

Mathematics teacher educators in the Spanish context: a glance at their beliefs and perceived competence

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Abstract: Mathematics Teacher Educators [MTEs] have been a subject of interest in recent years since researchers consider beliefs, and perceived competence to be a fundamental part of MTEs as they affect MTEs' teaching practice. In this study, a mixed analysis is carried out regarding the beliefs and perceived competence of MTEs in Spain under the theoretical framework of the Mathematics Teacher Educator's Specialised Knowledge [MTESK] model. Additionally, the relationships between these aspects are examined. The results indicate that some of the MTESK's subdomains rank higher in the MTEs' relevance. Results exhibit differences regarding MTEs' perceived competence but similar beliefs. Consequently, MTEs could be classified into three clusters based on their perceived competence.

Keywords: beliefs, mathematics, perceived competence, Spanish context, teacher educators

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1 Introduction

Research in mathematics education over the past decades has started to focus on MTEs (Sikko & Grimeland, 2021). As highlighted by Chapman (2021), MTEs are deemed responsible for aiding future mathematics teachers in developing and attaining the mathematical and pedagogical content knowledge they need for the teaching profession. As a consequence, researchers have started to study the specialised knowledge, professional competences, and training backgrounds of MTEs (Appova & Taylor, 2019; Beswick & Goos, 2018; Giadas et al., 2023; Pascual et al., 2019), coming out with different profiles of MTEs. The existence of different profiles of MTEs (Giadas et al., 2024) makes it harder to find a single model that characterises their knowledge.

In this regard, several models have been proposed for MTEs' knowledge, such as the MTESK (Martignone et al., 2022) or the one proposed by Goodwin et al. (2014). Despite literature indicating that beliefs are relevant aspects of MTEs, these are not always included in the knowledge models (Beswick & Goos, 2018). Escudero-Ávila et al. (2021) argue that beliefs and affective domain “go beyond the view of mathematics as a science and object of teaching and learning” (p. 24), and Chapman (2021) advocates beliefs show MTEs' relationship with the teaching process. We assume that

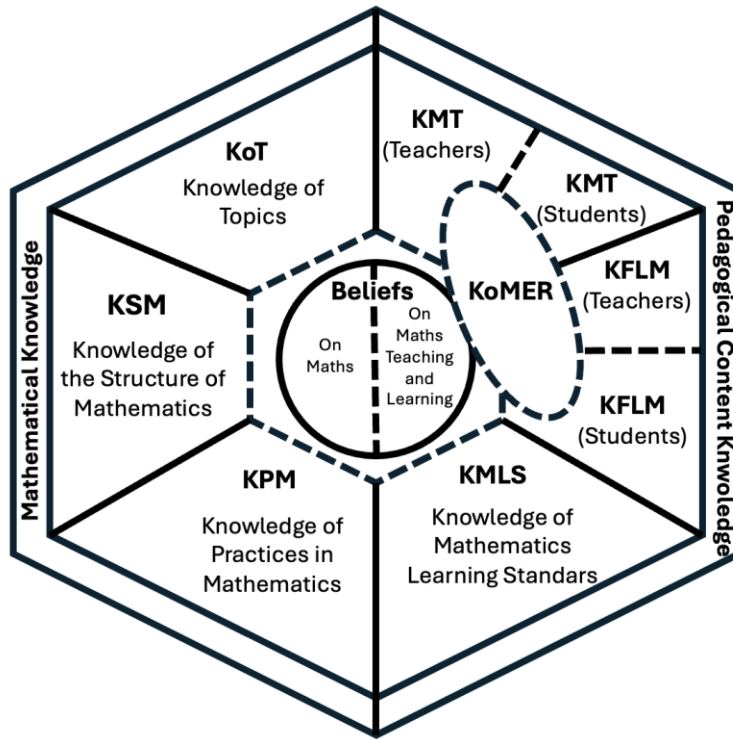


research on MTEs knowledge should also extend to the analysis of their beliefs about what they teach to their students as Beswick and Goos (2018) have declared in their research. The reason behind is that MTEs received little support in their training programmes (Olanoff et al., 2021) and there is not a common shared vision about what and how MTEs should teach (Appova & Taylor, 2019). Consequently, our research goals are, on the one hand, to describe MTEs' beliefs and perceived competence and, on the other hand, to analyse the possible relationships among these aspects. Note that, in line with Cueli et al. (2023), in this research perceived competence was measured through MTEs' knowledge and readiness level.

