Teaching as a system: Insights from a teacher professional development program

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Abstract: The influence of a change in teachers' practice on teachers' knowledge and beliefs has seldom been investigated in the literature. In the present work, we aim to explore the influence of a change in practice forced by participating in a Teacher Professional Development (TPD) program on beliefs, knowledge and practice. To this end, we consider the case of Katia, a teacher involved in a TPD program focused on mathematical modelling with digital technologies, and we employ the teaching system model to document the evolution of her practice, beliefs and knowledge. The performed analysis shows a change of practice induced by the TPD program, highlighting its influence on beliefs and the essential role of knowledge in sustaining teachers in further expanding their practice.

Keywords: teaching system, teacher professional development, digital technology, math views.

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1 Introduction

The relationship between teachers' practice, knowledge and beliefs has been known for a long time. While the majority of works explored the influence of teachers' knowledge and beliefs on teachers' practice (Ball, 1988; Leder et al., 2002), other works have shown that teachers' beliefs change can also be a consequence of a change in practice and knowledge (Guskey, 1986; Liljedhal, 2016). On the path of such works, Brunetto et al. (2022) proposed a theoretical model, on which we will elaborate in Section 2, that accounts for the systemic nature of teaching, the teaching system model, made of practice, beliefs, knowledge and their connections. They employed this model to document the evolution of the teaching system of three teachers at the university level as a consequence of the change of practice from in person teaching to online teaching during the pandemic.

This work is part of a more exhaustive research (Brunetto & Dello Iacono, 2023; Bassi et al., 2024) which aimed at investigating mathematics teachers' use of digital technologies in design learning activities during a Teacher Professional Development (TPD) program. Recent research shows the effectiveness of TPD programs in supporting the development of knowledge, beliefs and practices (e.g., Sztajn et al.,

2017) and on teacher's technology-related beliefs (e.g., Thurm & Barzel, 2020). However, more research is needed on these issues, particularly with respect to the intertwining between practice, beliefs and knowledge.

In the present work, on the path of Brunetto et al. (2022), we aim to explore if and how the teaching system evolves as a consequence of the change of practice introduced by a TPD program, in which teachers are involved as learners of mathematics education in a constructivist environment. To this end, we consider the case of Katia, one of the teachers who participated in the TPD program.

The paper is structured as follows: in Section 2, we present the theoretical framework and the research questions; in Section 3, we report on the research context, the data gathering, and the method of analysis. In Section 4, we report the results, and finally, in Section 5, we discuss the findings according to the research questions.

2 Theoretical framework

We consider the teaching system model proposed by Brunetto et al. (2022) as a lens of analysis. The teaching system is made of three elements: practice (P), beliefs system (B) and knowledge (K), and the interactions between the elements.

We share the definition of teacher practice (P) proposed in Da Ponte & Chapman (2006) as the work teachers do when carrying out their professional tasks. Teachers' practice includes, for example, lesson development, selection of examples, choice of lesson format, the tasks proposed, the use of digital tools, lesson planning and the choice of classroom setting (Dougherty, 1990; Brunetto et al., 2022). Moreover, participation in TPD programs is part of teachers' work. Thus, we can consider it as part of teachers' practice.

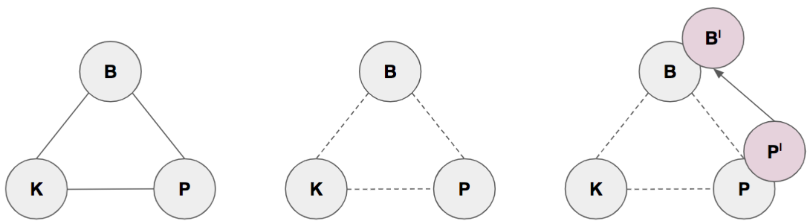
The second element of the teaching system corresponds to teachers' beliefs, which have a predominant role in influencing what and how teachers teach (Lortie, 1975; Ball, 1988). Since "beliefs always come in sets or groups, never in complete independence of one another" (Green, 1971, p. 41), we refer to a belief system (B) including interest, values and other affective dimensions. Since beliefs are hidden (Leder et al., 2002), they can be inferred from observable behavioural consequences (Di Martino & Zan, 2011), like a change in teachers' practice (Leder et al., 2002).

