Teachers' beliefs on scientific models and the role of mathematisation

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Abstract: The definition and application of models and modelling differ significantly in the different STEM subjects. This may lead to learning-related challenges for the students, especially in upper secondary school. One perspective that might shed a light on the differences and similarities between the subjects' models and modelling processes is the role of mathematics in them as well as the nature of a possible mathematisation. In an educational context, the teachers' perception of this matter is central. In this paper, we investigate how two upper secondary school science teachers characterise models in their respective subjects, biology and chemistry. By comparing the teachers' statements from an interview partly to existing literature and partly to statements from an expert informant, we analyse their beliefs about the role of mathematics and mathematisation within these models, leading to a discussion of not only the differences in the concept of model, but also of how teachers' beliefs may influence the students' understanding of models and modelling in the two scientific subjects.

Keywords: teachers' beliefs, modelling, science models, mathematisation, upper secondary school

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1 Introduction

Models and modelling are key elements of mathematics and science, not least from an educational perspective. Incorporating modelling in teaching may lead to more meaningful learning for students (Gilbert & Treagust, 2009), contribute to students' reflection on their learning process (Louca & Zacharia, 2012), as well as offer them opportunities to participate actively in scientific investigations of natural phenomena (Taber & Akpan, 2017). Nevertheless, there might be a vast difference between the perception of these concepts in the different STEM subjects. Where mathematical modelling entails a process in which a phenomenon from the real world is mathematised, for example, through a differential equation, models in biology and chemistry are often conceptual, describing complex, qualitative phenomena. These differences may be the cause of learning-related challenges, leading to some degree of confusion for the students, particularly, perhaps, in upper secondary school, where models concerning, for example, economy or communication are also introduced. Such challenges related to science teaching and students' comprehension are the focus of a larger design-based research study concerning the variation in the concepts of models and modelling in STEM education.

A possible way of investigating differences in the concepts of models and modelling in STEM subjects may be to explore the role of mathematics and mathematisation within the models. Part of this issue may be related to teachers and their perception of models and modelling in their subjects. Studies show that teachers' knowledge of and views on models and modelling in science may be limited and sometimes inconsistent (van Driel & Verloop, 2002).

In this paper, we specifically investigate two science teachers' statements concerning mathematics and mathematisation in the models of their respective subjects: biology and chemistry. Based on an interview with the teachers, we qualitatively analysed their beliefs on this matter. We compare these partly to existing literature and partly to statements from an expert informant. We thereby form a ground for discussion of not only the differences in the concept of model, but also of how teachers' beliefs may influence the students' understanding of models and modelling in the two scientific subjects.

More specifically, our research question is: *What characterises upper secondary school science teachers' beliefs about the concept of model as well as the role of mathematics and mathematisation within the models of in their respective subjects?*

Given that we only focus on two teachers, the generalisability of the findings may of course be limited. However, their perspectives on the concept of model may serve as indicators of implicit issues and challenges regarding the teaching of scientific models as well as potential guidelines for further research.

2 Models and modelling

The concept of model is not necessarily consensual nor unambiguous across scientific disciplines. Furthermore, the term *model* may cover a diversity of scientific representations, ranging from 3D plastic replication of the human anatomy over a drawing of a nitrogen cycle to an equation illustrating the chemical process of densification. In general, a model can be defined as "a simplified representation of a phenomenon (an object, system, event, process)" (Justi et al., 2009, p. 286). To distinguish between models in science, different categories based on how they are expressed or what they represent have been made in literature (e.g., Bryce et al., 2016). For instance, models in chemistry, specifically, can represent different levels of knowledge, such as the macroscopic level, the sub-microscopic level, and the representational or symbolic level (Johnstone, 1982). For biology, this categorisation has been elaborated on with the addition of the cellular/subcellular level (Treagust & Tsui, 2013).