This study expands research on MTEs' beliefs and perceived competence, identifying those more prominent in the Spanish context and creating the opportunity to explore through observations and interviews how these detected beliefs influence the MTEs' teaching practice, which is an understudied aspect (Beswick & Goos, 2018).

2 Theoretical background

As highlighted by Beswick and Goos (2018), beliefs are considered cognitive aspects that differ from knowledge in the degree of consensus they generate. Beliefs are influenced by the educational background of MTEs and their research so they could vary between different MTEs' profiles. In addition, beliefs could limit MTEs' practice (Chapman, 2021) due to differences between MTEs' theoretical beliefs and MTEs' practical beliefs (Martignone et al., 2022). So, analysing MTEs' beliefs is a good practice for examining how they affect MTEs' planning and decision-making, ultimately impacting their teaching practice (Beswick & Goos, 2018). Also, it is pointed out that beliefs about mathematics teaching process must be discussed when preparing prospective teachers (Marshman, 2021). The MTESK model proposed by Martignone et al. (2022), based on the MTSK model by Carrillo-Yáñez et al. (2018), includes MTEs' beliefs as part of their knowledge because MTEs' teaching practice is influenced not only by their knowledge but also by their beliefs. The MTESK model consists of three major domains: Mathematical Knowledge [MK], Pedagogical Content Knowledge [PCK], and Beliefs. Each is further divided into subdomains, as seen in Figure 1.

Figure 1. MTESK model (adapted from Martignone et al., 2022, p. 319).

Note. The figure represents the MTESK model, in which the different sub-domains of the model can be observed.

According to the MTESK model, the Knowledge of Mathematics Teaching [KMT] is divided into teaching teachers and students, and the Knowledge of Features of Learning Mathematics [KFLM] is divided into how teachers and students learn. Additionally, the MTESK assumes the existence of the subdomain Knowledge of Research in Mathematics Education [KoMER].

3 Methodology

This paper presents the results of an exploratory study, through a mixed methodology, conducted with MTEs in Spain. Through non-probabilistic sampling but trying to reach the maximum quota of the population (Giadas et al., 2024), 405 MTEs were invited to participate in this study, of which 157 accepted, constituting a response rate of approximately 39%. The research instrument was a questionnaire designed within the MTSK Network (<https://redmtsk.net/>), comprising 67 items about MTEs' training and professional experience and 66 items about MTEs' conceptions about initial teacher training and perceptions about their readiness level (Rojas & Pascual, 2023). The questionnaire was validated within the MTSK Network by experts in

MTEs' knowledge and the MTESK knowledge model. This study benefited through this anonymous questionnaire from the quick, comprehensive and honest responses of MTEs in the Spanish context. It was administered via Microsoft Forms®. In this paper, a set of 30 items related to the research goals is analysed. The items correspond to:

- Subset 1 in Table 1 (six items): MTEs' beliefs about initial teacher training were measured using the ordered Likert-scale from 1 (*Strongly disagree*) to 4 (*Strongly agree*).

Table 1. Items of subset 1.

To what extent do you agree or disagree with the following statements?
B1. It is important that student teachers' assignments always have an objective situated in the pre-primary/primary/secondary classroom.
B2. During initial teacher training, it is important for student teachers to become aware of the vision of the mathematics they project.
B3. If student teachers surpass the mathematical and didactic content of the courses, they will be able to articulate them autonomously in any future teaching situation.
B4. Initial teacher training should not address issues related to value judgments about teaching styles in mathematics.
B5. The best approach in initial teacher training is to ensure that they master the mathematical knowledge they will teach.
B6. The planning and evaluation of mathematical activities are processes common to other teaching disciplines and therefore should be addressed in general didactics courses.

Note. In this table, the items related to the MTEs' beliefs are observed.

- Subset 2 in Table 2 (nine items): MTEs' perception of the relevance of different activities in initial teacher training was measured. MTEs must order these items from most to least relevant to them. An average order was determined by using the Borda method (Pérez-Fernández & De Baets, 2018).

