects, and we have (or have not) sufficient theoretical backbone to support our approaches. In reconstructing insights about beliefs and identity, we can zoom in (or out) to different levels of action and interaction in learning and teaching mathematics. The field of affective aspects is much wider here beyond identity and beliefs. The findings that researchers in mathematics education have mainly agreed on concerning the development of beliefs and identity are that core or central beliefs seem to be a stable construct, whereas identity is considered a dynamic and evolving phenomenon (Skott, 2022). However, both affective constructs are influenced by a range of social and external factors. This brings up the question of whether both – beliefs and identity – can be challenged by educational trends and social desirability phenomena.

Andreas: It seems that we are at different points on a continuum, from observing actions to describing a set of internalised reifications linked to actions. Tracy positions herself at one end, Ralf on the other, and maybe I find myself somewhere in the middle. Yet human lived experience seems to be a common ground in relation to teacher and teacher educator decision-making, whether we choose to describe actions, the

linguistic means attached to these actions, or the variables that guide these actions. This is why it is so important to appreciate one another's perspectives, before we can engage fully in a discussion on affect within mathematics education.

Research ethics

Author contributions

A.E.: conceptualization, investigation, methodology, writing—original draft preparation, writing—review and editing
R.E.: conceptualization, investigation, methodology, writing—original draft preparation, writing—review and editing
T.H.: conceptualization, investigation, methodology, writing—original draft preparation, writing—review and editing

All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

References

Achtziger, A., & Gollwitzer, P. M. (2018). Motivation and volition in the course of action. *Motivation and Action*, 485–527. https://doi.org/10.1007/978-3-319-65094-4_12

Apostolidou, A., & Daskalaki, I. (2021). Digitally performing duoethnography: Collaborative practices and the production of anthropological knowledge. *Entanglements, 4*(1), 51–69.

Battey, D., & Franke, M. L. (2008). Transforming identities: Understanding teachers across professional development and classroom practice. *Teacher Education Quarterly, 35*(3), 127–149. http://www.jstor.org/stable/23478985

Blumer, H. (1969). *Symbolic interactionism: Perspective and method*. University of California Press.

Claxton, G. (2000). The anatomy of intuition. In T. Atkinson & G. Claxton (Eds.), *The intuitive practitioner: On the value of not always knowing what one is doing* (pp. 32–52). Open University Press.

Ebbelind, A., & Helliwell, T. (2023). The role of dissonance in mathematics teacher education. In P. Drijvers, C. Csapodi, H. Palmér, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)* (pp. 3219–3226). Alfréd Rényi Institute of Mathematics and ERME.

Eisenhart, M. (1991). Conceptual frameworks for research. Ideas from a cultural anthropologist; implications for mathematics education researchers. In R. Underhill (Ed.), *Proceedings of the 13th Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. (pp. 202–219). Virginia, USA.

Ejersbo, L. R., & Leron, U. (2005). The didactical transposition of didactical ideas: The case of the virtual monologue. *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME4)*. http://www.erme.tu-dortmund.de/~erme/CERME4/CERME4_WG11.pdf#page=143

Erens, R., & Eichler, A. (2019). Belief changes in the transition from university studies to school practice. In M. Hannula, G. C. Leder, F. Morselli, M. Vollstedt & Q. Zhang (Eds.), *Affect and Mathematics Education Fresh Perspectives on Motivation, Engagement, and Identity* (pp. 345–373). Springer.

Fives, H., & Buehl, M. (2011). Spring cleaning for the "messy" construct of teachers' beliefs: What are they? Which have been examined? What can they tell us? In S. Harris, T. Urdan, S. Graham, J. Royer & M. Zeidner (Eds.), *APA educational psychology handbook* (Vol. 2, pp. 471–499). American Psychological Association.

Gee, P. (2011). *An introduction to discourse analysis: Theory and method* (3rd ed.). Routledge.

Giddens, A. (1984). *The constitution of society: Outline of the theory of structuration*. Polity Press.

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Edward Arnold.

