Exploring the Internal Structure of the Achievement Emotion Questionnaire: Mathematics (AEQ-M) in Finnish Secondary Education Data

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Abstract: The control-value theory of achievement emotions (CVT) posits that students experience a range of emotions in academic contexts that are organized setting-dependant ways. Our aim was to examine the internal structure of the Finnish version of the Achievement Emotion Questionnaire –Mathematics (AEQ-M) with two datasets from upper secondary education. Our sample comprised a total of 2235 students (1418 students in 2021 and 1042 students in 2022 datasets) from 19 upper secondary schools across Finland. Confirmatory factor analysis showed identification issues with the seven-emotions-three-settings factor model, primarily due to issues with the hopelessness factor. After excluding hopelessness, the six-emotions-three-settings factor model fit our data well. These results provide evidence for the organization of achievement emotions in setting-dependent patterns. Further research is needed to determine whether the issues with the complete CVT factor model are attributed to, for example, cultural influences or the appropriateness of the measurement instrument.

Keywords: Mathematics, Achievement emotions, Control-value theory, AEQ-M, Internal structure.

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1 Introduction

Emotions play a pivotal role within academic settings, influencing learning and academic achievements (Pekrun, 2006). In the literature, there are several definitions for emotions stemming from three distinct traditions: emotions as an outcome of evolution, psychoanalytic research, and cognitive tradition (cf. Hannula, 2012). Emotion theories from the evolutionary tradition focus on the physiological characteristics to define a small number of basic emotions (e.g. Buck, 1999; Ekman,1992). On the other hand, cognitive theories, such as control-value theory (Pekrun, 2006), focus more on the cognitive appraisals in the situation and often identify a larger number of emotions.

While basic emotions are likely to be universal across cultures, it is likely that the more cognitively defined emotions are more strongly influenced by cultural norms

and learning, and, hence, may be less universal. Despite the recognized significance of emotions, limited attention has been devoted to understanding their universality or potential cultural variations. The control-value theory of achievement emotions (CVT; Pekrun, 2006) posits a universal relation between appraisals and achievement emotions across cultures, while acknowledging variations in mean emotion levels. This was observed in a cross-cultural comparison between China and Germany (Frenzel, Thrash, et al., 2007). On the other hand, emotions have been argued to be a cultural construct, where the same factor structure is not necessarily achieved in, for example, individualistic and collectivist cultures (Bofah & Hannula, 2015; Tuohilampi et al., 2015).

1.1 Control-value theory of achievement emotions

CVT categorizes students' achievement emotions into two main types: activity and outcome emotions (Pekrun, 2006). Activity emotions, such as enjoyment and boredom, are associated with ongoing academic activities, while outcome emotions, like pride and anxiety, stem from academic outcomes such as test results. These outcome emotions can be further categorized as prospective (e.g., anxiety before an exam) or retrospective (e.g., pride after successful performance). Additionally, achievement emotions are divided based on valence (positive versus negative) and activation (activating versus deactivating) in Pekrun's (2006) taxonomy.

The theory recognizes the situational and temporal dimension of the achievement emotions (Pekrun et al., 2011). Achievement emotions are influenced by the current situation: whether the student is attending class, studying or participating in an exam. These different settings are important to distinguish since they differ in function and social structure (Pekrun et al., 2002). On the other hand, achievement emotions can be conceptualized as momentary (state-like), such as momentary anxiety before an exam, or typically experienced (trait-like), such as enjoyment when facing mathematical challenges (Pekrun, 2006).

CVT posits that the emergence of achievement emotions is influenced by two self-assessment processes: subjective control and subjective value (Pekrun, 2006). For instance, students are likely to experience enjoyment when they perceive a high level of mastery in their studies (subjective control) and attach importance to their academic activities (subjective value). Conversely, anxiety may arise when students doubt their competence (subjective control) but value success highly (subjective value).