Table 2. Items of subset 2.

Rank the following items from most to least relevant according to the relevance you think they should have in initial teacher education.
R1. Detailed reconstruction of school mathematical content (algorithms, definitions, properties, etc.), through a thorough and detailed analysis.
R2. Analysis of the fundamentals, applications, and phenomenology, enabling an understanding of the different representations and relationships between their elements.
R3. Reflection on different mathematical practices (defining, proving, conjecturing...), correct use of them, and assessment of their inclusion in compulsory education.
R4. Mathematical situations that enhance inter- and intra-conceptual connections.
R5. Potentials and limitations of different manipulative and virtual resources.
R6. Characterisation of different types of problems and discussion on their potentials and limitations.
R7. Learning difficulties and assessment of students' common errors.
R8. Analysis of different learning standards. Assessment of curriculum changes in mathematics in recent years.
R9. Analysis of different diagnostic tests (PISA, TIMSS, TEDS-M, ...), potentials and limitations, including proposals for improvement.

Note. In this table, the items related to MTEs' perception of the relevance of different activities in initial teacher training are observed.

- Subset 3 in Table 3 (eight items): MTEs' knowledge was measured using a Likert scale from 1 to 4, with 1 being the lowest level and 4 being the highest.

Table 3. Items of subset 3.

Please give a value from 1 to 4 to assess your knowledge of the contents described below.
K1. Management of classroom climate in initial teacher training.
K2. Learning dynamics of student teachers.
K3. Common difficulties in the learning of student teachers.
K4. Professional development of mathematics teachers.
K5. Mathematical knowledge for teaching.
K6. Knowledge of teaching strategies in initial teacher training.
K7. Knowledge of how and why to assess in initial teacher training.
K8. Knowledge of initial teacher training standards.

Note. In this table, the items related to MTEs' knowledge are observed.

- Subset 4 in Table 4 (seven items): Readiness level as MTE was measured using the ordered Likert scale from 1 (*I don't feel prepared*) to 4 (*I feel very well prepared*).

Table 4. Items of subset 4.

In your experience as a teacher educator, how well prepared do you feel today to carry out the following activities in initial teacher education?
RL1. Analysing the current curriculum materials in mathematics to work with your students.
RL2. Exploring manipulative materials (concrete or digital) with your students to solve mathematical problems.
RL3. Developing a class based on the mathematical knowledge and skills that your students exhibit during the session.
RL4. Teaching how to develop mathematical skills in children (problem-solving, reasoning...).
RL5. Modelling or demonstrating specific mathematical practices that students need to learn to perform as teachers.
RL6. Analysing the teaching practice in mathematics of another teacher.
RL7. Exploring research literature regarding the teaching or learning of mathematics with your students.

Note. In this table, the items related to readiness level as MTE are observed.

MTEs' responses were collected to conduct a descriptive analysis and, subsequently, examine the relationships between them using RStudio. The analysis of the relationships between the items of subsets 1, 3 and 4 was carried out in two phases. The analysis of the items of subset 2 was conducted separately using the Borda ranking method due to its ranking nature. Firstly, following the guidelines outlined in Muñiz-Rodríguez et al. (2020), the suitability of our data for conducting a Principal Component Analysis [PCA] was checked to reduce the data dimensionality, achieving greater interpretability of the results in terms of the considered components. This verification involved the Kaiser-Meyer-Olkin [KMO] sampling adequacy test and Bartlett's test of sphericity. Secondly, following Maffia et al. (2020), an agglomerative hierarchical clustering (assisted by *ggplot2*, *stats*, and *cluster* RStudio packages) was applied to detect clusters of MTEs with similar behaviours. Graphical representations were generated to characterise the MTEs of each cluster.

Additionally, the items considered in this research were linked to the different subdomains of the MTESK model through a deductive classification based on their description. The authors carried out a triangulation process.

4 Results

4.1. Description of the items and their relationship with MTESK

This section contains a detailed description of each item considered in this research. This description shows how the MTEs of the Spanish context behave in each of the subset of items considered.

Table 5 presents the results of subset 1. In this regard, the highest level of agreement is observed in B2. This is followed by items B1, B3, and B5, respectively. B4 and B6 are the items with which the MTEs are most in disagreement.

