Intervention and intended change: A search for mathematical views that can be agreed upon as didactically desirable

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Abstract: There is a broad consensus in mathematics beliefs research that teachers' beliefs influence their classroom practice significantly. Therefore, several studies have attempted to change beliefs by using specific mathematical-didactical interventions. Aside from being interpreted as stable, or difficult to change, beliefs are generally classified as descriptive. An intervention to change beliefs, however, implies some predefined goals that are considered didactically desirable and, therefore, normative. Consequently, the question arises which beliefs can be considered as desirable, and which should be changed through targeted interventions. As a starting point for this discussion, this paper elaborates a potential theoretical framework for the domain-specific mathematical normative approach to beliefs. This framework is based on the ideas of the "Nature of Science" research and is referred to as "Nature of Mathematics". Additionally, to support the theoretical framework, representative beliefs are cited that have either been found to be beneficial or detrimental to learning in other studies. Through the combination of the theoretically based framework and empirically evaluated desirable and non-desirable beliefs, this paper facilitates the localization and differentiation of mathematical beliefs research, especially those which focus on beliefs-change through intervention, while opening a discourse on beliefs' normativity.

Keywords: beliefs, beliefs change, nature of mathematics, mathematics education

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1 Introduction

Mathematics beliefs research has consistently shown that teachers' beliefs have a strong influence on their classroom practices (Eichler et al., 2023, p. 1490). As Skott (2015) points out, beliefs are generally interpreted as action-leading or as the default of classroom practice since they filter individual perceptions like lenses (Philipp, 2007). Furthermore, educators' beliefs are seen as having a significant impact on students' beliefs, which, for instance, influence their learning strategies (Eichler & Erens, 2015). In light of these findings, several studies (e.g., Eichler et al., 2023; Safrudiannur & Rott, 2022; Weygandt, 2021; Fives & Buehl, 2016) have been conducted to examine the potential changes in students', pre-service teachers' and teachers' beliefs with or without interventions. When attempting to change beliefs through didactic interventions, certain beliefs may be considered as desirable, i.e., useful for learning and teaching mathematics, and others as hindering (e.g., “unproductive beliefs” - Jankvist & Niss, 2018; “negative beliefs” - Di Martino & Sabena, 2010). An implicit normative interpretation of beliefs, however, is incompatible with the descriptive nature of the theoretical construct. This necessitates a further theoretical development of the beliefs concept or a concretization of this already “messy construct” (Pajares, 1992).

Research on the Nature of Science (NOS), which can be interpreted as a domain-specific scientific epistemological research field, and its similarities and differences with domain-general epistemological beliefs research (Neumann & Kremer, 2013) provides a valuable starting point for examining the distinction between descriptive and normative approaches.

According to Hofer and Pintrich (1997), epistemological beliefs, defined as “individual representations of knowledge and knowing” (Mason & Bromme, 2010), can evolve from a naïve to a sophisticated form through education and experience (Schreck et al., 2023, p. 1637). For example, domain-general beliefs that perceive knowledge as absolute or unchangeable are considered naïve, whereas beliefs such as “knowledge is relative” or “knowledge is uncertain” are regarded as sophisticated (ibid.). Furthermore, Schreck et al. (2023, p. 1637) emphasize that beliefs are only truly sophisticated if they are supported by sophisticated justifications. This classification suggests an implicit hierarchical structure, which is typically considered descriptive rather than normative (Neumann & Kremer, 2013).

Neumann and Kremer (2013) discuss several distinguishing features between NOS research and epistemological beliefs research, including discipline specificity, content, personal versus scientific perspectives, and the distinction between knowledge and views. Notably, they also highlight the difference between descriptive (epistemological beliefs) and normative (NOS) approaches.

This article builds on this discussion by analysing domain-specific mathematical beliefs and views, focusing on the descriptive-normative distinction. It aims to establish a potential foundation for interventions by discussing a possible normative theoretical framework and exemplarily identifying empirically supported desirable and non-desirable views from a didactical perspective through a literature analysis. Drawing on Østergaard’s (2024) systematic review, which examines students’ beliefs about mathematics, their empirical tendencies, and their valuation, this article primarily investigates the valuation of beliefs as a basis for evaluating desirable views in educational mathematical contexts.

