Factfulness: theoretical and empirical considerations within affect-related research in mathematics education

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Abstract: The study aims to investigate the role of mathematical beliefs in the context of environmental and social issues, when people have to rate the incidence of certain phenomena. The framework of Factfulness is intertwined with mathematics-related beliefs and the emergence of conceptions is investigated. In this research, we asked a group of students in Environmental Sciences and a group of researchers in mathematics education a series of multiple-choice questions retrieved from the Factfulness website. The results reveal that mathematics- and/or environmental-knowledgeable people tend to be less pessimistic than the average population with respect to social and environmental facts such as pollution, cost of solar energy, people perceiving climate change as a threat, and the like.

Keywords: beliefs; critical thinking; mathematics and the environment

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1 Introduction

The aim of this paper is to shed light on the relationship between affect-related aspects in the teaching and learning processes in mathematics, such as beliefs (in mathematics and beyond) and out-of-school practices and actions, in an attempt to address Coles' (2023) plea for a school mathematics that is more connected to socio-ecological issues. The motivation for this research has been given by a recently published book: in April 2018, Hans Rosling, a Swedish doctor and statistician, with his colleagues wrote the book *Factfulness*, an essay of statistical interpretation of reality that aims to bring out the huge amount of conceptions most individuals hold when prompted to interpret data and news of the world in which we live. The book opens with a series of 13 close questions with 3 possible choices, asking about the evolution of various aspects of reality over the last years, including poverty, demography, environmental and social challenges. The reader is prompted to answer mathematical facts related to the aforementioned phenomena, such as the rate of girls in the world that have received elementary education, or the percentage of people vaccinated in their first year of life. Specifically, being multiple-choice questions, the author of the book invites the reader to take all the necessary time to think about finding the alternative response he/she believes to be the correct one. The results are shocking: most people (with error rates of up to 98% in certain questions) got most of the answers wrong, regardless of their social context and school level. Even journalists and documentary filmmakers specialised in the topics covered, which are presumed to be competent as disseminators, have reached rates of error higher than the rate of 33%, namely the one corresponding to perfect random choice.

Pessimism, extreme simplification and bipolar vision of reality, these are some of the aspects that Rosling and his colleagues identify as justifications for this large number of wrong answers, as well as a narrative of reality, implemented by newspapers and mass media, that draws individuals' attention to a pessimistic vision of the world: that is, what the author calls "dramatic instinct". Reading the book with an expertise in mathematics education and in its affect-related aspects, we see a possible link between Rosling's et al. (2018) considerations and the theories developed within our field of research, thus the goal of this paper is to initiate a dialogue between these two realms, claiming that Factfulness can become a powerful tool to understand students' and teachers' conceptions of mathematical modelling related to climate change, poverty, pollution, over-exploitation of resources, and so on. Conceptions, in fact, are seen as clusters of beliefs and we recall that, according to Furinghetti and Pehkonen (2002), beliefs are the conclusions that an individual draws from their perceptions and experiences in the world around them. Beliefs can be understood as subjective knowledge: they are propositions about a certain topic that are regarded as true (Philipp, 2007). Being continuously subject to new experiences, beliefs can change and new beliefs can be adopted (Furinghetti & Pehkonen, 2002). When a new belief emerges, it never comes in isolation from other beliefs, but becomes part of, what has been called, an individual's belief system. According to Green (1971), in fact, beliefs tend to form clusters, as they "come always in sets or groups, never in complete independence of one another" (Green, 1971, p. 41). These clusters form a system, which is organised according to the quasi-logical relations between the beliefs and the psychological strengths with which each belief is held (Green, 1971). Belief clusters are, thus, almost coherent families of beliefs across multiple contexts: for example, beliefs about the nature of mathematics and about its learning tend to cluster in a quite coherent way, for a student. This has probably led Furinghetti and Pehkonen (2002) to conclude that "an individual's conception of mathematics [is] a set of certain beliefs" (p. 41), namely to understand conceptions as clusters of beliefs. For this reason, in this paper we decided to use the construct "conceptions", which is meant as an umbrella concept, namely: "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p.259). Hence, conceptions may have both affective and cognitive dimensions and serve the purpose of capturing students' ideas and dispositions (Philipp, 2007).