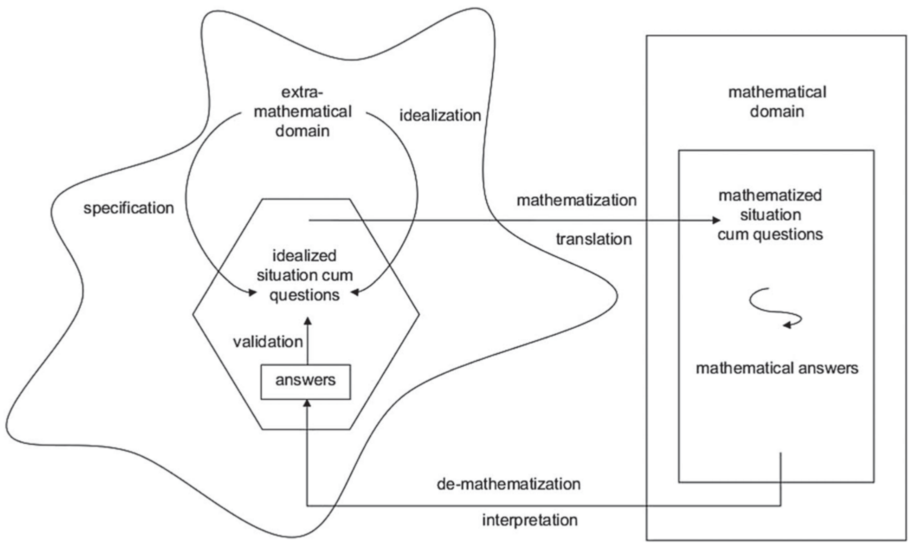
Also, the purposes of models play an essential role in how they are made and applied (Gilbert & Justi, 2016). In a communicative and educational context, models can stimulate learning and conceptualization, simplify complex objects and concepts, and explain scientific phenomena (Coll & Lajium, 2011). This allows for the investigation of natural phenomena that are typically difficult to access. Gilbert (2004) sees models as a "bridge between scientific theory and the world-as-experienced" (p. 116):

> They can be simplified depictions of a reality-as-observed, produced for specific purposes, to which the abstractions of theory are then applied. They can also be idealisations of a possible reality, based on the abstractions of theory, produced so that comparisons with reality-as-observed can be made. (Gilbert, 2004. p. 116)

Hence, the purpose of a model may be either depiction or idealisation. Gouvea and Passmore (2017) distinguish between models *of* and models *for*: "Models are not simply knowledge representations *of* the world, they are epistemic tools *for* making sense of the world" (p. 56). Several researchers thus distinguish between an *explanatory/descriptive* and a *predictive* purpose of a model (e.g., Baek et al., 2011; Gilbert, 2004; Justi et al., 2009). Where the former generally focuses on expressing what is known about a certain phenomenon and reproducing established knowledge, the latter encompasses dynamic tools that may generate new knowledge (Gouvea & Passmore, 2017). Considering models as epistemic tools for generating knowledge, emphasises the interaction between user and models where models may become independent from the real-world phenomenon (Knuuttila & Boon, 2011).

One type of model across scientific subjects is the mathematical model (e.g., Bryce, 2016), which involves *mathematisation*. Mathematisation is a process of mathematical modelling (Figure 1), where an idealised and specified version of a problem from the real world is translated into mathematics (Niss, 2010).

Figure 1. Mathematical modelling and its processes



Jankvist & Niss, 2020, p. 469. Originally in Niss, 2010 (with permission from the author).

The cyclical representation of modelling may not capture all important aspects of the modelling process such as the roles modelling and models plays in sociopolitical problems and in relation to digital transformations (Doerr et al., 2017). As an example of such phenomenon, Skovsmose (2021) analyses the roles that mathematical models play in relation to crises. Models can of course *depict* and characterize a crisis, but models can also *format* a crisis in the sense that the consequences of the crises are only apparent through the models themselves. Furthermore, models can even create or *constitute* a crisis. That can happen in cases where mathematical-based algorithms "provide an underlying dynamic which brings about the crisis [...] it becomes an essential ingredient in a crisis" (Skovsmose, 2021, p. 370, using the financial crash in 2008, and its relation to trading complex financial instruments based on mathematical models, as an example). Thus, mathematical models can also become independent agentic artefacts that may influence real-world phenomena. This dynamic is hard to capture in a cyclic interplay between question, mathematisation, calculation and interpretation cycle as depicted in figure 1.

Where mathematical modelling always entails mathematisation, this is not necessarily the case in a scientific modelling process, and thus not all scientific models include mathematics. In this study, we investigate whether the role of mathematisation in a scientific model can contribute to an understanding of the differences and similarities of the model concept in biology and chemistry, respectively.

