Transposition of the Thinking Classroom approach: cultural and institutional factors

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Abstract: In this paper, we investigated the cultural transposition of the Thinking Classroom approach to the teaching and learning of mathematics in the context of a professional development program for in-service Italian teachers. This approach, originating from a different cultural context, diverged in many aspects from the usual practices of Italian teachers. The participants in our study are 17 (lower- and upper-) secondary school mathematics teachers following the program for the second year. We collected data from a written anonyms questionnaire with open and Likert-scale questions about teachers' evaluation of various aspects of the Thinking Classroom approach. We conducted a qualitative content analysis of teachers' answers, from which we derived three main emerging themes: Affect, Method, and Task and mathematical content. From a cross analysis of these themes, several teachers' unthoughts about mathematics teaching and learning approaches emerged, which we relate to both cultural and institutional characteristics of the Italian school system.

Keywords: cultural transposition, teacher professional development, thinking classroom, unthoughts

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1 Cultural transposition of educational practices

Different cultural contexts give rise to different educational practices within mathematics teaching (Bartolini Bussi & Sun, 2018). The encounter of these differing educational practices lies at the heart of the Cultural Transposition (CT) perspective (Mellone et al., 2020). This encounter is viewed as a potential occasion for reflection and development of awareness among researchers, educators, and teachers. The essence of the CT perspective isn't to conduct a comparative analysis or to blindly transfer mathematics education practices between nations. Instead, it aims to foster dialogue between two distinct cultural contexts.

The rationale of the study lays in the awareness that introducing an educational practice via the CT perspective, which enables teachers to engage with diverse values and beliefs regarding mathematics education, can prompt them to recognize some of the "invisible" cultural beliefs – *unthoughts* - concerning teaching and learning that they have assimilated and internalized within their own cultural context (Mellone et al., 2020).

In this study, we examine what a group of Italian teachers think about an approach to mathematical problem-solving that diverged from their usual practices, originating from a different cultural context. Specifically, we designed and carried out a professional

development (PD) program for them, focused on the Thinking Classroom (TC) approach, as outlined by Liljedahl (2016, 2020).

The research question addressed in this paper is: What are the unthoughts concerning teaching and learning that influence the cultural transposition process of the Thinking Classroom approach for our group of Italian teachers?

2 Thinking Classrooms in mathematics education

Building Thinking Classrooms (Liljedahl, 2020) addresses the issue that usually only a small portion of students engage in thinking during math lessons: indeed, norms in classrooms often discourage thinking behaviours. These norms are deeply rooted in school tradition, shaping how classrooms work. According to Liljedhal (2020), to foster thinking, these norms need to change.

Through years of experimentation involving hundreds of teachers, several practices emerged to encourage student thinking. These practices challenge traditional teaching norms and have proven effective. They address various aspects of classroom dynamics, from task types to group formation and workspace arrangements.

The framework emphasizes three main components:

1. *Thinking Tasks:* tasks should stimulate and sustain student thinking, transitioning from non-curricular to curriculum-based tasks.

2. *Visibly Random Groups* (VRG): grouping students randomly and frequently promotes a mindset of active contribution and breaks down social barriers.

3. *Vertical Non-Permanent Surfaces* (VNPS): working on vertical surfaces like whiteboards enhances engagement and prevents disengagement, transforming passive learning into active thinking.

Maintaining student engagement over time is crucial. For this reason, hints and extensions are used, providing students with a sequence of increasingly challenging tasks.

3 Methodology

3.1 The PD program and the sample of the study

The teachers of our sample are participating, on a voluntary basis, in a PD program focused on the TC approach, organized by the University of Pavia, since September 2022. They are 17 in-service mathematics teachers of lower (9) and upper (8) secondary schools, with different levels of experience. The PD program provides approximately one two-hour meeting per month, guided by a researcher in mathematics education (the first author), during which the teachers are involved in problem-solving activities, designed following the three main components of the TC approach described in the previous section. The room used for the PD program, not being equipped with a sufficient number of whiteboards for all the groups, is equipped with transparent plastic sheets that serve as vertical non-permanent surfaces along all the walls.