Concerning teachers' knowledge (K), we resort to Pedagogical Content Knowledge (PCK) (Shulman, 1986), which refers to teachers' understanding of how to effectively teach specific subject matter Pedagogical Content Knowledge (PCK) (Shulman, 1986) which refers to teachers' understanding of how to teach specific subject matter effectively. In particular, for technological knowledge, we consider the TPACK model (Mishra & Koehler, 2006), which extends the Pedagogical Content Knowledge to describe teachers integrating technology into their pedagogy.

As highlighted in Brunetto et al. (2022), the interaction between the system's three elements, beliefs, practice and knowledge, occurs when a change in one element provokes a change in the other. In this regard, Eichler et al. (2023) show considerable changes in mathematics teachers' central beliefs due to one-year practical experience. Namely, the fact that a change in practice provoked a change in the belief system indicates the interaction between these two elements. We specify that change has different meanings concerning practice, beliefs and knowledge. Roughly speaking, a belief change might imply that a new belief emerges, new connections in the belief system are established, or an implicit belief becomes explicit. A change in knowledge occurs when teachers acquire new facts, while a change in practice can be, for example, a change in how a lesson is planned or carried out.

The teaching system can be represented as a connected graph (see Figure 1) composed of three elements: Practice (P), Beliefs (B) and Knowledge (K).

Figure 1. Teaching system's change representation



A change in P, B or K is identified with a prime P^I, B^I or K^I. The initial teaching system, once perturbed, is identified by dashed lines (Figure 1-center). Furthermore, a directed edge (Figure 1-right) represents one element's influence on another. Given our specific interest in the impact of practice changes on the teaching system, each occurrence of a practice change is marked by a change in the color of the system's representation.

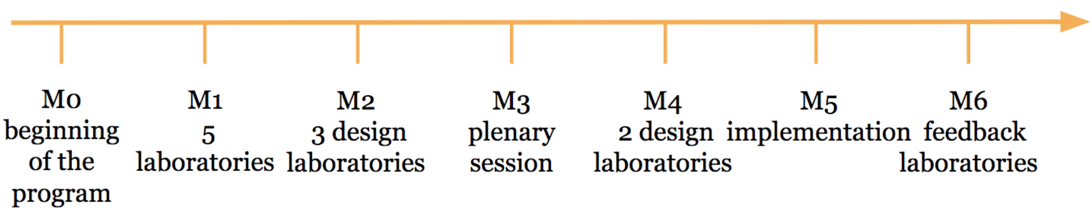
In this work, we consider the possible change in the teaching system provoked by participation in a TPD program. More precisely, this work aims at addressing the following research questions: (1) What is the effect (if any) of the TPD program on teachers' knowledge, beliefs and practice? (2) Does the teaching system analysis also confirm the systemic nature of teaching in the case of a change of practice induced by a TPD program?

3 Method

In the present section, we first provide an overview of the first year of the TPD program in which our research took place and then describe how data were collected and analysed according to the teaching system model to answer the research questions.

This research has been conducted in the framework of a three-year TPD program aimed at supporting teachers in designing and implementing activities focused on mathematical modelling with digital resources. The program involved 40 in-service teachers from grades 9 to 13 from all over Italy. The first year of the TPD program focused on an activity centred around mathematical modelling. Figure 2 provides an overview of the first year of the course in its various moments.

Figure 2. Timeline of the first year of the TPD program



"Moment M0" marked the start of the TPD program, where teachers were initially asked to complete a detailed survey. This survey delved into their background, motivations, expectations, prior TPD experiences, and use of digital resources and tools in their teaching practices. Moreover, they engaged in a design-focused activity on the ellipse. In "Moment M1", teachers, working in small groups with a tutor's guidance, carried out a mathematical modelling activity with digital technology (i.e., MIT App Inventor). In "Moment M2", teachers collaborated on developing lesson plans for integrating the activity into their classrooms. In "Moment M3", teachers participated in a plenary session in which they were provided a detailed task as guidance for completing the lesson plans. Moreover, at this moment, a logbook template to be compiled during the implementation phase has also been delivered. Finally, teachers were prompted to ask their students to write a scientific report at the end of the activity, following a template. In "Moment M4", teachers concluded the design phase. The implementation occurred during “Moment M5”; finally, in “Moment M6”, teachers provided feedback on the activity’s implementation in their classrooms. The duration of each moment ranges between 1 and 2 months.

Different kinds of data have been collected and analyzed: the initial questionnaire at M0, the laboratories’ recordings in M2, M4, and M6, the lesson plans produced in M0 and M4, and the logbooks produced in M5.