3 Teachers' beliefs

Ernest (1989) identifies beliefs as one of the key elements essential to the practice of teaching. A teacher's beliefs, both about the subject, the topic, and about teaching and learning, constitute a highly influential factor that affects what happens in the classroom and thus shapes the students' beliefs and learning outcomes. A teacher's unconscious beliefs that do not match the intentions behind the curriculum may even become what Furinghetti (1996) calls "ghosts in the classroom", which can cause a discrepancy between the intended and the enacted curriculum. Hence, as pointed out by Wilson and Cooney (2002, p. 128), an important reason for studying teachers' beliefs is that they are "a significant determiner of what gets taught, how it gets taught and what gets learned in the classroom".

Beliefs are formed through enculturation (experiences) as well as social construction (adaption of other people's beliefs) (e.g., McLeod, 1992; Pajares, 1992; Richardson, 1996). This applies equally to teachers and students, which, in relation to this study, means that the two teachers' perception of models and the role that mathematics plays in them may affect the students' perception of these matters, both through the activities and experiences that the teachers stage in the classroom and through a possible transfer of beliefs.

In relation to an intervention process, as the one which is initiated in the before-mentioned larger project about models in STEM, teachers' beliefs play a central role. Coburn (2003) defines four interrelated dimensions in relation to *scale* in educational reforms: depth, sustainability, spread, and shift in reform ownership, including beliefs as a crucial element of the *depth* dimension. Not only do teachers meet and interpret reforms or interventions according to their beliefs, but most implementation processes also challenge the teachers' existing beliefs and often require a change in them. In this light, we seek to gain an understanding of the two participating science teachers' beliefs about models and modelling prior to the design of the intervention.

Furthermore, the two teachers may be considered as a kind of specialists in their subjects, as they have not only an educational background within the respective scientific fields but also possess practical knowledge on and experience with the use of models and mathematisation in the teaching of biology and chemistry. Since we—as researchers in mathematics education—to some extent are exploring the hypothesis that the role of mathematisation within models in the different scientific subjects may contribute to an understanding of the differences between them, the teachers' considerations about this hypothesis may possibly qualify our research.

4 Method

Answering the research question is part of a larger project about models and modelling in STEM education, and the two teachers selected for the interview participated in the project and are involved in the design of several teaching modules. The 35-minute-long interview was performed and audio-recorded as part of the initial introductory meeting concerning the larger project and was in essence a conversation between the two teachers and the two researchers (first and second author of this paper) present.

The interview guide included the following questions relevant to the present study:

- How would you characterise the concept of model in your subject?
- What similarities do you see between (typical) models in biology and chemistry?
- How are they different?
- Which role does mathematics play in the models of the two subjects?
- Which role does mathematisation play in the modelling processes of the two subjects?

During the interview it became clear that the teachers had different views on models and mathematisation which support our hypothesis. For further investigation of this, we compare the teachers' statements to those of an "expert informant", whom we interviewed regarding the role of mathematics and mathematisation in scientific models. This informant is an experienced and well-renowned professor of mathematics education focusing on mathematical modelling, who also has a master's degree in biology. His statements may therefore be considered to rely on a high degree of knowledge compared to beliefs and thus constitute a base of somewhat certainty upon which we can analyse the teachers' beliefs through a qualitative interpretation.

Accessing beliefs is, as pointed out by several researchers, a complex matter (e.g. Rokeach, 1968; Schoenfeld, 2015). It often requires triangulation of several forms of data to capture a person's beliefs, which can be both conscious and unconscious, expressed in both statements and actions, as well as subject to social or psychological limitations. Hence, the interview data in this study may be considered a rather brief and peripheral insight into the teachers' beliefs.

5 Analysis and findings

In the following, we first present the statements of the expert informant concerning the role of mathematics in scientific models, the possible differences between models in biology and chemistry, and the nature of mathematisation within them. Second, data from the interview with the teachers are compared and analysed, based on the literature as well as the expert informant's statements.