According to CVT, emotions are related to achievement in studies (Forsblom et al., 2022; Pekrun et al., 2017; Pekrun et al., 2023). It is assumed that positive activating emotions (enjoyment, pride, and hope) support, and negative deactivating emotions (boredom and hopelessness) hinder study performance (Pekrun, 2006; Pekrun et al., 2011). On the other hand, positive deactivating emotions (relief) and negative activating emotions (anger, anxiety, and shame) have a twofold effect on performance in studies (Pekrun, 2006; Pekrun et al., 2011). For example, anxiety can be negatively related to performance, but on the other hand, it can increase extrinsic motivation to avoid failure and thus affect performance (Pekrun, 2006; Pekrun et al., 2011; Pekrun et al., 2023).

Achievement emotions are organized in subject-specific ways, since they stem from differences in expectancies, values, and goals (Goetz et al., 2007). Mathematics is considered a subject of interest since previous research has identified many emotions when studying mathematics (e.g., Frenzel, Pekrun, et al., 2007; Goetz et al., 2008; Holm et al., 2017), and the key components of CVT are associated with mathematics-related emotions such as control and value appraisals, environmental factors, and achievement (Biese et al., 2024; Buff, 2014; Holm et al., 2017).

1.2 The Achievement Emotions Questionnaire – Mathematics (AEQ-M)

AEQ-M (Pekrun et al., 2005) measures seven emotions (enjoyment, pride, anxiety, boredom, anger, shame, and hopelessness) in three academic settings covering attending class, learning, and test situations. AEQ-M has been validated in different cultural contexts like Germany (Bieleke et al., 2023) and Portugal (Moreira et al., 2019), as well as analysed for cross-cultural comparability (Frenzel, Thrash, et al., 2007). A study by Bieleke et al. (2023) reported results on the validity of the internal structure and external relations of the AEQ-M instrument with data used to develop the questionnaire as well as additional data. That study distinguished seven discrete achievement emotions that are setting specific (Bieleke et al., 2023). Similarly, setting dependency has been observed in a study by Moreira et al. (2019). Both of these studies were conducted in the lower secondary level (mean age of the students varied from 12.5 years to 14.3 years). In the Finnish context, AEQ-M has been previously used in its entirety (e.g., Holm et al., 2017; Holm et al., 2020) or focused on a specific setting (e.g., Biese et al., 2024). In the study by Holm et al. (2020), eight items were eliminated based on poor model fit in the confirmatory factor analysis.

1.3 Current study

In this study, we used CVT as a theoretical framework and focused on the internal structure of the Finnish version of the AEQ-M. Our aim was to assess whether discrete achievement emotions (enjoyment, pride, anxiety, boredom, anger, shame, and hopelessness) are organized in setting-dependent ways (attending class, learning, and taking tests) according to the assumptions of CVT (Pekrun et al., 2011), in the Finnish upper secondary school context.

2 Method

2.1 Participants

The data in this study was gathered from Finnish upper secondary schools during the spring semesters of 2021 and 2022. We used stratified sampling to obtain a geographically representative sample. Participating schools came from all jurisdictions, mostly from either Southern or Western and Inland, which are the largest in terms of population (Official Statistics of Finland, 2024). The sampling method has been described in more detail in Sydänmaanlakka et al. (2024).

In the initial data collection phase, 1,428 students from 19 schools participated, while in the subsequent phase, 1,066 students from 18 schools participated. Notably, one school withdrew from participation after the first data collection. Removing this school from the first data set would not have significantly changed the distribution of any of the variables, so it was retained in the data. Overall, 2,266 students contributed data across the two collection periods, of which 228 students participated in both data collections. Students with more than 50% of achievement items missing were excluded from the analysis (see Missing values). Thus, the final sample used in this study consisted of 1418 students (38.9% male, 59.9% female; M_{age}= 16.8 years, *SD* = 0.82) in the first and 1042 students (37.4% male, 59.7% female; M_{age}= 16.9 years, *SD* = 0.86) in the second dataset.