2 Theoretical background: NOS

The research in NOS is characterized by an emphasis on the features of scientific knowledge and the process of scientific inquiry (Lederman, 2007; McComas & Olson, 1998; Osborne et al., 2003; Schwartz et al., 2008; Neumann & Kremer, 2013). There is also a discussion of how to define NOS in science didactics. Specifically, whether to refer to "the", i.e., a single NOS, has been a matter of debate (e.g., Alters, 1997; Smith et al., 1997). Defining this topic is difficult due to its broad scope. In Kircher (2010), epistemological, scientific-theoretical, and scientific-ethical issues are addressed.

The didactic approach, however, does not focus on which properties are specific to science, but rather on those aspects that are relevant to teaching science in the classroom (Neumann & Kremer, 2013, p. 211). The first development of these characteristics or aspects was achieved by McComas and Olson (1998) and Osborne et al. (2003). McComas and Olson (1998) examined educational policy documents in five English-speaking countries and identified epistemological components in these documents that were widely accepted by the science-didactic community (see Table 1). An alternative approach can be found in Osborne et al. (2003), who surveyed experts from science, history, philosophy, sociology, and teaching staff as part of a Delphi-study on "ideas about science". As shown in Table 1, the results are highly similar to those of McComas and Olson (1998).

Lederman (2006, p. 303; see also Lederman et al., 2002; Lederman, 2007) refers to "the epistemology of science", "scientific knowledge as a method of knowing", or "values and beliefs inherent in scientific knowledge" as typical reference fields for NOS development of scientific knowledge and examines NOS aspects of students in relation to the following three questions:

- Is knowledge of the aspect of NOS accessible to students (can they learn and understand)?
- Is there general consensus about the aspect of NOS?
- Is it useful for all citizens to understand the aspect of NOS? (Lederman, 2006, p. 304)

Lederman's (2006) aspects, e.g., “scientific knowledge is tentative”, refer to the properties of scientific knowledge and are called “aspects of nature of science”.

Additionally, Schwartz et al. (2008) discuss the characteristics of scientific knowledge acquisition in the context of the “Nature of Scientific Inquiry” (NOSI). Neumann and Kremer (2013) suggest that overlaps in the list of aspects from McComas and Olson (1998), Osborne et al. (2003), NOS (Lederman, 2006) and NOSI (Schwartz et al., 2008) may indicate a certain level of consensus (Table 1).

Table 1. Comparison of exemplary NOS aspects (based on Neumann & Kremer, 2013, p. 215)

Category	**McComas & Olson (1998)**	**Osborne et al. (2003)**	**Lederman – NOS (2006)**	**Schwartz et al. – NOSI (2008)**
Tentativeness	Scientific knowledge is tentative	Science and certainty	Scientific knowledge is tentative	-
Empirically based approach	Science relies on empirical evidence	Analysis and Interpretation of Data	Distinction between observations and inferences / [scientific knowledge is] empirically based	Justification of scientific knowledge / Distinctions between data and evidence
Scientific quality criteria for research	Scientists require replicability and truthful reporting	Scientific method and critical testing	-	-
Obtaining knowledge as a goal	Science is an attempt to explain phenomena	Hypothesis and prediction	-	Scientific questions guide investigations
Creativity	Scientists are creative	Creativity / Science and Questioning	[scientific knowledge] necessarily involves human inference, imagination, and creativity	Community of practice
Social embeddedness	Science is part of social tradition	Cooperation and collaboration in the development of scientific knowledge	[scientific knowledge is] subjective	Community of practice

It is widely acknowledged within the scientific-didactic community that an adequate understanding of NOS is an essential component of a qualitative science education, as stated by Neumann and Kremer (2013, p.223). In order to implement this normative approach, it is necessary to identify aspects that are consensually agreed upon, can be found in educational policy documents, and meet the three criteria outlined by Lederman (2006) (Neumann & Kremer, 2013, p. 224). Based on a subject didactic perspective, NOS research focuses on the scientific views of students and teachers and how those views can be translated into an adequate understanding, which is normatively determined in advance (ibid.).