Concerning the initial questionnaire at M0, the questions were the following: (i) Why did you decide to attend this TPD program? (ii) What do you expect from this TPD program? (iii) Have you already attended TPD programs in mathematics education? If yes, which ones? (iv) During your lessons, what do your students do?

While (i)-(iii) were open-answer questions, question (iv) was a Likert-scale question with six options (they work alone, they listen to the teacher and they take notes, they ask questions to the teacher, they ask questions to their peers, they work in small groups, they discuss with each other and with the teacher) and with a Likert scale going from 0 (never) to 3 (each lesson). Moreover, the initial questionnaire also contained the 24 items on mathematical views proposed by Grigutsch et al. (1998). Examples of items are: “the ideas of mathematics are of general and fundamental use to society” (Application-oriented view); “mathematics is a logically coherent edifice free of contradiction consisting of precisely defined terms and statements which can unequivocally be proven” (Formalist view) “there is usually more than one way to solve a task or problem in mathematics” (Process-oriented view), and “Mathematics consists of memorising, recalling and applying procedures” (Schema-oriented view).

Questions (i) and (ii), together with the 24 items questionnaire, were proposed to investigate the teachers’ belief system at the beginning of the TPD program (M0), while questions (iii) and (iv) aimed at gaining information about teachers’ knowledge and practice at moment M0, respectively.

The lesson plans at M0 and M4 provide insights about teachers’ knowledge and practice components. With respect to knowledge, as in Brunetto & Dello Iacono (2023) and Bassi et al. (2024), we looked for indicators of the different components of the TPACK model in the lesson plans, identifying excerpts that belonged to the components. For example, the excerpt “going from the circumference equation to the ellipse equation with GeoGebra” has been classified as TCK because the teacher exploits technology to convey specific mathematical content. Still, she does not report how she plans to use these technological tools with students; she does not specify the pedagogical use of technology. The excerpt “the teacher’s monitor is used to lead the discussion" is an example of TPK because the teacher links the digital resource to a specific pedagogical purpose, namely for carrying out a discussion.

Moreover, the analysis of lesson plans also informed us about teachers' practice as designers. For instance, a lesson plan may contain only mathematical content while seldom mentioning the role of students.

Finally, the laboratories' recordings have been transcripted and qualitatively analyzed to find indicators of the different elements of the teaching system. Examples of indicators are reported in Table 1.

Table 1. Examples of excerpts containing indicators (underlined) for each element of the teaching system.

Practice	**Beliefs**	**Knowledge**
- After 10 minutes <u>I made visible</u> a worksheet with questions as a guide to discover the game - <u>I focus</u> on the game and on mathematical modelling - <u>I let</u> students <u>discuss</u>	- <u>I feel</u> insecure - This morning <u>I was reflecting</u> with a colleague. How much more effective it would be to enter the classroom with a step-by-step lesson plan!	- Following his indications <u>gave me so much</u> from the professional point of view - After a first hour <u>wasted for technical problems</u>, [...]

In this exploratory study, we consider the case of Katia, one of the teachers involved in the TPD program. Regarding Katia's teaching system, each author interpreted the data independently and afterwards. The analyses were discussed until a consensus was reached.

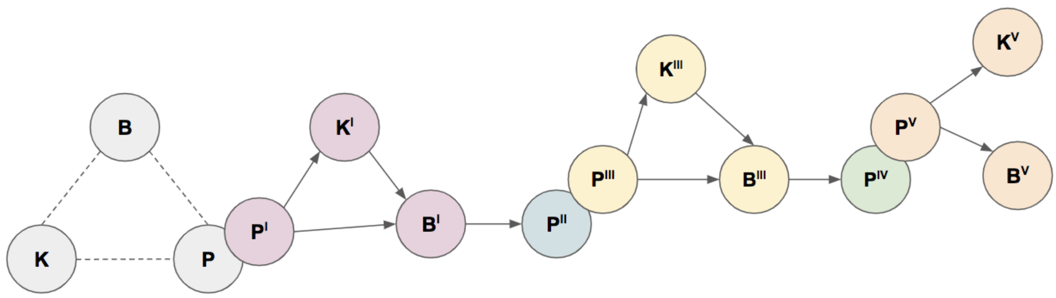
We stress that we have different data (initial questionnaire, lesson plans, laboratory recordings and logbook) analyzed qualitatively as previously described. Each piece of data contributes to informing the teaching system model in terms of its elements (practice, beliefs and knowledge), as performed by (Brunetto et al., 2022). To address the research questions, we investigate Katia's teaching system, highlighting whether and how a change occurs. Consequently, we can address the first research question, which is understanding the effect of the TPD program on the different elements. We specify that we are interested not only in seeing if Katia's teaching system at the end of the TPD program is different with respect to her system at the beginning of the program but also in whether and how it changes during the TPD program. The same analysis also allows us to answer the second research question by identifying the reciprocal influences between the different components.

