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Reading mathematical graphs about pollution: exploring the role of mathematical beliefs

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Abstract: Graphs are the most used tools to communicate data about pollution, climate change and other socio-environmental issues, but research in mathematics education warns us that people in general and students in particular find it difficult to read them. Our hypothesis is that affect-related elements play a key role in influencing how students read and understand mathematical graphs and in this paper we investigate the role of beliefs in shaping the way students approach graphs about pollution. Different views about mathematics emerge when students have to account for why they find a graph beautiful or clear.

Keywords: aesthetics, mathematics-related beliefs, mathematical graphs, transparency

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1 Introduction

Mathematical graphs are well acknowledged to be an effective tool to convey information about, e.g., pollution issues, increasing temperatures, and climate change, by environmental scientists, especially when interacting with people outside the scientific community (Grainger, Mao & Buytaert, 2016). However, Ainley (2000) warns us that mathematical graphs are not always transparent to those who read them. In accordance with previous discussions by Doria and Amico (2023), transparency in a general sense is the quality that allows for easy perception or detection of specific information, as well as the associated aesthetic appreciation. A meta-analytic review by Thielsch et al. (2019) highlighted that the visual aesthetics of websites, software and other interfaces have a positive impact on user performance, enhancing attention and concentration. As articulated by David and Glore (2010), "design and aesthetics profoundly influence how users perceive information, learn, assess credibility and usability, and ultimately assign value to a product" (David & Glore, 2010, p.3). From these words one can infer that beliefs play a central role in both approaching and interpreting a graph, so in this paper we investigate which beliefs emerge when students are asked to justify why they prefer one graph over another in terms of aesthetics or transparency. The theoretical framework is thus threefold and builds on the literature on mathematical beliefs, as classified and understood by Grigutsch et al. (1998), about transparency and about aesthetics of graphs.

2 Theoretical framework

2.1 Mathematics-related beliefs

Beliefs are propositions about a certain topic that are regarded as true (Philipp, 2007), and tend to form clusters as they “always come in sets or groups, never in complete independence of one another” (Green, 1971, p. 41). According to Green (1971), belief clusters are coherent families of beliefs across multiple contexts. Thus, beliefs have a systemic nature. This can be understood in terms of “world views” (Grigutsch, Raatz & Törner, 1998), or epistemological beliefs about mathematics (Hofer & Pintrich, 1997), including its teaching and learning. According to Grigutsch et al. (1998), it is possible to outline four different world views (see also Liljedahl, Rolka & Roesken, 2006): a process-oriented view that represents mathematics as a creative activity consisting of problem solving using different and individual ways; an application-oriented view that represents the utility of mathematics for real world problems as the main aspect of the nature of mathematics; a formalist view that represents mathematics as characterised by a strongly logical and formal structure; a schema-oriented view that represents mathematics as a set of calculation rules and procedures to apply for routine tasks. Each individual’s beliefs belong to (at least) one world view (Erens & Eichler, 2019), and the rationale for this is explained in the following. One aspect of a belief system relevant for our research is that beliefs are organised in clusters that are not necessarily logically connected. The fact that beliefs can be contradictory (Fives & Buehl, 2012) allows the possibility for people to hold beliefs that belong to different clusters. Skott (2015) suggests, however, to interpret possible contradictions in individuals’ belief systems not merely as incoherences, but rather to consider the different contexts in which beliefs are evoked and taking into account that they cannot be exhaustively described by one cluster of central beliefs. These considerations shed light on two intertwined features of beliefs: they are subjective in nature and individually held, but at the same time they are (or can be) socially and contextually shaped. The context plays a crucial role in evoking beliefs (e.g. Fives & Buehl, 2012). In our research, two contexts are considered: transparency of graphs and aesthetics of graphs.