5.1 Scientific models and mathematics according to an expert informant

Our expert informant (EI) is as mentioned an experienced researcher and university educator within mathematics education in particular, but also within the natural sciences. As part of the initial phase of the larger project concerning modelling, we interviewed him to gain an increased understanding of models and modelling in biology and chemistry. Since not only the EI, but also we as interviewers, are researchers in the field of mathematics education, we have an underlying common knowledge about and possible reference to mathematical modelling. Hence, a discussion of the role of mathematics in scientific models and modelling is not unexpected. During the interview, the EI suggests that certain standards related to the role of mathematics might apply especially to models in biology, one of them being the often descriptive nature of these:

> *EI:* [...] you can say that many [...] models in biology are conceptual, but they are probably also descriptive in one way or another; they cannot be used to predict anything in particular. [...] they define, to a certain extent, the meaning of the concepts, and then they describe some basic mechanisms, phenomena, and sometimes they can give rise to mathematising some of it. But [...] it typically doesn't have a character so that it becomes predictive [...]. Some of the models can be mathematised, but [...] it is not with the aim of testing the models that they are mathematised, it is more with the aim of providing a more refined description of what the mechanisms are [...].

Hence, when it comes to mathematisation, it often happens with an explanatory purpose. The EI points out that, in biology, the real world to a higher extent "exceeds" the models, as models can only capture generalities:

> *EI:* [...] in chemistry, you can make models, you can calculate how fast this reaction must proceed, and then you can check when you do it, and if it doesn't [fit], then [...] there must be something you have overlooked. If you have a population in biology that does not grow according to a specific model, then nothing

> happens […]. It doesn't give rise to any anomaly; you don't need to change your understanding.

He elaborates on this matter, connecting it with the role of mathematisation:

> *Interviewer:* Is that the biggest difference between the two subjects—what can be mathematised?
>
> *EI:* Yes. And what consequence it has when the models do not fit. […] you cannot falsify a biological model—perhaps you can do that in some extreme cases […]. [Lets us say that] you have some observations, with which you can follow the expectation of how things would go. If there were species that competed with each other, [then] we can set up a differential model. [If it does not fit], then it's just because we didn't include all the mechanisms involved, then there's something you didn't do well enough, or… […] you can't say that the theory doesn't fit. But in chemistry, if you have a reaction that proceeds in one way or another… It should go [in a certain way], and if it does not go like that, then something is not right. You haven't got a handle on all the physical and chemical parameters: the temperature, or something else, what do I know… or that there is another process, another reaction involved that was not understood at first.

In other words, models in chemistry should always be accurate. If they are not, they can be falsified and re-modelled, using mathematisation. In contrast, models in biology generally only suggest how a certain process happens, but reality, or nature, may behave differently, without that having consequences for the genuineness of the model.

5.2 Science teachers' characterization of a model

When asked what characterises models in their subjects, both teachers seem to mostly focus on the educational perspective of models. The biology teacher (BT) mentions *visualisation* as one of the primary purposes of models, thus enhancing the students' understanding of a phenomenon through some form of spatial illustration:

> *BT:* What is a model…? […] So, well, it's a visualisation of something […] Connecting it with a visualisation—now, I imagine something like Lotka-Volterra[1] […] So, that is where you meet the students, right? In that spatial understanding of some things.

[1] A set of differential equations describing how populations of predator and prey influence each other, often represented graphically.

Judging from this statement, it seems that the BT views models primarily from a descriptive or explanatory perspective, depicting reality (Gilbert, 2004), which is in line with the EI's considerations on the descriptive nature of models in biology. Following up on her comment about the Lotka-Volterra model, which is a set of differential equations that can, in fact, be used predictively, the BT affirms that she is referring to the graphic visualisation of these. Also, the chemistry teacher (CT) has a didactical view on the concept of model and how different representations of a chemical phenomenon might at the same time support and challenge students' learning. The CT reports that her students often experience great difficulties when having to shift between the levels of knowledge in chemistry (Johnstone, 1982):

> *CT:* So, if we have done a clinical experiment with the students, [...] and then go to the [sub-]microscopic level and try to make them see that it is actually an ion, or where the electrons are [...]. It is enormously difficult for them to go from one quantity to the second and the third quantity in the triangle; it presents them with huge challenges.

She mainly focuses on the switch in representation forms and the challenges related to the students' learning. The phrase "... make them see that it is actually an ion..." indicates that her perspective is somewhat focused on depicting reality. Furthermore, she mentions that, in contrast to biology, it is often difficult for the students to relate chemical phenomena to their personal experiences, e.g., with nature or daily life. This, too, is related to representing the *world-as-experienced*.