4 Data analysis

The evolution of Katia's teaching system is reported in Figure 3. Concerning the initial state, she reported at M0 that, in almost all of her lessons, students discuss with each other and with the teacher and ask questions to the teacher; she also affirmed that, often, students work in small groups and ask questions to their peers and that, only seldom, students listen to the teacher taking notes. Concerning the use of technology, Katia stated that she often uses digital resources in her practice, such as slides, videos, online platforms and educational software. As a consequence, we can infer that participative lessons with the use of technology characterize Katia's practice (P). Moreover, the analysis of the lesson plan produced at M0 showed that Katia's practice as a designer (P) is mainly focused on mathematical content ("After having introduced the conics as sections of a conic surface, the planes that determine sections which are ellipses are pinpointed"). At the same time, the role of students is mentioned only in some phases ("A discussion in the classroom is made on how to obtain an ellipse"), while in other phases is not completely clear ("Successively a GeoGebra file is built to explore the equation").

Katia's interest in innovative methodologies and collaboration with colleagues belong to her belief system (B). Indeed, answering the question (i) "Why did you decide to participate in this TPD program?", she wrote: "Because I'm interested in innovative methodologies to use in my day-to-day teaching, in new experimentations and in sharing experiences with other colleagues". Moreover, the 24-item questionnaire on mathematical views revealed the formal view as her primary view of mathematics, which is part of her belief system (B).

Figure 3. Teaching system's change representation



Concerning knowledge (K), and more specifically technological knowledge, the analysis of Katia's lesson plan at M0 showed that she also has good technological knowledge in relation to specific mathematical contents (TCK). She indeed reported:

“A file on GeoGebra is built to explore the ellipse’s equation”. However, she appears to be less aware of how to exploit digital tools from a pedagogical point of view. Indeed, no reference was found in the lesson plan on how she planned to use the technological tool with her students.

The TPD program (moments M1 and M2) perturbed Katia’s system, forcing a change of practice P^{I} (see Figure 3): Katia was indeed involved in carrying out (M1) and in designing (M2) a complex mathematical modelling activity using MIT App Inventor, a new digital tool for her. This change in practice caused a change in Katia’s belief system (B^{I}): firstly, the change of practice P^{I} made explicit Katia’s sectoral view of the teaching of mathematics, for which jumping from one content to the other is to be avoided. Katia indeed stated: “I’d hate to stop the [curricular] activity, this [introducing the conics in polar coordinates] could be a way to connect to what I’m doing right now [trigonometry]”. Katia showed low self-efficacy with respect to implementing the new activity. Indeed, she stated: “for me these are new topics, I have this feeling of insecurity. I hope not to get confused. I had a peek at the other lesson plans, but then I said: no, I end up in a crisis!”. We highlight that Katia’s low self-efficacy partially hides Katia's interest in new methodologies and collaboration with her colleagues at the beginning of the TPD program.

Moreover, Katia showed low self-efficacy with respect to the technological tool and a lack of knowledge (K^{I}) with respect to it, saying: “Since the beginning, my problem has been the part with the App”. Katia’s low self-efficacy with respect to the new technological tool, together with the belief that its introduction would require too much time for students who have never used it before, led Katia to change her practice (P^{II}) further. She chose to focus on the first part of the activity, namely the mathematical modelling part, and not propose the second part, which involved MIT App Inventor as a programming tool. Moreover, her sectoral view of mathematics teaching pushed her to focus on the conics in polar coordinates to establish a link with trigonometry, the curricular topic she was dealing with.

At this point, Katia’s system was further perturbed by the plenary session at moment M3. More precisely, we consider the requests to complete the lesson plan following a detailed task of asking students to produce a scientific report and compiling a logbook as a change in teachers’ practice (P^{III}).