2.2 Transparency and aesthetics of mathematical graphs

Graphs are mathematical entities that are comprehensible through the interpretation of their representations rather than in a direct manner: grasping a concealed concept within a chart involves a cognitive process that connects the signifier (information) to its signified (the graph) (Duval, 2006). Roth (2003) posits that "graphs, as entities, do not exist in isolation; rather, they are intricate networks that integrate various components and processes" (p. 305). Enhancing these cognitive processes, as discussed by Roth, could potentially enhance the transparency of graphs. The term *transparency* refers to the characteristic of seeing through an object, namely to the ease with which an observer can access the information hidden by what is visible in the foreground. There is an object, definite and circumscribed, and a story ‘lying beyond’ that; the term transparency denotes how easy the user can access such hidden significance. Lave and Wenger (1991) describe transparency as the combining of two characteristics: invisibility and visibility. In order to explain this dual nature, they resort to the metaphor of the window: a window is highly visible in contrast to the wall that contains it, but the hidden meanings, represented by what lays behind the window, is clear if the glass is invisible enough (Ainley, 2000, p. 366).

In the case of graphs, transparency is a characteristic that determines how easily data can be read (see e.g., Ainley, 2000); in this case the role of the window is played by the graph and the hidden meaning is represented by the detectable information. The slope of the curve, the relationships between the variables, the variation with time: these are just some examples of the information hidden behind a graph. Roth and McGinn (1996) consider the relationship between reality and graph as bidirectional,

> because it is assumed that a literate person can read the specifics of the walk from the graph or construct a graph after making (or imagining) a walk. However, there is evidence that this relationship must be constructed in the same way as the relationship between the word 'cat' and some furry creature that meows. (p.96).

These considerations, however, entail that students become literate in reading graphs. This is connected to Glazer's (2013) argument that, in order for students to use their knowledge and acquire critical thinking, it is necessary to do more than study the core subject. This holds in general, for mathematics, and in particular for graphs. Coles (2023) proposes a curricular innovation for mathematics, more linked to the real world, namely connected to data and experiences; we think that this should also apply to those tools that mathematics makes available to describe the world in which we live. Graphs in particular are crucial for conveying information to the general public, especially about the environmental sciences and the changes that occur every day in the world (Grainger, Mao & Buytaert, 2016). However, Ainley (2000) warns us that data is not always accessible from a graph for those who read it. This could have dramatic consequences when environmental, social, political or economic problems need to be communicated through graphs: as Demeritt and Nobert (2014) observe, ineffectiveness of communication through graphs may prevent comprehension and, thus, create misunderstandings and inconsistent or biased messages.

It has been observed that students in mathematical classes learn both how to read graphs and how to create them (Roth & McGinn, 1996), in strong connection with the real context they refer to (Ivanjek, Susac, Planinic & Andrasevic, 2016), in the sense that they are able to attach some specific meaning to a certain graph in a precise context, but seem to lack to have a general skill to understand a graph detached from the context, or two same graphs representing different data may look different.

From one side, it seems that graphs are strongly bonded to both the context and the subject they emerge from, but at the same time their readability heavily depends on the cognitive ability of those who read and interpret them. In fact, mathematical objects are accessible through representations such as graphs, and not in a direct way (Duval, 2006): understanding a concept hidden inside a chart is a cognitive act which relates the signifier (information) to its signified (the graph) (Duval, 2006). Roth (2003) states that: “graphs as objects do not exist as independent entities, but are a complex network that integrates entities and processes” (p. 305).

Moreover, Thielsch, Scharfen, Masoudi and Reuter (2019) observe that a fundamental feature of graphs is their aesthetics: “users not only feel better if they use aesthetically pleasant interfaces, but, on average, they also perform a little better as well” (p.208). *Aesthetics*, a term derived from the Greek "aisthesis", meaning perception, represents a philosophical construct imbued with profound reflections on art, beauty and sensorial experiences (Zingarelli, 2020). It is not only a matter of personal experience, but involves the philosophical and conceptual understanding of what arouses emotions and appreciation in the observer: studying the reasons behind such perceptions represents the very meaning of aesthetic research. In his *Critique of Judgment*, Kant (1790) argues that beauty cannot be reduced to specific rules or criteria, but is rather a subjective experience, based on the harmony between intellect and imagination. In *Lectures on Aesthetics*, Hegel (1835) maintains that the aesthetic experience is a phase of the evolutionary process of the spirit, linked to the understanding of the intrinsic meaning of a work of art. In general, therefore, we can say that the function of beauty is to make the viewer feel pleasure.