Hence, both teachers apparently view the purpose of presenting different models in their teaching as primarily didactical, arguing that multiple representations may contribute to a deeper and more nuanced understanding of a scientific phenomenon. From this perspective, models are seen as representations *of* reality rather than models *for* predictive idealisations.

Consequently, the role of mathematics within the models primarily becomes likewise illustrative and explanatory, and mathematical models are mainly seen as yet another representation where knowledge is reproduced rather than generated. Especially the BT primarily mentions mathematical models as tools for visualisation or description, exemplified in her before-mentioned comment about the Lotka-Volterra model, as well as in her description of the mathematical elements in a model of the menstrual cycle:

> *BT:* [...] we also have that menstrual cycle, where [...] one curve goes up, and the second curve goes down, without relating to anything other than that it is just a visualisation of something that rises and falls.

The two teachers agree that there are differences between the models in biology and chemistry. Like the EI, they believe that some of these differences may be connected to the subjects' relations to reality, giving mathematics a somewhat different role in the models of the subjects. The CT—who is also a mathematics teacher—suggests that one of the challenges related to mathematics in biology may be the difference in the premises or the nature of the two disciplines. She gives an example where the restrictions of mathematics do not necessarily apply when examining real-world data:

> *CT:* As a mathematics teacher, I also [teach] logistic differential equations. And [here] we actually get a complete solution [...]. [But then we] have this biological system, and then [...] there is something with a carrying capacity, and therefore we reach an upper limit—and thus translate the mathematics. But in reality, if you measure it, you can probably get over [the upper limit]. But the maths says [that] you will never [exceed the limit]. So, I think it's difficult for the students to relate to—that mathematics is also like that; that... it's very much a system of order.

With this statement, the CT addresses the issue also raised by the EI, namely that in contrast to chemistry, models in biology are what might be called "subordinate" to nature or reality. Where the mathematics in the models is clear and precise—or, with the CT's words "a system of order"—, nature is not always consistent and does not always behave as expected.

During the interview with the teachers, we present them with our hypothesis—that the role of mathematisation in models may be used to explain the differences between the two subjects' models and that mathematisation might even play a greater role in chemistry than in biology. Both teachers support this hypothesis, and the CT again bases her statements on her classroom experience:

> *CT:* [...] I think you may be right about that. [...] I think it is extremely difficult to go from reality to mathematics. And it's the same thing I see in chemistry [...]. I am thinking about a task, [...] where [the students] [...] make a Bjerrum plot[2] [...]. They have to measure the absorbance and the [...] *btb* [bromothymol blue]. And it is difficult for them to go from measuring an absorbance to how it is related to [a certain] *acid fraction*, which you need in the Bjerrum plot. So,

[2] A Bjerrum plot is a graph of the concentrations of the different species of a polyprotic acid in a solution, as a function of pH, when the solution is at equilibrium.

> [what is difficult is] this understanding of going from the absorbance that they measure—there is colour, and something that changes colour along the way—and then combine the measurements with the pH and translate all that into a curve and use the curve to read the *pKs* value.

The CT here elaborates her beliefs about the role of mathematics within models in chemistry, in particular regarding the students' challenges with the mathematisation in a process involving a model (in this case, the Bjerrum plot). However, it is also clear that mathematics and mathematisation is very present in her beliefs about models in chemistry education, and she appears to perceive the role of mathematisation as embedded in the experiment and the learning process.

The BT also uses an example from her teaching. However, in contrast to the CT, this example renders the mathematical elements as mainly illustrative:

> *BT:* We have a lot in biology, [...] where the students [...] have some data to work with. [...] They have to plot some kind of graph, which might end up with something logistical. Then they must relate to it mathematically to make sure that the numbers match. And then they explore [...] whether it makes sense from a biological perspective [...]

The BT here describes an activity, which might be interpreted as a mathematical modelling process, where the students translate empirical biological data into mathematics, work on it, and then interpret the result from a biological perspective. Also the CT's example resembles elements of mathematical modelling. Still, neither of the teachers appear to mention or include active modelling as part of their teaching. Nor do their didactical use of models seem to incorporate any form of meta perspective on models, for example in terms of how models are created, how they can be used, or what may cause changing a model.