This further change in teachers’ practice caused a perturbation in all the components of the teaching system. Indeed, both beliefs (B^{III}) and knowledge (K^{III}) changed. In particular, a negative belief’s cluster and a positive beliefs’ cluster (B^{III})

emerged. On the one hand, Katia stated: “in other TPD programs, you attend the courses and after, when you think it is appropriate, you use what you learned to make a specific topic more appealing. But only when I want to use it. I found the course really exaggerated. I can’t mess up all the planning I decided at the beginning of the year”. Katia’s words made it explicit that a TPD program should not interfere too much with curricular planning. This belief is also linked to the sectoral view of mathematics teaching that had already emerged, and Katia’s words further confirmed that: “I would have proposed this activity when I’m dealing with the ellipse. I can’t be dealing with exponential equations and then, suddenly, saying: now we consider the conics.”. Moreover, Katia also saw the program instructors as not being aware of the school context. She stated: “the whole plenary session seemed not to consider what we have at school. Because when we enter the classroom, there are other realities: it’s not how we imagine it!”.

Regarding the request for the scientific report, Katia said: “I’ll give the homework. I don’t think that I will do as professor Brunetto suggested [working together with the students for revising the report multiple times]. They will do what they can”. The sentence “I’ll give the homework. I don’t think that I will do as professor Brunetto suggested” suggests that she did not consider the report as very valuable: she planned to ask her students to write the report just because the instructors asked it but investing as little time as possible. Moreover, the sentence “they will do what they can” highlights a specific belief about her students, namely that Katia doubted that they would have been able to write the scientific report.

The change in practice P^{III} also caused a change in Katia’s knowledge (K^{III}). In particular, concerning lesson planning, Katia said: “Following Prof. Brunetto's instructions [for writing the lesson plan] gave me so much from the professional point of view”. This change in the knowledge component was also important for the emergence of a belief belonging to the positive belief cluster (B^{III}), namely the importance of careful lesson planning for enhancing teaching effectiveness. In Katia’s words, “This morning I was reflecting with a colleague how much more effective it [teaching] would be if one would enter the classroom having everything planned step by step!”. In the positive beliefs cluster (B^{III}) we also find the importance for Katia to not give up on what she had started (“Giving up isn’t in my style”) and the importance of attending TPD programs that produce a shock: “This course caused an internal change in me! It shocked me, but at the same time, I must say that the fact that I was shocked is a positive thing! In general, during my professional life, the greatest changes happened when I felt shocked”.

All the mentioned beliefs made Katia persevere in attending the TPD program and in further modifying her practice as a designer (P^{IV}), as her words highlight: “So, if the course was meant to shock us, it reached this goal. If you take a look at my lesson plan, it has changed completely: I work on it, I revise it, I review it, I polish it”.

The logbook analysis showed that Katia’s practice (P^{V}) during the implementation of the activity was participative with group work and classroom discussions, confirming the practice P at the beginning of the TPD program. Moreover, Katia reported that she left the preparation of the scientific report as homework. With respect to knowledge, the change of practice P^{V} highlighted, as in K^{II}, Katia’s lack of knowledge, and in particular technological knowledge TK, in managing the MIT App Inventor tool, since in the logbook, she reported having wasted an entire hour due to technical problems.

As a consequence of the change of practice P^{V}, Katia’s belief system also changed (B^{V}). Firstly, Katia seems to have overcome her low self-efficacy with respect to implementing the new activity, a belief belonging to beliefs cluster B^{I}. She said, “In the end, this experience made me feel good”. Moreover, the belief belonging to B^{III} about the lack of ability of her students to write the report was radically challenged. Indeed, she was positively impressed by how students, particularly students that she considered “weak”, dealt with the task. Katia indeed affirmed: “I’m satisfied because, despite students being very busy also with other activities, they worked hard both in the activity and in the report. [...]” and “This activity allowed me to know more about my students. Some students, who are not usually very collaborative, surprised me with their intuitions and hard work, while other particularly outstanding students did not give it their all in writing the scientific report.” The last sentence also shows how Katia was reevaluating the scientific report to better know her students.

5 Discussion and conclusions

We recall that the present paper analyzes the case of Katia with the purpose of addressing the following research questions: (1) What is the effect (if any) of the TPD program on teachers’ knowledge, beliefs and practice? (2) Does the teaching system analysis confirm the systemic nature of teaching also in the case of a change of practice induced by a TPD program?