2.2 Missing values

In 2021 a total of 1.36% and in 2022 a total of 2.86% of achievement emotion items were missing, stemming from 138 and 163 incomplete responses. Considering all achievement emotion variables, missing values ranged from 0.49% to 2.94% in 2021 and from 0.94% to 5.63% in 2022. We imputed data using an expectation-

maximisation (EM) algorithm (Dempster et al., 1977). Before imputation, we removed participants with more than 50% of items missing (n = 10 in 2021, n = 24 in 2022).

2.3 Measures

Students' mathematics-related achievement emotions were measured with Achievement Emotions Questionnaire – Mathematics (AEQ-M; Pekrun et al., 2005). The AEQ-M is a widely used instrument based on CVT (Pekrun, 2006) that can be used to analyse multiple emotions typically experienced in a learning environment using a single questionnaire. The AEQ-M has been translated into Finnish by a bilingual expert and pilot tested previously (see Holm et al., 2017). Both in Finland and internationally, the AEQ-M has been found to have good internal reliability (e.g., Bieleke et al., 2023; Frenzel, Thrash, et al., 2007; Holm et al., 2020).

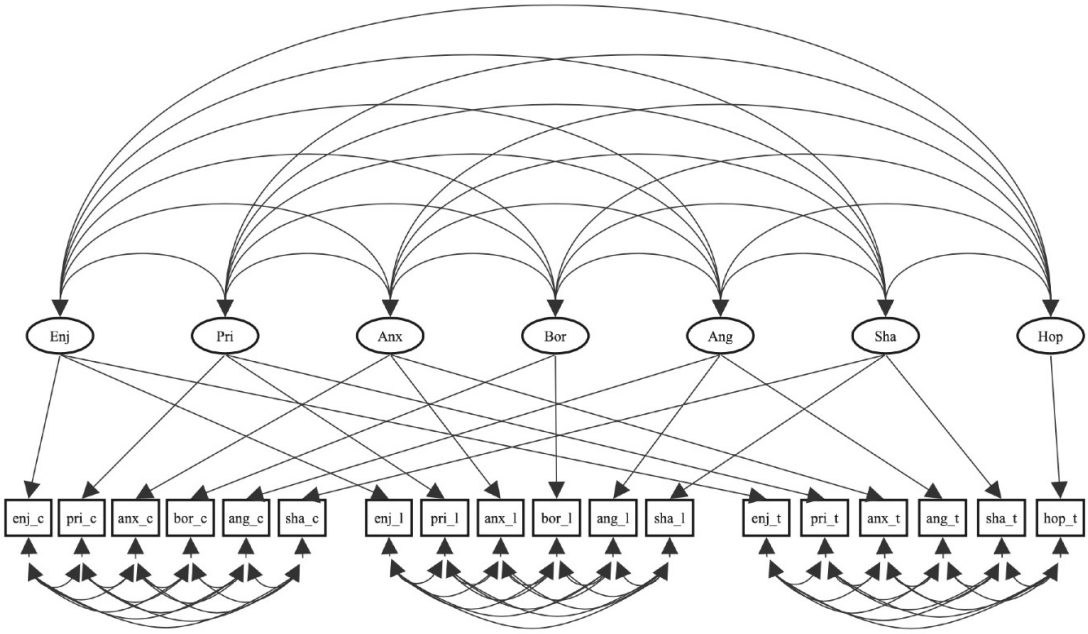
Using a 5-point Likert-scale (1 = strongly disagree, 5 = strongly agree), students responded to 60 items covering seven emotions (enjoyment: 9 items, pride: 8 items, anxiety: 15 items, boredom: 6 items, anger: 8 items, shame: 8 items, and hopelessness: 6 items). Items covered students' habitual mathematics-related emotions in class, learning, and test setting.

2.4 Statistical analysis

We conducted the analysis using R programming language (version 4.3.1; R Core Team, 2023), with "lavaan" package (Rosseel, 2012). We analysed the structural relationship between achievement emotions in both samples with a series of CFA, using a maximum-likelihood estimator with robust standard errors (MLR). We followed the procedure described by Pekrun et al. (2011) as follows. First, we created a one-factor model across all emotions and settings (M1). Next, each emotion was represented as its own factor (M2). Third, we formed a model where three factors represented each setting (M3). Lastly, seven factors represented each emotion with correlated uniqueness within settings (M4). In M4, latent achievement emotion factors were formed as a combination of settings connected to the emotion[1], and the settings were formed as scale averages from questionnaire items (see Figure 1). The effect of the setting was considered by correlating the error residuals within each setting.