6. Discussion

In the introduction, we posed the research question: What characterises upper secondary school science teachers' beliefs about the concept of model as well as the role of mathematics and mathematisation within the models of in their respective subjects? The analysis of interviews with the two science teachers indicates that their beliefs about scientific models are highly embedded in their educational practice, as they mainly view models as didactical tools for supporting the students' understanding of scientific phenomena. Models thus become representations of reality, offering various perspectives of a phenomenon, and the purpose of models becomes primarily descriptive, illustrative and explanatory. With Gilbert's (2004) terminology, the teachers only view—and present—models as "depiction of reality-as-observed" and not as "idealisations of a possible reality" (p.116). Consequently, active modelling is not expedient in the students' learning process, which is supported by the fact that none of the teachers include the more active part of modelling at any point in the interview. They both seem to view models merely as something that already exists or as already constructed, and not something that the students may develop themselves. From a mathematics education point of view, this is quite extraordinary, as working with models in mathematics highly concerns active modelling (Niss, 2010). Judging from this study, however, active modelling does not necessarily play a large role in upper secondary science education, at least when it comes to biology and chemistry. This aspect constitutes a future focus in our research as part of the larger project on models and modelling in STEM education. Through the design and implementation of interdisciplinary activities, we will further investigate how an inclusion of active modelling in the teaching is possible and to what extent such an approach may strengthen teachers and students' understanding of models in the scientific subjects.

The teachers agree that mathematics plays a somewhat different role in the models of the two subjects, suggesting—in line with the EI—that a reason for this might be connected to the nature of the two disciplines and the relation between models and reality. Especially the BT seems to view the role of mathematics in models as something that is mostly illustrative and may depict a biological process or phenomenon in a different form of representation. The CT, to a larger degree, appears to include mathematics in the learning processes and perhaps even views mathematics as fundamental for the models in chemistry. Of course, the variance in the two teachers' beliefs may very well be related to the fact that the CT also teaches mathematics and hence is more attentive to its potentials in other contexts.

Naturally, these beliefs influence the two teachers' approaches to teaching and working with models in the classroom, which again affects the students' images of and beliefs about what models are, which potentials they have, and which role they play in science. A merely descriptive view of models thus becomes a "ghost in the classroom" (Furinghetti, 1996). For example, it is unlikely that teachers use and display models with a prescriptive purpose in their teaching. A consequence may be that the students' beliefs about models also become restricted to a descriptive purpose. Furthermore, if the potentials and possibilities of including and emphasising the mathematics within the models are not addressed, the students might not become aware of these.

Following Gouvea and Passmore (2017), the teachers take a "models *of*" perspective, omitting a presentation of models as tools *for*, for example, generating new knowledge or making sense of the world. Hence, the students are not offered opportunities to develop an understanding of models and modelling as epistemic tools or as a crucial and influential element in society. If the students are only presented to models as depicting, for example, a crisis (Skovsmose, 2021), then they are deprived the opportunity to conceptualise models as formatting and constituting crises, thus limiting their understanding of potentials and pitfalls connected to models in society. The combination of a mathematical focus on modelling as a cyclic epistemic process (as we describe is the consensus from the literature), and a science (chemistry and biology) focus on models as simplifications and representations, that we document with the interviews, is likely to suppress the agentic aspects of models and modelling.

Finally, our study only reports upon two interviews, one with two science teachers and one with an expert. Therefore, the results are merely indications of science teachers' beliefs about models and the role of mathematisation within these models and cannot be generalised. We find that the interview with the expert provided depth to the analysis of the teachers' answers. Hence, it may be interesting and fruitful for further research to interview more experts with different foci, primarily experts with expertise within the scientific topics. Furthermore, a broader study involving more teachers of different subjects would also qualify the findings and discussion of this study. Still, the indications of differences between how models are used and presented in chemistry and biology and the role of mathematics and mathematisation within models are promising for further investigations in this direction.

Research ethics

Author contributions

M.K.Ø.: investigation, methodology, data curation, writing—original draft preparation,
U.T.J.: project administration, funding acquisition, data curation, writing—review and editing
M.M.: project administration, funding acquisition, writing—review and editing
M.K.P.: writing—review and editing.

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Conflicts of Interest

The authors declare no conflicts of interest.

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