Concerning the first research question, we can answer that the TPD program change of practice provoked a change also in knowledge and beliefs. First, we observe that, in some cases, the TPD program change of practice led to a change in Katia's belief system, mediated by a change in knowledge. For example, in the case of P^{I}, B^{I}, and K^{I}, the new practice of designing a complex mathematical modelling activity (P^{I}) provoked a change in knowledge in the sense that it highlighted a lack of knowledge in relation to the technological tool (K^{I}), which, in turn, caused in Katia a low sense of self-efficacy with respect to it (B^{I}). This mechanism led to a further modification of the practice (P^{II}) in the sense that Katia chose to eliminate the programming part involving the new technological tool from her lesson plan. Another example was given by the intertwining of P^{III}, B^{III}, and K^{III}, where the change in practice (P^{III}) induced by the plenary session in M3 (especially by the detailed task for the lesson plan) led to a change in knowledge (K^{III}), in the sense that new knowledge was acquired about lesson planning, which, in turn, contributed to change the belief system, with a new belief emerging about the importance of lesson planning (B^{III}). Also, in this case, we had a further modification of the practice of Katia as a designer (P^{IV}), with her involvement in refining the lesson plan. In the first example, a change in the belief system occurred mediated by a change in knowledge, in the sense that a lack of knowledge became explicit. The change in the belief system led to a change of the practice, which is a "shrinking" of the practice proposed in the TPD program: Katia indeed chose not to implement the programming part despite being technology one of the two milestones of the program, together with mathematical modelling. In the second example, the belief system change was mediated by acquiring new knowledge. As a result, the change in practice was an evolution of Katia's practice as a designer. These examples confirm the important interplay between the acquisition of new knowledge and the emergence of beliefs that, in turn, can sustain an expansion of practice.

Concerning the second research question, the analysis showed the intertwining of practice, beliefs and knowledge in the sense that the change in one of the components was reflected in the change of all the components of the system, thus confirming the systemic nature of teaching also in the case of a change of practice induced by a TPD program, namely attending the program, designing mathematical modelling activities with the use of technologies and carrying out the relative lessons.

Finally, we notice that the teaching system analysis has the potential to help identify possible critical points in a TPD program. For example, our case highlighted the necessity of providing teachers with greater technical support with respect to the new technological tool. Another critical point from the affective perspective was the change of practice introduced with the plenary session (P^{III}), which provoked the emergence of a negative belief cluster (e.g., the distance of the instructor of the course from the school reality, the presumed lack of ability of students in writing the scientific report) that could have potentially arrested the teaching system evolution or even have led Katia to drop the program. Fortunately, in our case, the same change of practice also led to the formation of a positive belief cluster (e.g., the importance of not giving up and of attending courses that produce a "shock") that sustained Katia in implementing the activity (P^{V}). The analysis also allowed us to identify the strengths of the program: the change of practice (P^{V}) related to the implementation of the activity in the classroom was indeed fundamental to overcoming the belief about the students and their ability to write the report.

The exploratory use of the PBK model (Brunetto et al., 2022) extends the previous work on teachers' use of digital technologies and how teachers designed their lessons during the TPD program (Brunetto & Dello Iacono, 2023; Bassi et al., 2024), in which the authors only focused on knowledge (K) and relied on the TPACK model in this respect. In this work, we went beyond knowledge (K) and analyzed how all three components, namely practice (P), beliefs (B), and knowledge (K), influence each other. On the other hand, we can argue that the present work also extends Brunetto et al. (2022) by considering a different change of practice: Brunetto et al. (2022) the forced change of practice from in-presence to online teaching during the pandemic considered in this work we focus on the change of practice "forced" by the participation in a TPD program. Despite the different changes in practice, we have found similar results. As in Brunetto et al. (2022) and Guskey (1986), the change of practice provoked a change in beliefs at different moments of the TPD program. Moreover, the importance of knowledge for teaching and the strength of the teaching system model in systematically identifying its fundamental role was confirmed. Finally, we argue that Katia's teaching system evolution is an example of what Brunetto et al. (2022) define as a dynamic scenario, where a change of practice triggers a dynamic process in which beliefs, knowledge and practice change and influence each other.

Together with the dynamic scenario, Brunetto et al. (2022) also provide two examples of teaching system evolution that are classified as static scenarios, in which the system resists change, trying to "repair the damage". In future work, we aim to perform the same analysis of this paper considering other teachers who participated in the TPD program to see if similar static scenarios can also be identified when a change of practice forced by a TPD program is considered.

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