3 Method

The initial part of this study consists of a lecture delivered three times: to two high school classes, once in Milan (23 students) and once in Novara (24 students), and once to a class of university students enrolled in Biology undergraduate course at the University of Eastern Piedmont (41 students). Participants were gender balanced and came from different educational backgrounds, resulting in a good representation of the sample, both for gender and mathematical skills. The lecture presents data about air and water pollution from textile factories dislocated in the territory of Biella. Some critical chemical substances are firstly defined, then their impact on rivers and atmosphere is discussed and then real data with thresholds imposed by the EU law are shown by means of graphs. During the lecture, which lasted three hours, two different types of tasks were used regarding the graphs presented.

The first type of task requires to rate, both in terms of transparency and in terms of aesthetics, 7 sets of four graphs each, for a total of 28 graphs (each question shows 4 graphs and the same group of 4 graphs is used both for aesthetics and for transparency). Figure 1 shows an example of 4 graphs representing the same data. The scale ranges from 1 to 4, where 1 stands for “not at all”, 2 for “more no than yes”, 3 for “more yes than no” and 4 for “absolutely yes”. It is the same for the 28 questions asking “Do you find this graph beautiful?” and for the 28 questions asking “Do you find it transparent?”.

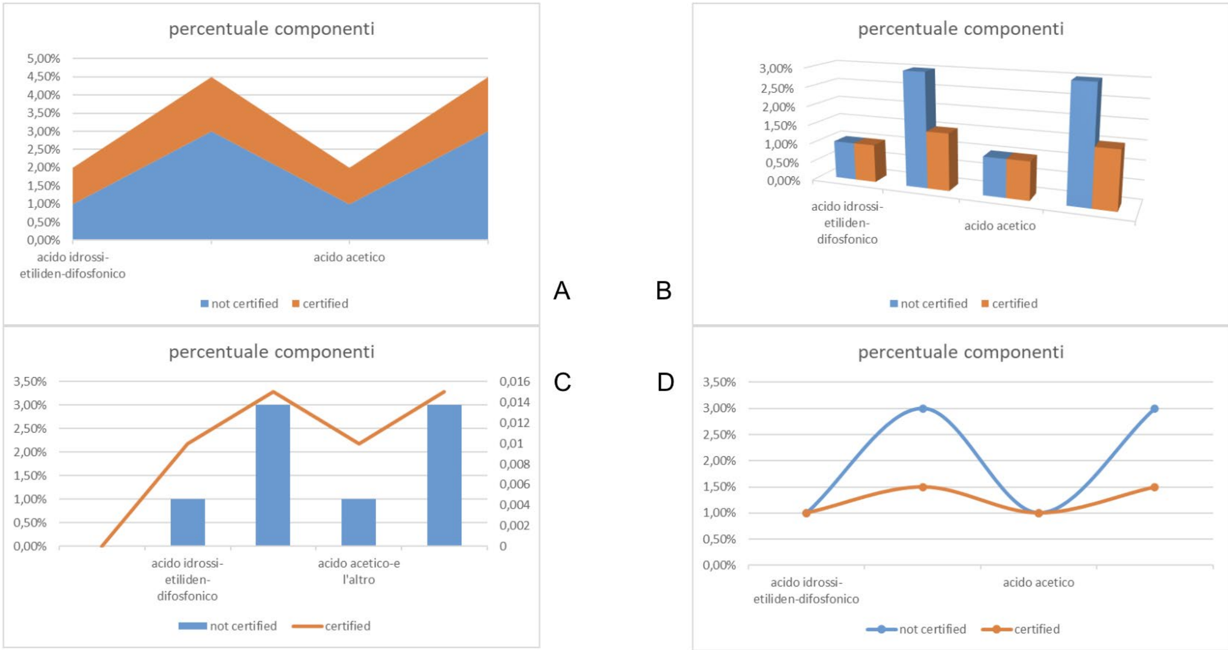
The second type of tasks consists of justifying one’s own choices, in the format of open-ended written questions that ask the students to say why they found a certain graph the most transparent and/or the most beautiful. This constitutes the data for the present study, while the first type of tasks has been analysed in Doria and Amico (2023). In analysing the written answers, we used a qualitative coding method (Mayring, 2015), based on Erens and Eichler's (2019) four deductive categories described in their coding manual. Examples, from Erens and Eichler's (2019) paper, of statements coded as application-oriented view are: "mathematics helps to solve tasks and problems that originate from daily life", "the ideas of mathematics are of general and fundamental use to society", and "a sound knowledge of mathematics is very important for students in their whole life". Examples of statements coded as formalist view are: "logical strictness and precision are very essential aspects in mathematics", "mathematics is a logically coherent edifice free of contradiction consisting of precisely defined terms and statements which can be unequivocally be proven", and "in mathematics students must use mathematical terms correctly". Examples of statements coded as process-oriented view are: "there is usually more than one way to solve a task or problem in mathematics", "in order to comprehend and understand mathematics, one needs to create or (re-)discover new ideas", and "everyone is able to (re)invent or to comprehend the central ideas of mathematics". Examples of statements coded as schema-oriented view are: "Mathematics consists of memorising, recalling and applying procedures", "doing mathematics demands a lot of practice in adherence and applying to calculation rules and routines", "nearly any mathematical problem can be solved by the direct application of familiar rules, formulas and procedures", and "to solve a mathematics task, there is mostly a unique way of solution which needs to be found". Each student's statement has been assigned a world view by one of the authors, and the other author independently agreed or disagreed. In case of disagreement, discussion among the authors took place, until consensus has been reached.