Table 5. Statistical summary of the items of subset 1 and their relationship with the MTESK model. Scale: (1) *Strongly disagree*, (2) *Disagree*, (3) *Agree*, (4) *Strongly agree*. SD=Standard Deviation.

Items (<i>MTESK subdomain</i>)	Min	Median	Max	SD
B1. It is important that student teachers' assignments always have an objective situated in the pre-primary/primary/secondary classroom. (<i>KFLM teachers</i>)	2	3	4	0.64
B2. During initial teacher training, it is important for student teachers to become aware of the vision of the mathematics they project. (<i>KFLM teachers</i>)	3	4	4	0.46
B3. If student teachers surpass the mathematical and didactic content of the courses, they will be able to articulate them autonomously in any future teaching situation. (<i>KFLM teachers</i>)	1	3	4	0.78
B4. Initial teacher training should not address issues related to value judgments about teaching styles in mathematics. (<i>KMT teachers</i>)	1	2	4	0.92
B5. The best approach in initial teacher training is to ensure that they master the mathematical knowledge they will teach. (<i>KoT and KFLM teachers</i>)	1	3	4	0.82
B6. The planning and evaluation of mathematical activities are processes common to other teaching disciplines and therefore should be addressed in general didactics courses. (<i>KMT teachers</i>)	1	2	4	0.95

Note. In this table, the items related to MTEs' beliefs are observed, their relationship with the sub-domains of the MTESK model, and a statistical description through the minimum, median, maximum, and standard deviation.

Table 6 shows the items of subset 2, ordered by their average relevance order. Note that the items were not shown in this order in the questionnaire.

Table 6. Predominant ordering of subset 2 and their relationship with the MTESK model.

Items (<i>MTESK subdomain</i>)	Relevance order
R4. Mathematical situations that enhance inter- and intra-conceptual connections. (<i>KSM and KMT teachers</i>)	1
R2. Analysis of the fundamentals, applications, and phenomenology, enabling an understanding of the different representations and relationships between their elements. (<i>KoT and KMT teachers</i>)	2
R1. Detailed reconstruction of school mathematical content (algorithms, definitions, properties, etc.), through a thorough and detailed analysis. (<i>KoT and KMT teachers</i>)	3
R7. Learning difficulties and assessment of students' common errors. (<i>KFLM students</i>)	4
R3. Reflection on different mathematical practices (defining, proving, conjecturing...), correct use of them, and assessment of their inclusion in compulsory education. (<i>KPM and KMT teachers</i>)	5
R5. Potentials and limitations of different manipulative and virtual resources. (<i>KMT teachers</i>)	6
R6. Characterisation of different types of problems and discussion on their potentials and limitations. (<i>KPM</i>)	7
R8. Analysis of different learning standards. Assessment of curriculum changes in mathematics in recent years. (<i>KMLS</i>)	8
R9. Analysis of different diagnostic tests (PISA, TIMSS, TEDS-M, ...), potentials and limitations, including proposals for improvement. (<i>KoMER</i>)	9

Note. In this table, the items related to MTEs' perception of the relevance of different activities in initial teacher training are observed, along with its relationship with the sub-domains of the MTESK model, and the statistical outcome of the Borda method.

Table 7 presents the results of subset 3. It is detected that mathematical knowledge for teaching is the least well-evaluated knowledge by MTEs.

Table 7. Statistical summary of the items of subset 3 and their relationship with the MTESK model. Scale: 1 (lowest level) – 4 (highest level).

Items (<i>MTESK subdomain</i>)	Min	Median	Max	SD
K1. Management of classroom climate in initial teacher training. (<i>not included</i>)	1	3	4	0.93
K2. Learning dynamics of student teachers. (<i>KFLM teachers</i>)	1	3	4	0.76
K3. Common difficulties in the learning of student teachers. (<i>KFLM teachers</i>)	1	3	4	0.67
K4. Professional development of mathematics teachers. (<i>KoMER</i>)	1	3	4	0.80
K5. Mathematical knowledge for teaching. (<i>MK</i>)	1	3	3	0.58
K6. Knowledge of teaching strategies in initial teacher training. (<i>KMT teachers</i>)	1	3	4	0.70

K7. Knowledge of how and why to assess in initial teacher training. (<i>KFLM teachers</i>)	1	3	4	0.78
K8. Knowledge of initial teacher training standards. (<i>KMLS</i>)	1	3	4	0.81

Note. In this table, the items related to MTEs' knowledge are observed, their relationship with the sub-domains of the MTESK model, and a statistical description through the minimum, median, maximum, and standard deviation.