3.2 Data collection

During the first months of the second year of the PD program, the 17 participants were given an anonymous questionnaire through an online form, consisting of 5 open questions and 1 Likert-scale question (points 1-5) with 5 items: 1) Which are the main potentialities of the TC approach? 2) Which are the main limits of the TC approach? 3) What would you incorporate into the TC approach to make it more effective for your classes? 4) What would you avoid into the TC approach to make it more effective for your classes? 5) What skills are you not able to promote through the TC approach? 6) How much importance do you attribute (points 1-5) to:
- forming visibly random groups?
- having students stand and write on easy erasable vertical surfaces?
- to the fact that the problem is divided into gradual steps of increasing difficulty?
- extensions and helps being customized for different groups?
- consolidating "from the bottom" in the discussion phase?

At the time the questionnaire was administered, many of the participants had already experimented with the TC approach in their classes, some of them multiple times.

3.3 Data analysis

In the first phase, a qualitative analysis of the answers to the open questions has been carried out, codifying the themes that emerged according to the principles of qualitative content analysis (Cho & Lee, 2014).

First, the two authors separately classified all the answers before comparing their findings and reaching an agreement. We classified the lexical units related to the same topic, and segmenting the data into semantic units. Initially, the semantic units were examined and a lexical unit frequency analysis was conducted, revealing certain semantic categories—units of analysis sharing similar meanings. Subsequently, the frequencies of lexical units within these categories were assessed, yielding three main themes characterized by higher lexical unit frequencies: *Affect*, *Method*, and *Task and mathematical conte*nt.

For the answers to the Likert-scale question, we recorded the number of teachers giving each possible score to each item. Finally, we classified the topic of each Likert-scale item according to the themes emerged from the answers to the open questions, to assure coherence to the whole analysis.

4 Results

4.1 Overview of the answers to the open questions

Analysing the answers to the 5 open questions of the questionnaire provided by the 17 teachers of our sample, we found that three main themes emerged - *Affect*, *Method*, and *Task and mathematical conte*nt - with different proportions according to the question

under examination (see Table 1). In addition, for questions Q3, Q4, and Q5, many teachers expressly answer "Nothing" or "None", or that they don't know.

Table 1. Themes emerged in teachers' answers (MC stands for Mathematical Content)

Question Q1 (Potentialities)		**Question Q2 (Limits)**		**Question Q3 (What to add)**		**Question Q4 (What to avoid)**		**Question Q5 (Skills not promoted)**	
Theme	**N**	**Theme**	**N**	**Theme**	**N**	**Theme**	**N**	**Theme**	**N**
Affect	13	Affect	3	Affect	0	Affect	0	Affect	0
Method	13	Method	12	Method	9	Method	2	Method	7
Task and MC	3	Task and MC	6	Task and MC	1	Task and MC	3	Task and MC	4
Nothing	0	Nothing	0	Nothing	2	Nothing	6	Nothing	4
Don't know	0	Don't know	0	Don't know	7	Don't know	6	Don't know	2

In Table 1, N represents the number of teachers' answers in which the theme has been retrieved. As one can notice, the sum of these numbers for the first three questions (columns Question Q1, Question Q2, and Question Q3) is greater than 17, because, in some answers, more than one theme has been retrieved.

These data show that affective aspects related to the TC approach are mainly regarded as potentialities by the teachers of our sample. Methodological aspects are seen both as potentialities and as limits, with many suggestions given regarding what to add in order to promote additional students' skills. Aspects related to the tasks proposed and their mathematical content are addressed as the most critical ones, with a little number of mentioned potentialities and a lot of limits and suggestions regarding what to avoid or to add for promoting students' skills.