[1] Other emotions cover all settings except boredom, which is measured in class and learning settings, and hopelessness, which is measured only in test setting.

Figure 1. Achievement emotion model with correlated uniqueness within setting (M4).



Note. Enj = enjoyment, Pri = pride, Anx = anxiety, Bor = boredom, Ang = anger, Sha = shame, Hop = hopelessness. The prefixed c = class setting, l = learning setting, t = test setting.

We evaluated the models using the Chi-square statistic (χ^2), the comparative fit index (CFI), the Tucker-Lewis index (TLI), the standardised root mean square residual (SRMR), and the root mean square error of approximation (RMSEA). CFI and TLI ≥ 0.90, SRMR ≤ 0.08, and RMSEA ≤ 0.08 were considered to demonstrate an acceptable fit (Marsh et al., 2004). Since our dataset contained partly the same participants (n = 225), we removed these from the first dataset before conducting the CFAs in order to obtain samples of equal size.

3 Results

Table 1 presents descriptive statistics and Pearson's correlations of the AEQ-M items, where no substantial difference between datasets was observed. Mean scores for positive achievement emotions were slightly higher compared to negative achievement emotions expect for hopelessness. The internal consistencies of the emotion scales were estimated with Cronbach's alpha coefficients. In both samples, the alpha coefficients ranged from good ($0.9 > \alpha \geq 0.8$) to excellent ($\alpha \geq 0.9$). Also, in both samples, the positive achievement emotions were positively correlated with each other, as were the negative achievement emotions. Instead, positive achievement emotions were negatively correlated with the negative ones. High correlations were

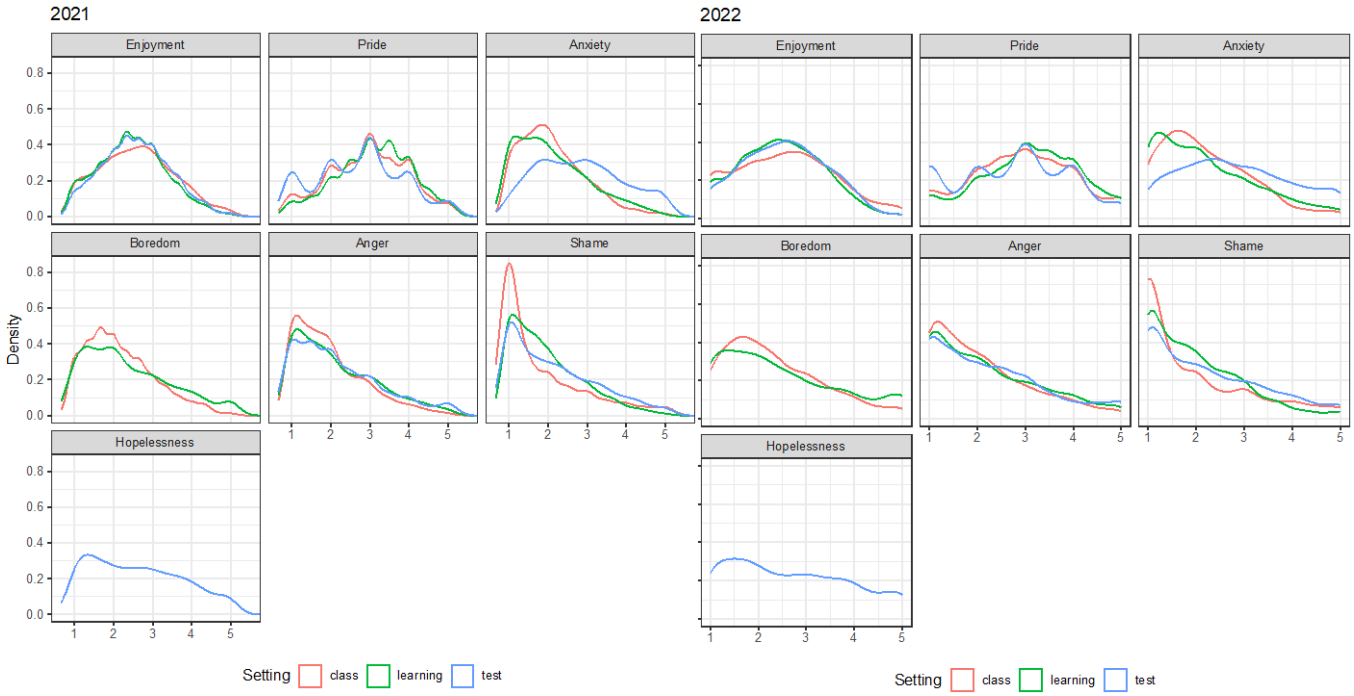
observed, especially between anxiety and hopelessness. Figure 2 shows the distributions of the emotion variables in both datasets. The distributions were highly similar in both years. Negative emotions, in particular anger, shame, and hopelessness, are visibly skewed due to a lack of high values.