Table 8 summarises the responses to the items of subset 4. As we can see, the tasks for which MTEs feel more prepared are RL2 and RL5, respectively. No differences in the behaviour of the MTEs are observed in the rest of the tasks.

Table 8. Statistical summary of the items of subset 4 and their relationship with the MTESK model. Scale: (1) *I don't feel prepared*, (2) *I feel somewhat unprepared*, (3) *I feel prepared*, (4) *I feel very well prepared*.

Items (MTESK subdomain)	Min	Me-dian	Max	SD
RL1. Analysing the current curriculum materials in mathematics to work with your students. (<i>KMLS</i>)	1	3	4	0.66
RL2. Exploring manipulative materials (concrete or digital) with your students to solve mathematical problems. (<i>KMT teachers</i>)	2	4	4	0.59
RL3. Developing a class based on the mathematical knowledge and skills that your students exhibit during the session. (<i>KMT teachers and KFLM teachers</i>)	1	3	4	0.63
RL4. Teaching how to develop mathematical skills in children (problem-solving, reasoning...). (<i>KMT students</i>)	1	3	4	0.73
RL5. Modelling or demonstrating specific mathematical practices that students need to learn to perform as teachers. (<i>KPM</i>)	2	3	4	0.61
RL6. Analysing the teaching practice in mathematics of another teacher. (<i>MK and PCK</i>)	1	3	4	0.80
RL7. Exploring research literature regarding the teaching or learning of mathematics with your students. (<i>KoMER</i>)	1	3	4	0.78

Note. In this table, the items related to readiness level as MTE are observed, their relationship with the sub-domains of the MTESK model, and a statistical description through the minimum, median, maximum, and standard deviation.

4.2. Relationship between items

This section contains a joint description of the subsets considered. As a result, it is found that the MTEs are distributed in 3 groups described at the end of the section. Three groups of MTEs were selected because this was the optimal number of groups from which a detailed description of each group could be made. Note, as mentioned

above, that due to the ranking nature of the items of subset 2, they were not considered in the joint description and therefore nor in the elaboration of the groups. As a preview of the results, MTEs are distributed in 3 clusters corresponding to high perceived competence (cluster 1), to standard perceived competence (cluster 2), and finally to low perceived competence (cluster 3).

The value of the KMO test was .79 and the p-value for Bartlett's test was .000, indicating that the application of PCA is appropriate. Thus, 21 principal components were identified. Table 9 shows the loadings of the different items on the first two principal components. Since a higher loading indicates a greater degree of association, it is observed that almost every item of subset 1 is represented in the second principal component which explains 2.10 % of the variance, while items concerning subsets 3 and 4 are mainly in the first principal component which explains 4.95 % of the variance.

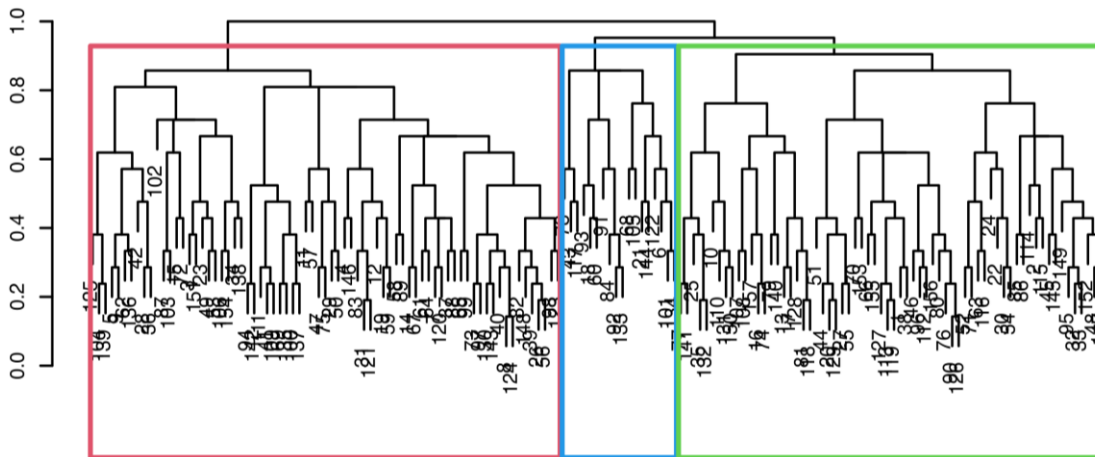
Table 9. Summary of the rotated component matrix.