Helliwell, T., & Ebbelind, A. (2024). Conceptualising (mathematics teacher educator) identity work: an enactivist-informed approach. *NOMAD Nordic Studies in Mathematics Education, 29*(3-4), 101–123. https://doi.org/10.7146/nomad.v29i3-4.150755

Holland, D., & Eisenhart, M. (1990). *Educated in romance: Woman, achievement, and college culture*. University of Chicago Press

Holland, D., Skinner, D., Lachicotte, W., & Cain, C. (1998). *Identity and agency in cultural worlds*. Harvard University Press.

Krammer, D., & Mangiardi, R. (2016). The hidden curriculum of schooling: A duoethnographic exploration of what schools teach us about schooling. In J. Norris, R. Sawyer & D. Lund (Eds.), *Duoethnography* (pp. 41–70). Routledge.

Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations of research in mathematics education. *ZDM – The International Journal on Mathematics Education, 37*, 457–467. https://doi.org/10.1007/BF02655854

Liljedahl, P., Oesterle, S., & Bernèche, C. (2012). Stability of beliefs in mathematics education: a critical analysis. *NOMAD Nordic Studies in Mathematics Education, 17*(3-4), 23–40. https://doi.org/10.7146/nomad.v17i3-4.148472

Lloyd, G. M., de Carle, A., & Coon-Kitt, M. J. (2021). When you're with me, I'm learning: A duoethnography of teacher educators' identities in relation to observing preservice teachers' emergent mathematics instruction. *International Journal of Science and Mathematics Education, 19*, 77–98. https://doi.org/10.1007/s10763-021-10162-5

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge. https://doi.org/10.4324/9780203471876

Maturana, H., & Varela, F. (1998). *The tree of knowledge: The biological roots of human understanding* (original work published 1987). Shambhala.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of curriculum studies, 19*(4), 317–328. https://doi.org/10.1080/0022027870190403

Oliveira, H., & Hannula, M. S. (2008). Individual prospective mathematics teachers. In K. Krainer & T. Wood (Eds.), *The International Handbook of Mathematics Teachers Education* (vol. 3, pp. 13–34). Brill.

Pajares F. M. (1992). Teachers' beliefs and educational research: cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332. https://doi.org/10.3102/00346543062003307

Proulx, J., & Simmt, E. (2016). Distinguishing enactivism from constructivism: engaging with new possibilities. In C. Csíkos, A. Rausch & J. Szitányi, (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (vol. 4, pp. 99–106). https://www.igpme.org/publications/current-proceedings/

Skott, J. (2015). The promises, problems, and prospects of research on teachers' beliefs. In H. Fives & M. Hill (Eds.), *International handbook of research on teachers' belief* (pp. 13–30). Taylor and Francis Routledge.

Skott, J. (2022) Conceptualizing individual-context relationships in teaching: Developments in research on teachers' knowledge, beliefs and identity. *Twelfth Congress of the European Society for Research in Mathematics Education* (CERME12), Feb 2022, Bozen-Bolzano, Italy. ⟨hal-03808889⟩

Törner, G., Potari, D., & Zachariades, T. (2014). Calculus in european classrooms: curriculum and teaching in different educational and cultural contexts. *ZDM - International Journal on Mathematics Education, 46*(4), 549–560. https://doi.org/10.1007/s11858-014-0612-0

Varela, F. (1999). *Ethical know-how: Action, wisdom and cognition*. Stanford University Press.

Wetherell, M. (2001). Themes in discourse research: The case of Diana. In M. Wetherell, S. Taylor, & S. J. Yates (Eds.), *Discourse theory and practice: A reader* (pp. 14–28). Sage.

Winter, R. (1988) Fictional-critical writing: An approach to case study research by practitioners and for in-service and pre-service work with teachers. In J. Nias & S. Groundwater-Smith (Eds.), *The enquiring teacher: Supporting and sustaining teacher research* (pp. 231–248). Routledge. https://doi.org/10.4324/9780203058855-23

Zazkis, R., & Koichu, B. (2015). A fictional dialogue on infinitude of primes: introducing virtual duoethnography. *Educational Studies in mathematics, 88(*2), 163–181. https://doi.org/10.1007/s10649-014-9580-0