Neumann and Kremer (2013) indicate that NOS research and epistemological beliefs research, as described in Section 1, have many points of commonality. However, besides its domain-specificity, NOS research is distinguished by its normative approach, which involves observing predetermined aspects as an integral part of scientific education. It is important to differentiate between these two directions based on their content and terminology. A detailed discussion of domain-specific and domain-general epistemological beliefs can be found for example, in Schommer-Aikins and Duell (2013) or Urhahne and Kremer (2023). The German terms "Ansichten" and "Überzeugungen" may be used to describe (epistemological) beliefs, whereas "Vorvorstellungen" or "Alltagsvorstellungen" may represent normative beliefs (ibid.). Within this text, the term "beliefs" refers to descriptive interpretations, while "views" refers to normative ones. The extent to which normative interpretations of beliefs can already be classified as knowledge depends on further development of the potential theoretical framework of NOM-research. Based upon Brock and Park (2022), it is possible to distinguish between NOS beliefs and NOS knowledge, where beliefs are defined as the mental state of accepting an NOS proposition without clear truth status or sufficient justification, and knowledge as a justified true belief. From the didactical viewpoint, "true" beliefs could be interpreted as desirable views within NOM-research.

3 Research context and research question

With the above considerations in mind, the aim of this article is to facilitate the localization and differentiation of mathematical beliefs research, especially those which focus on beliefs-change through intervention, while opening a discourse on beliefs' normativity. To achieve this, the following research question will be addressed:

- Which empirically evaluated exemplary aspects, and the embedded desirable and non-desirable domain-specific mathematical views of a potential theoretical framework can be analysed through an exploratory, interpretative literature analysis to propose a normative foundation for interventions and intended belief change in mathematical beliefs research?

4 Methodology

Identifying views that are considered desirable from a mathematics-didactical perspective is a challenging task, as they should be widely accepted within the mathematics education community. A possible starting point for specifying relevant aspects and their therein embedded views can be found in the already mentioned NOS research (Section 2).

To identify such aspects within mathematics education, this paper builds on Østergaard's (2024) systematic literature review, which analysed 292 studies published in peer-reviewed journals over the last 20 years, focusing on primary and secondary students' beliefs about the nature of mathematics. Of these, 18 studies met all inclusion criteria, and eight specifically examined the quality of students' beliefs (Østergaard, 2024, p. 51).

Furthermore, academic databases and search engines were used to locate relevant references with empirical findings for this paper. Additionally, all online-accessible MAVI proceedings since the inception of the research community, as well as the CERME-13 conference proceedings of the thematic working group "Mathematics Teacher Knowledge, Beliefs, and Identity" were reviewed. This process led to the identification of further relevant sources, whose references were also examined and analysed.

Search terms such as "change", "intervention", and indicators of a normative interpretation of beliefs were used to guide the search. To ensure the reliability and quality of the sources, only peer-reviewed journal articles were included. The researcher initially skimmed the texts to assess their relevance to the objectives of this study. The overall aim of the review was to explore how existing research contributes to the understanding of potential normative interpretations of beliefs within a mathematics education context.

To establish a curricular connection, the Austrian secondary school mathematics curriculum was analysed following the methodology described by McComas and Olson (1998) for examining scientific curricula and policy papers.

Following this exploratory approach, the empirical findings from the selected literature were interpreted within the normative framework of beliefs research. In summary, the methodology of this article can be determined as an exploratory, interpretative literature analysis.

5 Results and discussion

5.1 NOM – a theoretical normative framework

As mentioned in the introduction, the intent to change beliefs requires a specific objective - namely, a normative goal. In mathematical-didactical beliefs-research, however, there is no generally accepted theoretical framework or theoretical classification of normative approaches.

The NOS research provides a possible starting point here, stating that an adequate understanding of a consensually agreed-upon list of NOS aspects is an essential component of science education (Neumann & Kremer, 2013).

By addressing this gap within mathematic-didactics, “Nature of Mathematics”-research (NOM for short) attempts to provide a normative theoretical framework based on examples of desirable and non-desirable views and to initiate a debate on this issue. As with NOS research, NOM-research examines aspects of mathematics education that are perceived as adequate and, therefore, relevant to students' future lives. These aspects may be identified in both epistemological beliefs-research, e.g., certainty, simplicity, sources and justifications of knowledge, as well as NOS research, e.g., creativity, social integration, mathematics in the structure of society and culture. Several additional mathematical domain-specific considerations are presented, for instance, by Goldin (2002) and by Liu and Liu (2011). It should be noted that NOM is not limited to epistemological issues but also addresses learning-related topics (for example see Schoenfeld, 1992, p. 359).