Item	Component		Item	Component		Item	Component	
	1	2		1	2		1	2
B1	-.001	-.269	K1	.176	-.204	RL1	.247	.159
B2	.031	-.161	K2	.293	-.134	RL2	.219	.270
B3	.141	-.424	K3	.172	.063	RL3	.253	.077
B4	.033	-.245	K4	.261	-.079	RL4	.290	.019
B5	.016	-.367	K5	.174	.125	RL5	.286	.033
B6	.076	-.457	K6	.287	-.033	RL6	.297	.126
			K7	.279	-.152	RL7	.245	.292
			K8	.283	-.047			

Note. In this table, the presence of the different items considered in the study is shown across the two principal components.

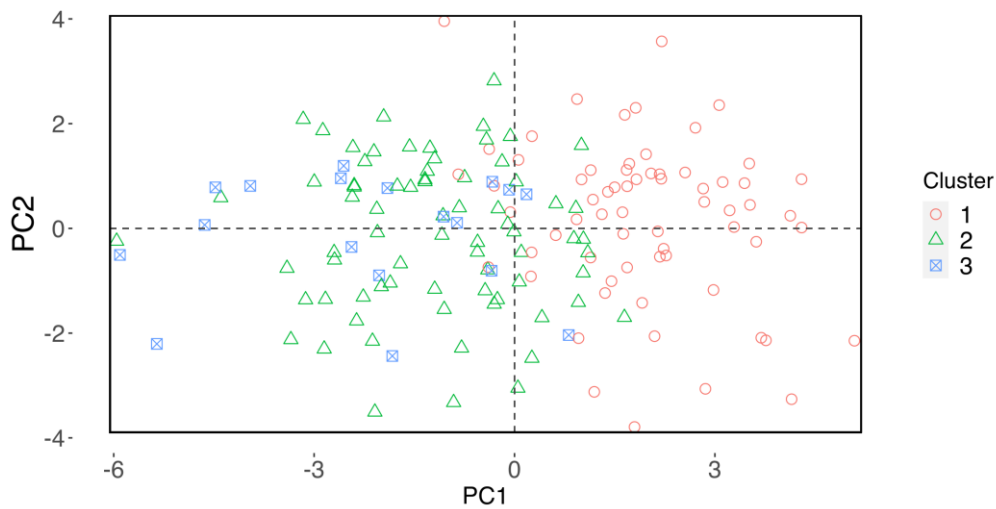
After applying hierarchical clustering, described on Figures 2 and 3, it is concluded that the MTEs can be divided into three clusters differentiated by colours in both figures. In Figure 3, it can be observed how the MTEs are distributed with respect to the two principal components considered. A similar distribution around the origin is observed in the second component, which means beliefs add no value when clustering MTEs.

Figure 2. Cluster analysis dendrogram of three clusters separated by colour. Red, green, and blue represent clusters 1, 2 and 3 respectively.



Note. The figure shows how the MTEs are gradually related by clusters through the dendrogram. Additionally, it outlines the three clusters of MTEs considered in the study.