The research questions we aim to answer are two. One has a theoretical nature and is: how is Factfulness related to conceptions, as they have been understood by researchers in mathematics education, especially in the field of affect? In order to answer it, we dwell on Factfulness and its principles in the next section. The second research question is empirical and is: how does Factfulness-related beliefs emerge in particular groups of people, who have mathematical and/or environmental knowledge?

2 Factfulness: theoretical considerations

In their essay, Rosling et al. (2018) identify ten different conceptions presumably responsible for the pessimistic vision of reality, of which six may concern maths education. According to Burton (1999), conceptions lie at the boundary between thinking and feeling and they should be understood in a subjective way. With this in mind, we focus on the six conceptions, pinpointed by Rosling et al. (2018), which relate to mathematics.

The *gap instinct* is the tendency to see the world divided into only two main categories. The example that the author brings for this conception concerns the level of wealth of the population in the various states of the world, which is commonly divided into rich and poor. The core of this conception is the loss of the majority, namely the large portion of the population that has a level of wealth between the two extremes, poor and rich. To overcome this, according to the author, it is necessary to overcome the tendency to compare exclusively the averages of two different populations, but also to take into account the median and especially the frequency distribution, identifying the majority in this way. Within mathematics education, the students' understanding of mean, median, and mode was the focus of an extensive review made by Garfield and Ben-Zvi (2007), who show a lack of conceptual understanding beyond the algorithm. For instance, many students know the algorithm for the mean, but do not know, nor pay attention to, the mathematical properties of the measure. Thus, students may develop conceptions about these measures (Bond et al. 2012). We underline that the main point is not about conceptions being correct or wrong, but about the complex role that beliefs and conceptions play in relation to knowledge. For example, partial knowledge may lead to holding beliefs of this sort, and these beliefs can cluster in conceptions.

The majority can be misleading, and so are the categories that are used in an explanation. Beware the *generalisation instinct*, which is responsible for the loss of exceptions. To describe the world in which we live it is necessary to remember that reality is distributed on a continuum basis and that the categories in which it is divided could be very arbitrary and fragile; Factfulness devotes particular attention to the comparison between groups, especially for what concerns the differences within the same category (a more divided vision could better describe a distribution), but also similarities between different groups. Questioning the arbitrary division of a population is a good way to highlight any features that would otherwise be hidden; what is true in one group may not be true for another. Again, we are not talking about being correct or wrong, but we take this as a description of how beliefs can deeply influence the way people see and interpret certain facts.

The third instinct deals with pure numbers (absolute data) and invites the readers to compare them with some other relevant number; this is the *size instinct*. Dealing with proportions and knowing how to apply them to the real world is a decisive skill to make big numbers more graspable; according to Rosling et al. (2018), for a correct understanding of the data, it is important to differentiate between rates and amounts. It seems that mathematical beliefs, developed in school years, push individuals to focus more on absolute numbers and to pay less attention to rates.

Not directly related to maths education, but crucial to collecting right information before the analysis, two instincts are also identified regarding acquired data: the *negativity* one and the single perspective one. The first has to do with our tendency to recognize and remember bad information more, which represents the major kind of news coming from the world (when things are getting better we often don’t hear about them). This gives us a systematically too-negative impression of the world around us, which is very stressful. Mitigating the effects of this conception seems to be the most difficult challenge for *Factfulness*, because it concerns, in most cases, trends that are not of public interest. First of all it is necessary to distinguish between levels, which can be good or bad, and direction of change, better or worse, and that a situation can be both better and worse at the same time. Then, for any bad news spread by mass media, there may be other good news or slow improvements not yet recorded; above all, it is important to remember that more negative news does not always mean an increase in suffering, but could also be due to better surveillance of suffering.

Data are not always available and do not always exhaustively describe reality, for this reason Rosling et al. (2018) identifies the *single perspective* conception, inviting readers to question their beliefs and thus acquire a scientific mind, free from external conditioning; beware from ideas and simple solutions, it is better to confront with those who do not think like us.