4.2 Details of the answers to each question.

4.2.1 Q1: Which are the main potentialities of the TC approach?

As regard to the potentialities of the TC approach (Q1), 13 teachers out of 17 mention aspects related to the theme *Affect* and 13 teachers out of 17 mention aspects related to the theme *Method*. For 9 of these teachers, the two themes appear intertwined in the same answers. For example, Lia cited both the involvement of all students and the promotion of a positive approach towards mathematics (*Affect*), and the students centred, cooperative, inclusive, and laboratory approach (*Method*) in the same sentence.

> Involvement of all students, active role in their own learning, development of cooperative learning, inclusiveness, development of a positive approach towards mathematics, laboratory-type approach (Lia)

Also in the case of Silvia, she intertwined aspects related to the theme *Method*, such as heterogeneous groups, inclusive work and dynamic lessons with aspects related to the

theme *Affect*, such as the absence of boredom.

> Heterogeneous groups ensuring inclusive work, dynamic lessons without the risk of getting bored, for both me and the students. (Silvia)

Only 3 teachers cited aspects related to the theme *Task and mathematical content*, and they do it in a very general way, mentioning "the content to be learned" (Paola) and "logical-mathematical reasoning" (Pietro), without specifying a precise content or the structure of the tasks.

4.2.2 Q2: Which are the main limits of the TC approach?

As regard to the limits that the teachers of our sample acknowledge to the TC approach (Q2), the most recurrent theme is *Method* (present in 12 answers), with particular reference to the amount of time (8 teachers) and space (4 teachers) needed to carry out the activities.

> The time. I only have an hour at a time and preparing the "blackboards", dividing them into groups and starting the problem takes up time and leaves little for the actual work. (Eri)

> Spaces not always adequate, classes sometimes too large. (Diana)

Another aspect emerges in the theme *Method*, mentioned by 3 teachers: that of students' evaluation. These teachers highlight the difficulty of carrying out a traditional, summative evaluation (that is, giving marks) in the context of the TC approach.

> I have more difficulty in evaluating the learning level of each student. (Simo)

> The evaluation. Teachers have evaluation among their obligations. But how to evaluate in a fair and objective way? (Emma)

The theme *Task and mathematical content* appears in 6 teachers' answers, in most cases referred to the difficulty in retrieving suitable tasks referred to curricular topics (4 teachers). One teacher referred to the need of simplifying (in respect to what proposed during the PD program) the objectives for middle school. This appears particularly in line with the literature reporting teachers' reluctance to engage students in solving problems with no obvious solution, especially at the lower school levels (Leikin et al., 2006, Pocalana et al., 2023).

> It is not always easy to find activities related to the curriculum. (Lia)

> The material. The lack of availability of adequate teaching resources. (Emma)

> At the secondary school, I have to simplify the objectives. (Silvia)

Only 3 teachers mention aspects related to the theme *Affect*, and they did it in very different ways, one mentioning distraction, one mentioning differences in students' level of participation and one mentioning the fear of being judged by the classmates.

> It can intimidate some particularly shy and insecure students who are very afraid of the judgment of their classmates. (Anto)
>
> Distraction, thinking it's just an opportunity for entertainment. (Fra)

4.2.3 Q3: What would you incorporate into the TC approach to make it more effective for your classes?

As far as the answers to Q3 are concerned, 7 teachers said that they don't know what they would add to the TC approach to make it more effective, 2 of them asserting that it is fine as it is. Of the remaining 10, 9 provide suggestions attributable to the *Methods* theme. In particular, 2 of them suggest adding digital technologies and multimedia resources.

> In order to work on the digital aspect, I would place a digital device next to the blackboard on which to write, on which, at the end of the work, the students must summarize the solutions found and the observations that emerged, all in a shared mode with the other groups, so to create a final product available to everyone. (Andy)
>
> The use of innovative technologies and multimedia resources and self-assessment sheets to make students reflect on their own thinking processes. (Paul)

3 teachers suggest giving more space to the collective discussion after the activity, in which students' can argue their solutions and the teacher can consolidate the topics.

> I would leave more space for the final discussion, the students could argue about the choices and strategies adopted. (Emma)

Only one teacher gives a suggestion related to the theme *Task and mathematical content*, in particular proposing to introduce interdisciplinary tasks.