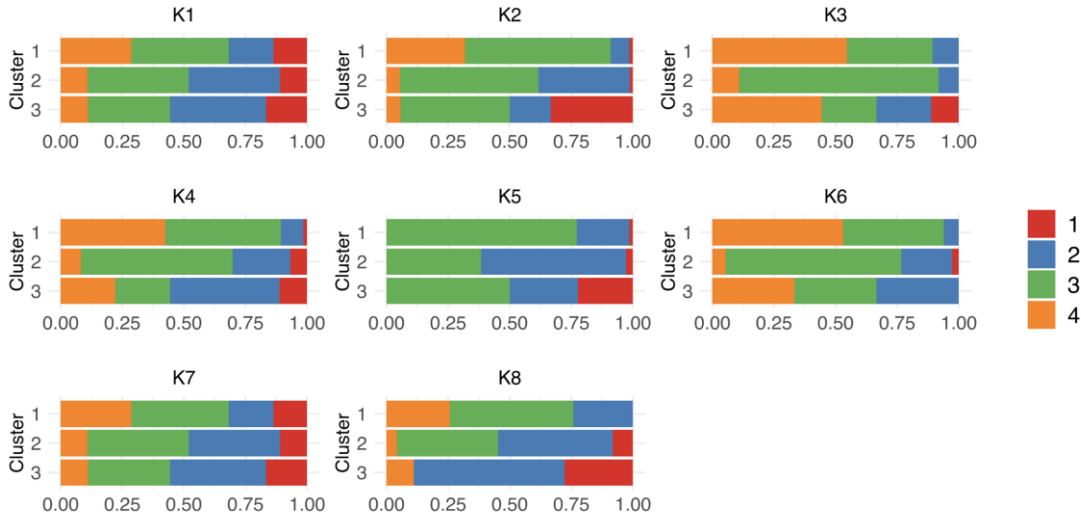
Figure 3. Graphic representation of the three clusters through the two principal components.



Note The figure shows how the MTEs are distributed in the coordinate plane formed by the first two principal components. Additionally, each MTE is characterized based on the cluster to which it belongs.

Figure 4 displays the percentage of MTEs who indicated each of the possible values for the items of subset 3, considering the cluster they belong to.

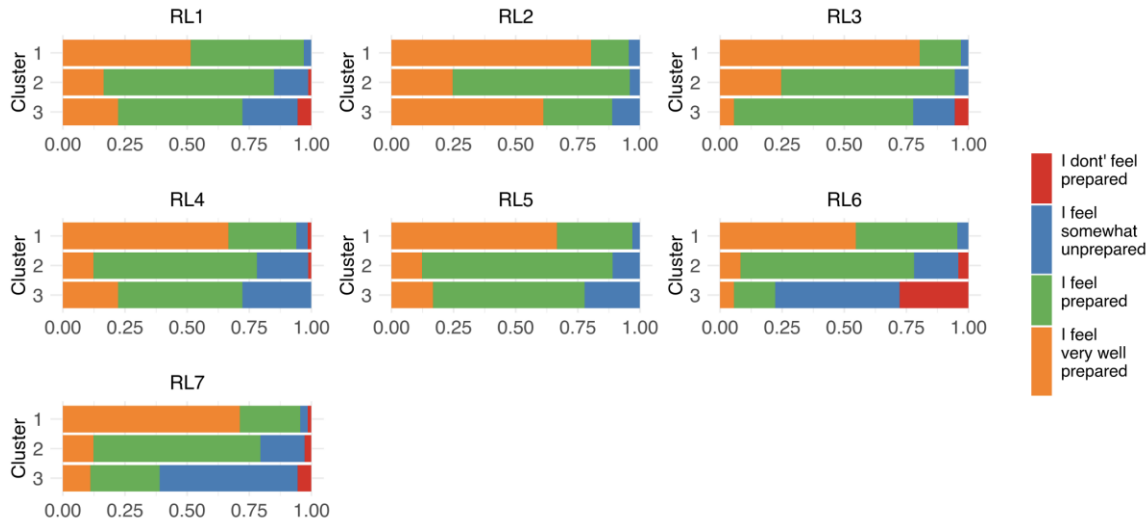
Figure 4. Answers (in Likert-scale 1-4) to the items of subset 3, depending on the clusters.



Note. The figure displays 8 relativized statistical graphs describing each of the clusters considered based on the responses to the MTEs' knowledge items.

The percentage of MTEs who indicated each of the possible values for the items of subset 4 are depicted in Figure 5, considering the clusters.

Figure 5. Answers (in Likert-scale) to the items of subset 4, depending on the clusters.



Note. The figure displays 7 relativized statistical graphs describing each of the clusters considered based on the responses to the readiness level as MTE items.