Finally, not for importance, but deeply related to maths education, there is the *straight line* conception. This instinct has to do with the tendency to assume that trends (schooling, vaccinations, etc.) do not always follow straight lines but curves of a more complex nature. Linear relationships within certain ranges could not be in others or if the initial conditions changed; even the height varies linearly with age, but this does not mean we expect children to continue to grow endlessly.

Rosling et al. (2018)'s elaboration is deeply linked to the over-simplification that, both at school and in the field of information, builds what we define as a conception. Factfulness in short is a way of thinking that aims to build a mentality based on facts; this would allow students to put news from around the planet into context and understand how easy it is to leverage dramatic instincts with hyperdramatic stories; these skills fit into critical thinking.

3 Towards empirical considerations: method

In order to see how Factfulness can be connected to research in affect-related issues in Mathematics Education, a pilot explorative study has been conducted through a questionnaire that consists of 7 short *Factfulness*-style questions taken from the Gapminder website (see Figure 1). The questions are intended as a tool for thinking about how the world has changed over a period of time.

The questions have been administered at the beginning of a lecture given to two classes of 42 (14 and 28, respectively) university students enrolled in Environmental Science undergraduate courses at the University of Eastern Piedmont. The participants were gender balanced and came from different educational backgrounds, resulting in a good representation of the sample, both in terms of gender and mathematical skills. This specific sample of students was chosen because it is thought that they may be more sensitive and informed than other samples that can be studied.

During the lecture, just before a slide presentation, the aforementioned questionnaire was given to the students.

Moreover, for sake of strict time constraints, a shorter version of the questionnaire was given to 32 people attending the Research Forum “How socio-ecological issues are urging changes in curriculum (and beyond)” at PME46 (see Amico *et al.*, 2023) presented in 2023 at the annual PME 46 International Conference held in Haifa. This sample is studied because they participate in this specific Research Forum and are assumed to be mathematicians and therefore experts in numbers. In addition to this, this Research Forum deals with topics closely related to the environmental field, so it is assumed that they are also sensitive to environmental and climate issues. This specific questionnaire contains only the three underlined questions in Figure 1.

Figure 1. The seven questions given to the students and the 3 questions given at PME46 (underline

1. Compared to 2016, how has the area occupied by marine protected areas around the world evolved?
a) Increased by 75%*
b) Stayed about the same
c) Decreased by 75%

2. <u>Compared to 1970, what has happened to the number of oil tanker accidents?</u>
a) Decreased by a factor of ten*
b) Stayed about the same
c) Increased by a factor of ten

3. <u>Considering the total mass of all mammals currently living on planet earth, what percentage represents those free in the wild?</u>
a) 5%*
b) 25%
c) 50%

4. <u>In the last 30 years, the area occupied by forests is:</u>
a) Decreased by 10%*
b) Decreased by 30%
c) Decreased by 50%

5. What percentage of the world's population sees climate change as a threat to their country in the next 20 years?
a) Around 65%*
b) Around 45%
c) Around 25%

6. Compared to 1980, the cost of energy produced by a solar panel is:
a) 1% compared to 1980*
b) 21% compared to 1980
c) 41% compared to 1980

7. How many of the 198 states present have signed the 2015 Paris climate and environmental agreements?
a) 191*
b) 141
c) 91

For each question, as can be read in Figure 1, there are three possible answers: an extremely pessimistic one, a moderately pessimistic one and a positive one, with the latter being the correct one (it is marked with an asterisk in the above list).

The analysis was conducted by calculating the frequency of answers for each question, then compared with Rosling’s et al. (2018) study.

4 Data analysis and results

During the analysis, it emerged that most of the answers given by the respondents tended to be wrong, which may highlight the strong presence of misconceptions about the world around us. It can be seen that the percentage of correct answers varies between a minimum of 2.7% and a maximum of 39.2%, with an average answer of 20.8%. With reference to the most neutral answer, but still wrong, the percentage of answers varies between 6.8% and 66.2%, with an average answer of 44.4%. If we look at the third answer choice, namely the very wrong answer choice, the response rate varies between 18.9% and 66.2%, with an average response rate of 34.8%. Table 1 shows the responses to the three questions given to both groups of respondents and the results from Rosling's et al. (2018) survey.