4.2.4 Q4: What would you avoid into the TC approach to make it more effective for your classes?

In regard to question Q4, the large majority of the teachers (12) claim that they wouldn't avoid anything about the TC approach (6) or that they don't know what they would avoid (6). Among the other 5 teachers, 2 refer to the *Methods* theme, saying that they would avoid too complex technologies and too small groups, while 3 refer to the *Task and mathematical content* theme, saying that they would avoid tasks that are too complex or too far from the curriculum. This last aspect is particularly coherent with the well-known

teachers' reluctance to give students challenging tasks (Leikin et al., 2006, Pocalana et al., 2023), as said before.

> To make it more effective in the 1st, 2nd and 3rd classes of middle school [grades 6-8] I would try to use the method while maintaining a close link with the topics covered in class to support the development of new skills. Probably secondly I would use it to enhance the skills of logical-abstract reasoning disconnected from the purely didactic context. (Angy)
>
> Tasks that are too complex, I would start with simple requests with gradual complexity. (Fra)
>
> I would give simpler problems, given the level of the students. (Anto)

4.2.5 Q5: What skills are you not able to promote through the TC approach?

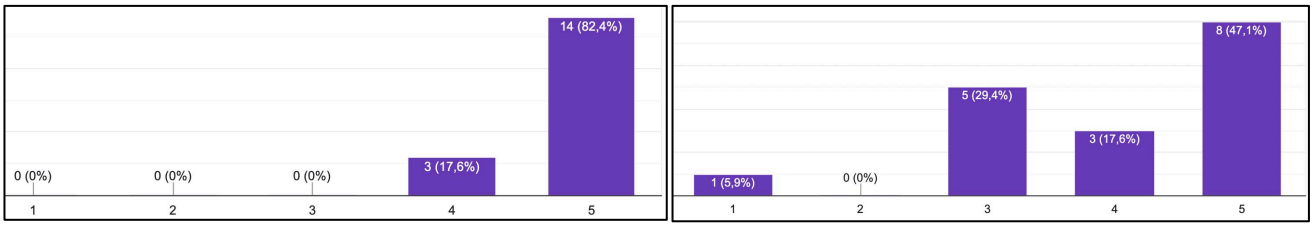
As regard to the question Q5, 4 teachers claim that there are no students' competencies included in the Italian national curriculum (MIUR, 2010; 2012) that are not promoted through the TC approach, while 2 teachers say that they don't know because they haven't experienced TC enough yet. Of the remaining 11 teachers, 7 refer to aspects relating to the *Methods* theme, 4 to aspects relating to the *Task and mathematical content* theme, and 1 teacher refers to both the themes. Among those who refer to the *Methods* theme, some highlight critical issues relating to inclusion and the participation of everyone in the task resolution process. Some refer to the failure to promote argumentative competencies (1), digital competencies (1) and the ability to concentrate (1). Among those who refer to the *Task and mathematical content* theme, 3 claim that the TC approach does not favor the acquisition of basic calculation skills and application of procedures, while 1 says that the TC approach does not promote the formalization of the discovered concepts in mathematically correct terms.

> Calculation skills, memorization of calculation techniques, solving standard problems. (Diana)
>
> Argumentative and synthesis skills are promoted less than the others. (Paul)

4.2.6 Likert Scale question

Analyzing the answers to the items of the Likert Scale question, which are focused on aspects of the TC approach related to the themes *Methods* and *Task and mathematical content*, we derive that, in teachers' opinion, the most valuable aspect is that the task is divided into gradual steps of increasing difficulty. In fact, the average score attributed to it is 4.82 and 14 teachers give the maximum score (5) to this aspect, that is referred to the theme *Task and mathematical content* (Fig. 1a). On the contrary, the less valuable aspect for them is the subdivision of the students in visibly random groups, that is referred to the theme *Methods*. In fact, the average score attributed to this aspect is 4, only less than half of the teachers (8) give it the maximum score, and 1 teacher gives it the minimum score (1).