4 Data

The research question that we aim at answering in this paper is: when the students have to justify their choices for beautiful and transparent graphs, is it possible to use the lens provided by Erens and Eichler (2019) and which feature of mathematical beliefs emerge?

A set of four graphs representing the same data is shown in Figure 1 and the students found graph B as the most beautiful and the most clear (see Doria & Amico, 2023).

Figure 1. An example of a set of 4 graphs representing the same data.



Students' written statements that have been coded as formal-oriented view and that emerge when they justify their choice for aesthetically pleasant graphs (not necessarily the graph B in Figure 1), are:

- S1: the graph is clear and orderly
- S2: the order
- S3: it is symmetric and I like the colours
- S4: it is the clearest
- S5: it is the most beautiful and dainty
- S6: it is more precise and tidy

All in all, 8 out of 23 answers have been coded as formal-oriented and we can notice that it emerges, from these students' quotes, that they value order, cleanliness, clarity and precision the most, which is in line with a formal-oriented view of mathematics.

Students' written statements that have been coded as schema-oriented view and that emerge when they justify their choice for aesthetically pleasant graphs (not necessarily the graph B in Figure 1), are:

- S7: the graph is interpretable quickly.
- S8: it is easy to interpret.
- S9: it is the most related to my school routine.

It emerges, from these students' quotes, that they value easiness and quickness of interpretation, but also routine the most, which is in line with a schema-oriented view of mathematics. Schema-oriented quotes are quite rare: only 4 over 23 have been found.

Only one quote has been classified as a process-oriented view, for its focus on the fact that all the people can access and use mathematical ideas, and it reads: "this graph is at the reach of everyone". As well, also only one quote has been classified as an application-oriented view, for its focus on the real data, and it reads: "the differences among the measures are more visible in this graph". Finally, only one quote has been classified as both formal and application-oriented views, as it reads: "the graph is the most orderly and clean, but it also the simplest to interpret, thus at the reach of everybody".

The students' written statements that have been coded as formal-oriented view and that emerge when they justify their choice for transparent graphs (not necessarily the graph B in Figure 1) represent almost 25% of the statements and they all refer to clarity, as it is exemplified in what follows:

- S1: the graph is really clear
- S4: it makes clear the idea conveyed by the graph
- S5: it allows to have the clearest interpretation of data

The students' written statements that have been coded as schema-oriented view and that emerge when they justify their choice for transparent graphs (not necessarily the graph B in Figure 1) represent almost 30% of the statements and they all refer to immediacy of interpretation, the easiness with which it can be read, the use of basic mathematical objects (like rectangles), the fact that it is the graph they are more used to, the most linear one.

We can notice that more students elicit schema-oriented views when they are asked to justify their choice for a transparent graph, compared to when they justify their choice for a beautiful graph, when the majority of answers elicit formalism in mathematics.

Also application-oriented views emerge in 28% of statements, which represent a much higher percentage compared to the case of aesthetics. References to the adherence to data, to how much the differences are made visible, to the trend in the real situation that is represented by the graph, emerge.