Table 1. Descriptive statistics and Pearson's correlations (2021 below diagonal and 2022 above diagonal) of achievement emotions.

	2021			**2022**			**Item correlations**						
Scale	***M*(*SD*)**	**α**	**Skew (Kurt)**	***M*(*SD*)**	**α**	**Skew (Kurt)**	**1**	**2**	**3**	**4**	**5**	**6**	7
1 Enj	2.56 (0.81)	0.91	0.27 (-0.38)	2.56 (0.86)	0.91	0.25 (-0.49)		.80	-.49	-.64	-.54	-.39	-.58
2 Pri	2.94 (0.89)	0.87	-0.04 (-0.51)	2.94 (0.94)	0.87	-0.08 (-0.61)	.81		-.48	-.53	-.47	-.47	-.58
3 Anx	2.46 (0.88)	0.93	0.46 (-0.45)	2.52 (0.97)	0.94	0.46 (-0.61)	-.47	-.46		.69	.79	.80	.90
4 Bor	2.29 (0.93)	0.88	0.71 (-0.21)	2.42 (1.05)	0.90	0.67 (-0.42)	-.64	-.52	.62		.80	.55	.69
5 Ang	2.09 (0.88)	0.90	0.92 (0.28)	2.20 (1.00)	0.92	0.84 (-0.09)	-.53	-.45	.76	.75		.65	.78
6 Sha	2.01 (0.86)	0.87	0.90 (0.11)	2.08 (0.96)	0.89	0.85 (-0.04)	-.34	-.41	.78	.48	.61		.77
7 Hop	2.62 (1.16)	0.92	0.35 (-0.96)	2.69 (1.22)	0.93	0.33 (-1.06)	-.58	-.59	.88	.65	.75	.72	

Note. Enj = enjoyment (10 items); Pri = pride (6 items); Anx = anxiety (15 items); Bor = boredom (6 items); Ang = anger (9 items); Sha = shame (8 items); Hop = hopelessness (6 items)

Figure 2. Density plots of achievement emotions in both datasets.



3.1 Confirmatory factor analysis

The results of the CFA's showed significant challenges in model identification for both datasets. Subsequent investigations detected hopelessness as a primary concern, as the variance of its residual error was estimated as zero. This is indicative of collinearity with the other emotions, which was also supported by substantial correlations between hopelessness and anxiety (≥0.88, see Table 1). In response to these findings, we tried systematically removing individual items from the "hop_t" scale (see Figure 1). However, this approach revealed that the issue was more extensive than a single item.

Consequently, we made the decision to exclude hopelessness from the model entirely. Therefore, we conducted the model comparisons (M1-M4) using six latent emotions (enjoyment, pride, anxiety, boredom, anger, and shame). Table 2 presents the results from the CFA with six achievement emotions. The six-emotions-three-settings factor model had an acceptable fit between the model and the observed data in 2021 (CFI = 0.978, TLI = 0.953, RMSEA = 0.066, and SRMR = 0.037) and 2022 (CFI = 0.983, TLI = 0.965, RMSEA = 0.062, SRMR = 0.036). Standardized parameter estimates for the CFAs are presented in Figure 3.