Figure 1. Scores given by the teachers to the subdivision of the task into gradual steps (1a) and to the subdivision of the students in visibly random groups (1b)



a b

As for the other aspects mentioned in the Likert scale question, all related to the theme *Methods,* having students stand and write on easy erasable vertical surfaces received an average score of 4.47, extensions and helps being customized for different groups received an average score of 4.35 and consolidating "from the bottom" in the discussion phase had an average score of 4.31.

4.3 Cross case analysis

Considering the themes emerged across the answers to the various questions, we derive that our sample of Italian teachers attribute great value to the potentialities of the TC approach to promote positive affective aspects (theme *Affect*), such as students' involvement and contrast to boredom. Instead, the methodological aspects (theme *Methods*) are the most disputed, especially for logistical and contingent issues, such as lack of enough time and/or space to implement the TC approach properly. As regard to the theme *Task and mathematical content*, the teachers of our sample mainly challenge the topics addressed, that is a contingent aspect related to the specific tasks presented to them during the PD program, and mention the difficulty to find suitable task addressing their didactical objectives. As a positive aspect related to the theme *Task and mathematical content,* they greatly value the subdivision of the task into gradual steps of increasing difficulty.

5 Discussion and conclusions

Several teachers' unthoughts about mathematics teaching and learning approaches emerged from data analysis, which we relate to both cultural and institutional characteristics of the Italian school system. For example, the need, expressed by the teacher in regard to the theme *Task and mathematical content*, to promote students' calculation and procedural skills, combined with the reluctance in giving them tasks requiring higher order thinking, especially at lower-secondary school. This is not surprising for us as researchers, because the teachers' belief that challenging tasks are only suitable for high-achieving students while procedural tasks are more inclusive is well-known in literature (e.g., Leikin et al., 2006; Pocalana & Robutti, 2022; Pocalana et al.,

2023; Zohar & Dori, 2003). In any case, it can be considered a teachers' *unthought* which can be revealed and made explicit by introducing an educational practice originated in a different context via the CT perspective.

Strictly correlated with the aspect described above is the issue of the difficulty connected with the students' evaluation within the TC approach, raised by some teachers. We interpret it as a manifestation of a vision of the evaluation as a "measurement", expressed as a number, of the results of a procedural performance, typical of the majority of Italian secondary schools. This kind of vision is far from the idea of formative assessment (Black & Wiliam, 2009, Cusi et al., 2017) and from the evaluation of processes involving the development of problem-solving and social competencies, as it is intended in the TC approach. We interpret it as an example of cultural and institutional difference between the Italian context and the context of origin of the TC approach that can influence teachers' invisible cultural beliefs (Mellone et al., 2020).

Another element emerged from the data analysis is the need to integrate digital technologies in problem solving activities, at least in the consolidation phase. We interpret it partially as a consequence of the institutional requirement made by the Italian government to prompt students' digital competencies, also demonstrated by the number of training courses proposed to teachers with this focus.

As a final reflection, we would like to underline the need of devoting more time to the collective discussion and formalization after the group problem solving work and of better promoting students' argumentative competencies, expressed by several teachers. This can be considered as an *unthought* testifying to the importance that is culturally given in Italy to the mathematical discussion (Bartolini Bussi, 1996). Nevertheless, we would find it interesting to delve deeper into this aspect to understand whether teachers intend the discussion as a moment for students to merely recount what they have done, while the teacher *explains the* correct strategy, or if the discussion is viewed as an opportunity for students to analyse the effectiveness of their strategies, fostering dialogue on the reasons behind their success or failure, and potentially sharing alternative solutions.

In previous studies (Pocalana & Robutti, 2023; 2024), we have suggested that involving teachers in task design during a PD program could be a valuable strategy to reflect and have an influence on their beliefs and practices. This encourages teachers to integrate elements that they deem significant into the tasks and adapt them to the needs of their cultural and institutional context, potentially revealing some of their *unthoughts*. This, too, could be a venue for future research within the context of this PD program based on the TC approach and on the CT perspective.

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