5.2 Normative views concerning NOM

In the first step of the analysis, the findings of Østergaard (2024) are interpreted within the given theoretical framework. According to her findings, beliefs about mathematics as a discipline are evaluated based on their quality along a spectrum. The majority of the articles present a spectrum that ranges from a "static, rigid, rule-based discipline to a dynamic, relativistic, and applicable 'science of patterns'" (Østergaard, 2024, p. 54). According to Grouws (1996), this spectrum ranges from "dualistic" to "relativistic." Grigutsch (1998) describes these two poles as "schema-orientation" and "process/application-orientation." Additionally, Gattermann et al. (2012) categorize students' views as either "naive" or "sophisticated," where "sophisticated" refers to deep-processing learning (Østergaard, 2024, p. 55). There is a consensus, according to Østergaard (2024, p. 59), that beliefs belonging to the dualistic pole hinder learning. In contrast, the relativistic end of the spectrum is considered beneficial for learning.

To interpret these findings within the theoretical framework of normative mathematical views, and for the further analysis of desirable views within the NOM framework, it can be stated that dynamic, relativistic, application-oriented, process-oriented, or otherwise sophisticated views are generally regarded as conducive to learning and can therefore be considered normative goals for intervention studies. It should be noted, however, that an adequate understanding of mathematics cannot only include dynamic aspects, as this scientific discipline also covers static, schematic, recipe-like processes. Therefore, students should be given the opportunity to develop a comprehensive understanding of mathematics by gaining experience with both dynamic and static content. In this regard, Jankvist (2015) defines ideal beliefs as those that are based on examples, experiences, reasoning, etc. (Østergaard, 2024, p. 60).

Within the exploratory, interpretative literature analysis applied in this research project (see Section 4), the findings from the analysed references are interpreted within the given theoretical framework, allowing for the identification of desirable (normative) domain-specific views.

Based on Conley et al. 's (2004, see Table 2) domain-general definition of naive and sophisticated epistemological beliefs, Beumann and Geisler (2022) demonstrate that by implementing an intervention for mathematical experimentation in out-of-school learning situations, secondary school students' beliefs transform from a naive to a sophisticated level. As argued by the authors (ibid.), sophisticated beliefs can positively influence learning, whereas naive views can be hindering.

However, it is not possible to provide a mathematical-normative interpretation for all subcategories (“Certainty of Knowledge”, “Simplicity of Knowledge”, “Source of Knowledge”, “Justification of Knowing”; Hofer & Pintrich, 1997) of the domain-general epistemological beliefs theory. Rott et al. (2015) and Rott (2020) illustrate this point by demonstrating that mathematicians can make highly persuasive arguments in support of the theoretical stability of mathematics which is seen as naive within Hofer and Pintrich (1997).

Beumann and Geisler (2022, p.2) argue that the following epistemological views inhibit or promote learning, and therefore, according to the theoretical NOM-framework presented within this article, should be considered non-desirable or desirable goals for future interventions:

Table 2. Desirable and non-desirable mathematical-epistemological views based on Beumann and Geisler (2022) and Conley et al. (2004)

Epistemological dimensions	**Epistemological subcategories**	**Desirable (sophisticated) views**	**Non-desirable (naiv) views**
Nature of knowledge	Certainty of knowledge	There are several possible answers / solutions to complex problems.	There is only one possible answer/solution.
Nature of knowing	Source of knowledge	It is possible for an individual to actively construct knowledge on their own.	Knowledge is only provided by authorities.
	Justification of knowing	A systematic evaluation of knowledge can provide justification for knowledge.	Only authorities can justify knowledge.

Similar formulations of the above mentioned non-desirable views on the epistemology of mathematics can also be found in Schoenfeld (1992, p. 359; “typical student beliefs about the Nature of Mathematics”) and the literature referred to therein.

1. Mathematics problems have one and only one right answer.
2. There is only one correct way to solve any mathematics problem – usually the rule the teacher has most recently demonstrated to the class.
3. Ordinary students cannot expect to understand mathematics: they expect simply to memorize it and apply what they have learned mechanically and without understanding.
4. Mathematics is a solitary activity done by individuals in isolation.
5. Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.
6. The mathematics learned in school has little or nothing to do with the real world.
7. Formal proof is irrelevant to processes of discovery or intervention.

As Bernack (2011, p. 25) reports, several studies and teacher programs have demonstrated an effected change in beliefs, primarily towards mathematics as a process and toward a constructivist view of mathematics learning, by enabling participants to experience being a learner themselves by involving problem-solving tasks, open-ended problems, or new mathematical material. It should be noted that these didactical "objectives" can also be interpreted as desirable, whereas the related views that Schoenfeld (1992) mentions (e.g., 1, 2 and 5) can be interpreted as non-desirable within the potential NOM- framework.