Figure 3. Standardized coefficients for CFAs

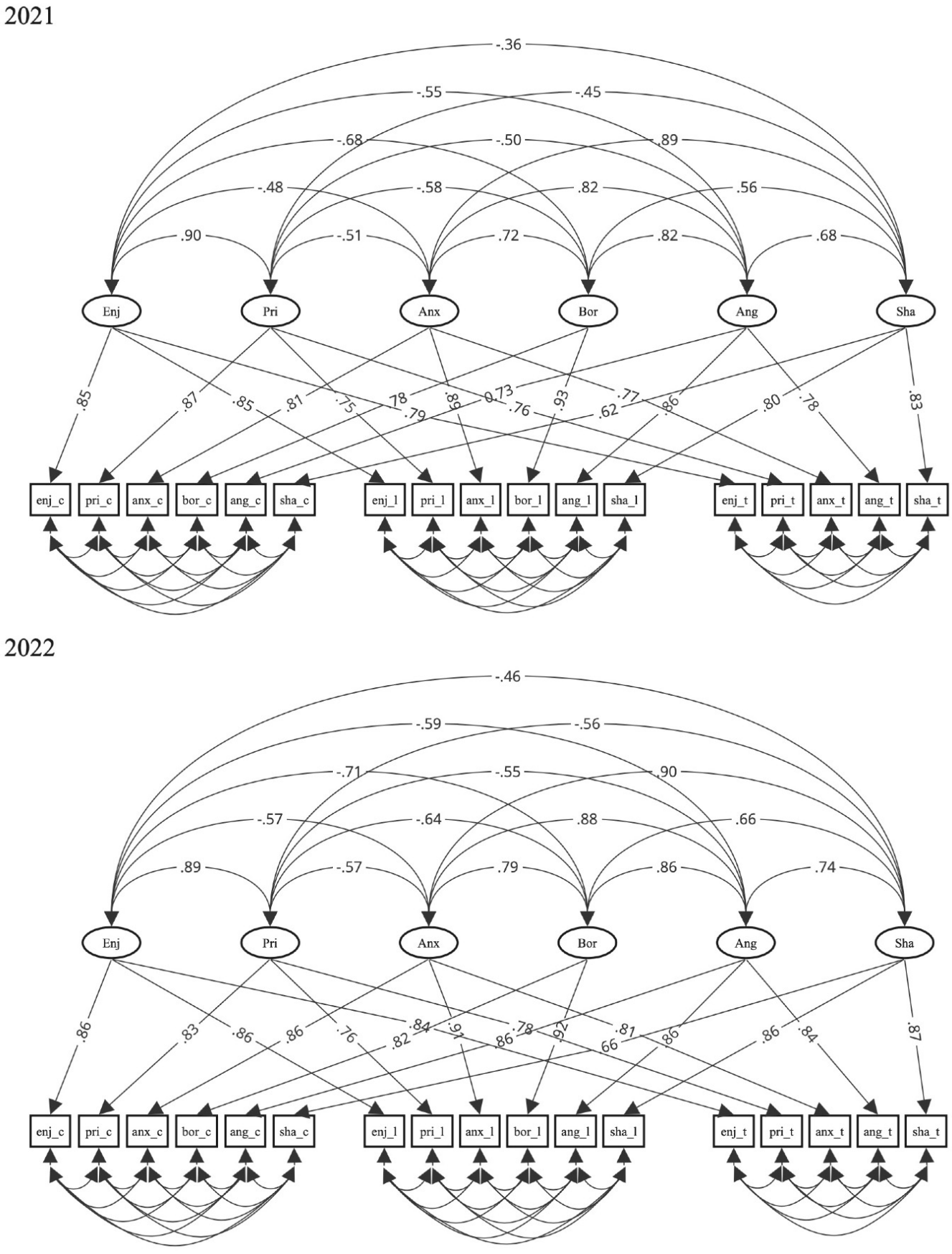


Table 2. Confirmatory factor analysis model comparisons for both datasets.