Based on Figures 4 and 5, the following characterisation of each cluster is assumed:

1. Cluster 1 consists of 66 MTEs who perceive their professional knowledge as high and exhibit an excellent readiness level to carry out their teaching practice. They excel in knowledge related to learning dynamics [K2], mathematics [K5], and standards for initial teacher training [K8]. Additionally, they consider themselves well-prepared to analyse other teachers' mathematics teaching practice [RL6] and to explore research in mathematics education with their students [RL7].
2. Cluster 2 consists of 73 MTEs who believe they possess good knowledge and readiness level in general, but with lower levels than MTEs in cluster 1. These MTEs stand out for their good knowledge regarding common student learning difficulties for teachers [K3] compared to the rest of the MTEs.
3. Cluster 3 includes 18 MTEs who exhibit lower knowledge and readiness level. These MTEs are characterised for having low knowledge about common student learning difficulties for teachers [K3], professional development of mathematics teachers [K4], and standards for initial teacher training [K8]. Additionally, they have a low readiness level in analysing other teachers' mathematics teaching practice [RL6] and exploring research in mathematics education with their students [RL7].

Apart from the clusters, we can observe (Figure 4) that none of the MTEs consider their mathematical knowledge [K5] to be very high and (Figure 5) that MTEs do not feel well prepared for analysing other teachers' teaching practices [RL6].

Discussion and conclusions

Regarding the first objective, the items related to KoT, KPM, KMT-teachers, and KFLM-teachers subdomains of the MTESK model receive higher ratings. Pascual et al. (2019) observed how MTEs showed a greater knowledge of the KMT and KFLM subdomains of the MTSK model. Concerning the second objective, we determined three groups of MTEs. The MTEs of these groups exhibit similar behaviour regarding their beliefs, but not when it comes to their knowledge and readiness level. Lovin et al. (2012) noted this aspect with MTEs from the American context. In the first group, MTEs excel in items related to the KFLM-teachers, KMLS, KMT-teachers, KoMER

subdomains, and the overarching domain related to mathematical content. The second group exhibits good behaviour with items related to KFLM-teachers subdomain. In the third group, MTEs highlighted the lack of knowledge and readiness they experienced during their teaching practice. These three groups constitute a relevant finding of the current research, since only two groups had been considered in previous literature: those who lacked preparation for teaching activities and those who did not (Appova & Taylor, 2019). Following the line by Sikko and Grimeland (2021), the existence of the third group can be attributed to a lack of programmes for training MTEs. It may also be due to the need for teaching staff faced by Spanish universities, and therefore, the hiring of MTEs without training for university teaching (Bolívar Botía & Caballero Rodríguez, 2008; García Calavia, 2015). However, since a small sample of MTEs participated in this study, perhaps the educational background or experience in the profession is also influencing this low perceived competence. In this way, regarding the research of Olanoff et al. (2021) and the low readiness level in aspects like RL6 and RL7 it would be interesting to form communities of practice by serving to these identified groups to support each other.

The amount of variance explained by the questionnaire used is a limitation of the study. It represents a moderate amount of variance and reinforces the fact that the questionnaire has a complex structure. Therefore, it is necessary to analyse these results using other different instruments, for instance, observations or interviews.

In our future research, we will analyse the reasons why MTEs consider their mathematical knowledge (K5) to be lower compared to the rest of their knowledge. Perhaps this aspect is influenced by the didactician profile of the MTEs surveyed in the questionnaire. We will also analyse the possible relationship between the educational background, the experience as a MTE and the lack of readiness. It will be also examined whether results for MTEs in Spain are consistent in other contexts such as Italy or Latin America.

Author contributions

Pablo Giadas: conceptualization, methodology, formal analysis, validation, visualization, writing-original, writing-review and editing.

Laura Muñiz-Rodríguez: conceptualization, validation, visualization, writing-review and editing.

Luis J. Rodríguez-Muñiz: conceptualization, validation, writing-review and editing, funding acquisition, project administration.

All authors have read and agreed to the published version of the manuscript.

Artificial intelligence

The authors declare that no artificial intelligence has been used in the research or writing the article.

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Informed consent statement

Informed consent was obtained from all research participants.

Conflicts of interest

The authors declare no conflicts of interest.

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