In addition, Schoenfeld (1992, p. 359) addresses the view that students possess predetermined fixed abilities that cannot be changed by school interventions or experiences (3).The belief that mathematical abilities and skills are anthropologically determined, i.e. that mathematics can only be understood by "brilliant people" or "math people", encourages so-called "fixed mindsets" among students (Chestnut et al. 2018, p. 4 sq.; Anderson et al. 2018, p. 3). People who have internalized these "mindsets" believe that their abilities and skills are constant and cannot be improved. It is therefore common for them to memorize rules and procedures more than those who believe their intelligence and problem-solving ability can be improved ("growth mindset"). Many studies (see Anderson et al., 2018 as well as the literature cited therein) have already demonstrated that the concept of "math people" is untenable and that all learners are capable of understanding math.

Another aspect of the theoretical NOM-framework can be found within Schoenfeld's (1992) point 4, which can be expanded to the "image of mathematicians". Grevholm (2010) asserts that it is crucial for students to gain an adequate impression of mathematicians, their way of working, their goals, etc. before considering mathematics studies or becoming mathematicians. As with NOS research, the process of inquiry and the characteristics of "the mathematical method" for gaining and securing knowledge (7) should be components of the NOM-framework.

Based on an analysis of Austrian secondary school curricula, it is evident that the legal mathematical requirements for regular classes partially incorporate the NOM-aspects mentioned earlier, and that most of these relate to the three fundamental experiences discussed by Winter (1995). As an example, SEK 1 (students from ten to fourteen) curriculum requires students to gain an understanding of the history of mathematics and the influence of its personalities in order to demonstrate that mathematics is not a static concept but rather is constantly evolving. The SEK 2 (students from fourteen to eighteen) curriculum includes an emphasis on the epistemological aspects of mathematics to facilitate the understanding of mathematics as a way of understanding the world. Furthermore, mathematics should also be regarded as a school of specific thinking that stimulates the imagination and develops creativity. In addition, both curricula emphasize the importance of the application of mathematics. Niss and Højgaard (2011, p. 2019), for example, have also identified these aspects in the Danish mathematics competence framework: "the actual application of mathematics in other subject and practice areas", "the historical evolution of mathematics, internally as well as in social context", "the nature of mathematics as a subject" (Østergaard, 2024, p. 60). As a result of this consistency, it may be possible to establish a consensus regarding these NOM-aspects. There is, however, a need to evaluate this hypothesis within the context of an analysis of further educational mathematics policy papers from different countries (see Mcomas & Olson, 1998). These aspects are incorporated into educational curricula and are thus considered desirable from the educational policy perspective.

To answer the research question posed in Section 3, the exploratory, interpretative literature analysis, combined with findings from Østergaard (2024), exemplarily identifies aspects (Section 5.1) and the therein embedded desirable and non-desirable views (Section 5.2), which can serve as a potential starting point for further discussions on the normativity of beliefs, as well as a theoretical foundation for interventions aimed at changing beliefs from a mathematics-didactical perspective.

6 Conclusion and limitations

Based on the literature review, it has been concluded that changing beliefs within mathematics-didactic approaches in order to achieve certain predetermined desirable outcomes is commonly theoretically positioned in the rather descriptive concept of domain-specific epistemology-research. Comparatively to NOS research, this article proposes a normative approach to locate the quasi-hierarchical but implicit normative interpretations of beliefs within the explicit normative framework of NOM-research.

Regarding normativity of beliefs, it is essential to note that cultural differences should be considered. Thus, the question arises whether there are universally desirable and non-desirable views. Using the methodology used by McComas and Olson (1998), a future analysis of educational policy documents from different countries and cultures might be a first approach to this question. Moreover, future studies on the normativity of beliefs could examine very specific didactical contexts such as problem-solving to identify desirable and non-desirable viewpoints within these settings (see Schoenfeld, 1992).

This study's findings are limited by its selection of exemplary views (NOM-aspects), which cannot claim to be exhaustive or to represent a consensus in mathematics didactics. However, by combining these aspects with those of Østergaard (2024) and with the previously analysed theoretical framework of NOM-research based on NOS research, the results of this article can be viewed as a starting point for further evaluations of normative approaches to domain-specific mathematical views.

In the author's opinion, it is essential to clarify that within the possible framework of normative beliefs, it is not the intention to define what students and prospective teachers should believe, rather to determine which beliefs could be beneficial in specific contexts thus may serve as a basis for intervention.

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