	Model	**χ^2**	**df**	**CFI**	**TLI**	**RMSEA**	**SRMR**
2021*	M1: One-emotion model	4863.14	119	.632	.580	.199	.132
	M2: Six-emotions model	1288.23	104	.909	.880	.106	.052
	M3: Three-settings model	4674.52	116	.650	.589	.197	.134
	M4: Six-emotions-three-settings model	354.70	64	.978	.953	.066	.037
2022	M1: One-emotion model	4061.10	119	.685	.640	.197	.123
	M2: Seven-emotions model	1087.46	104	.923	.899	.104	.048
	M3: Three-settings model	3953.00	116	.697	.645	.196	.127
	M4: Six-emotions-three-settings model	282.31	64	.983	.965	.062	.036

Note. * = Participants who were in both datasets (n = 225) were removed from the 2021 data.

4 Discussion and conclusions

The objective of this study was to analyse the internal structure of AEQ-M using two separate datasets collected from Finnish upper secondary schools. Specially, our aim was to examine whether achievement emotions manifest in a setting-dependent way, wherein emotional experiences are hypothesized to differ depending on whether student is attending class, learning or in test situation.

In our data, the seven-emotions-three-settings factor model had identification issues in both datasets due to problems with hopelessness. Therefore, the data used in this study did not validate the model assumed by CVT (Pekrun et al., 2011). This finding contradicts previous studies where the seven-emotions-three-settings factor model showed an acceptable fit (Bieleke et al., 2023; Moreira et al., 2019). However, the result is not exceptional because previous research has shown that for other affect instruments, the same factor structure is not necessarily achieved across different cultures (Bofah & Hannula, 2015).

Several aspects may explain why our data did not follow the same factor structure as previous studies. In previous research, hopelessness has been identified as a separate mathematics-related emotion among lower secondary school students, but this emotion was mostly experienced by lower-performing students (Holm et al., 2020). Thus, the sample used in this study may be too selective, and the factor structure should be examined in a larger population and at different school levels. On the other hand, hopelessness is the only emotion that has been measured in just one

setting — the test. While Finnish students tend to have low levels of anxiety (see, e.g. Lee, 2009), and the anxiety in this study was mostly experienced in test situations (see Figure 2), these two scales may have measured the same phenomenon, test anxiety. Including items measuring hopelessness in classroom and learning settings may enhance its distinction as a separate factor. Additionally, the potential impact of translation on the unexpected factor structure cannot be excluded.

Our results supported the six-emotions-three-settings factor model. Notably, this factor structure was consistently identified in both datasets, which demonstrates the stability of the results. This result further validates the assumption that mathematics-related achievement emotions are setting-dependent as assumed by CVT (Pekrun, 2006) and observed in the validation studies by Bieleke et al. (2023) and Moreira et al. (2019). This validation of the widely employed measurement tool and its theoretical framework is important in the Finnish educational context.

Research ethics

Author contributions

All authors have contributed to the study conception and design and have read and agreed to the published version of the manuscript.

A.S.: material preparation, data collection, conceptualisation, methodology, analysis, writing—original draft preparation, writing—review and editing. J.H.: conceptualisation, methodology, supervision, writing—review and editing. M.E.H.: conceptualisation, supervision, writing—review and editing. M.S.H.: conceptualisation, supervision, writing—review and editing.

Artificial intelligence

During the preparation of this work the author used ChatGPT 3.5 to improve language and readability. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Institutional review board statement

We followed the Finnish ethical principles of research in the humanities and social and behavioural sciences (National Advisory Board on Research Ethics, 2009). According to these principles, this study did not involve any of the circumstances that would require an ethics review.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

The participants of this study did not give written consent for their data to be shared publicly, so due to the sensitive nature of the research supporting data is not available.

Conflicts of Interest

The authors declare no conflicts of interest.

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