Table 1. Table 1: Frequencies of answers to three selected questions.

Question number	**Right answer**		**Wrong answer (neutral answer)**		**Very wrong answer**	
	Responses frequency (our study)	**Responses frequency (Rosling et al., 2018)**	**Responses frequency (our study)**	**Responses frequency (Rosling et al., 2018)**	**Responses frequency (our study)**	**Responses frequency (Rosling et al., 2018)**
Q 2	20/74 (~27%)	37%	2/74 (~3%)	18%	8/74 (~11%)	7%
Q 4	46/74 (~62%)	18%	37/74 (~50%)	27%	49/74 (~66%)	27%
Q 6	8/74 (~11%)	45%	35/74 (~47%)	55%	17/74 (~23%)	66%

Considering the rate of correct answers to questions on the Gapminder website (the analysis was conducted on UK citizens of all walks of life or backgrounds) there are similarities to our results. In the second question (Q2) the rate of correct answers is 27%, not far from the 37% found in the UK; this applies much less to Q4, where the results are even worse with a 3% compared to 18%, but percentages are closer for Q6, where the 11% we detected is close to the 7% on the website. Although the participants in the study were either students enrolled in a degree program where the environmental and ecological component is central or professors competent with the numbers and recognized in their role, the rate of correct answers is low. If, from the point of view of the correct answers, the situation is dramatic, it is not so if we consider the level of pessimism. As mentioned in the methodology, Gapminder divides the three possible answers to the questionnaire as right, wrong and very wrong. In Rosling's et al. study, Q2 recorded a rate of very wrong answers equal to 45% against 11% in our study, Q4 recorded a rate of 55% against 47% of our survey, and Q7 a rate of very wrong answers equal to 66% (23% from our sample). The majority of respondents in our study selected the mildly wrong answer.

5 Discussion and conclusions

In the present research, we used a sample that can raise concerns. Why not students in mathematics? Why not mathematics teachers? These are reasonable questions that can surface in the mind of the reader and the answer is twofold: on one side, it was a convenience sample, and on the other and more important side, we believe that investigating the beliefs of environmental students can be more representative of how students perceive facts. We also believe that investigating the beliefs of researchers in mathematics education can shed some light on how mathematics teacher education and professional development programs are shaped.

The empirical research question of our study concerned the emergence of conceptions in mathematical facts about the socio-ecological (Coles, 2023). As Rosling et al. (2018) has already pointed out in the first pages of their essay, the pessimistic conception of the world strikes all people. Our analysis showed that neither the undergraduate students in environmental sciences, who are supposed to be prepared or at least aware of the evolution that the human being has had in addressing ecological problems, nor the experienced professors in mathematics, who certainly have developed number-related skills, are exempted from the hyperdramatic vision of the world. It is evident that what Rosling et al. (2018) points out is a conception that people have of the rest of the world. One observation that, however, partly contradicts Rosling et al. (2018) is the level of pessimism that emerged in questions Q2 and Q6: if most people (according to the results of the UK survey of the Gapminder site) have a tendency to prefer the *very wrong* answers (and from which emerges a vision not only dramatic but even hyperdramatic), in the case of our sample it turns out that most of the answers reflect a more moderate vision closer to reality.

A conclusion that can be drawn is that people have pessimistic conceptions but mathematical knowledge might mitigate them. What can be the role of beliefs? Beliefs form a system, they originate from experience and influence the way people come to see and interpret data. It seems, from our study, that knowledge about environment and/or mathematics can be even hindered by beliefs, especially those identified by Rosling et al. (2018). We can also conclude that beliefs help us to describe how people use and apply their knowledge, rather than distinguishing between wrong and right beliefs.

Research ethics

Author contributions

A.A.: conceptualization, theoretical framework, writing—original draft preparation;
C.A.: literature review, writing—review and editing;
L.D.: data collection, data analysis, writing—original draft preparation.

All authors have read and agreed to the published version of the manuscript.

Artificial intelligence

No artificial intelligence has been used in the research or writing of the article.

Funding

No funding.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

Data is unavailable due to privacy restrictions.

Conflicts of interest

The authors declare no conflicts of interest.

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