5 Discussion and conclusions

Our research examines the differences that emerge in the justifications students provide when choosing aesthetically pleasing graphs versus transparent graphs. We can answer positively to the first research question, namely that it is possible to use the lens provided by Erens and Eichler (2019) to analyse the justifications and to sort them by the prevalent mathematical view(s) that emerge. With respect to the second research question, the results show that there is a prevalence of references to mathematical formalism when discussing the beauty of graphs, while this tendency decreases when evaluating graphs based on transparency.

These results can be linked to the theoretical framework we mentioned, where beliefs are organised in clusters and can belong to different mathematical worldviews. Mathematical formalism, which emphasises order, precision and clarity, reflects a formal-oriented vision typical of the world of mathematics. Our study reveals that this view of mathematics emerges significantly when students have to justify their choice for a beautiful graph. Differently, when evaluating the transparency of graphs, other perspectives such as schema and application orientations emerge, indicating greater consideration for immediate interpretation and practicality in the context of real data use. Since the schema-oriented view is the prevalent one at school, it seems that when students have to judge a graph based on its transparency, they refer to school practice and beliefs developed during their school years.

In summary, although aesthetics and transparency may seem linked, our results suggest that there are significant differences in the beliefs that emerge from the justifications provided by students, with a predominance of mathematical formalism in the context of the beauty of graphs and a greater variety of perspectives when transparency is evaluated.

Research ethics

Author contributions

L.D.: conceptualization, data curation, data analysis, methodology
C.S.: conceptualization, data analysis, writing

All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare no conflicts of interest.

References

Ainley, J. (2000). Transparency in graphs and graphing tasks: An iterative design process. *Journal of Mathematical Behavior, 19*, 365–384

Andrà, C., Amico, A., Scalvini, C., Doria, L., Liljedahl, P., & Pezzutto, M. (2023). A method for meta-reviews of research unfolding the unavoidable systemic nature of affect. *Quadrante, 32*(2), 6–24.

Coles, A. (2023). Teaching in the new climatic regime: steps to a socio-ecology of mathematics education. In: M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-34). Haifa.

David, A., & Glore, P. (2010). The impact of design and aesthetics on usability, credibility, and learning in an online environment. *Online Journal of Distance Learning Administration, 13*(4), 43–50.

Demeritt, D., & Nobert, S. (2014). Models of best practice in flood risk communication and management. *Environmental Hazards, 13*(4), 313–328.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics, 61*, 103–131.

Erens, R., & Eichler, A. (2019). Belief changes in the transition from university studies to school practice. Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity, 345–373.

Fives, H., & Buehl, M. M. (2012). Spring cleaning for the “messy” construct of teachers’ beliefs: What are they? Which have been examined? What can they tell us?.

Glazer N.(2011): Challenges with graph interpretation: a review of the literature, *Studies in Science Education, 47*(2), 183–210

Grainger, S., Mao, F., & Buytaert, W. (2016). Environmental data visualisation for non-scientific contexts: Literature review and design framework. *Environmental Modelling & Software, 85*, 299–318.

Green, T. F. (1971). *The activities of teaching*. McGraw-Hill Kogakusha

Grigutsch, S., Raatz, U., & Törner, G. (1998). Einstellungen gegenüber mathematik bei mathematiklehrern. *Journal für mathematik-didaktik, 19*, 3–45.

Hegel, G. W. F. (1835). *Lectures on aesthetics*. Newcomb Livraria Press.

Hofer, B. K., & Pintrich, P. R. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of educational research, 67*(1), 88–140.

Ivanjek, L., Susac, A., Planinic, M., Andrasevic, A., & Milin-Sipus, Z. (2016). Student reasoning about graphs in different contexts. *Physical Review Physics Education Research, 12*(1), 010106.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

Liljedahl, P., Rösken, B., & Rolka, K. (2006, November). Documenting changes in pre-service elementary school teachers beliefs: Attending to different aspects. *In Proceedings of the 28th international conference for Psychology of Mathematics Education-North American Chapter* (Vol. 2, p. 279–285).

Kant, I. (2008). *Critique of judgment*. Newcomb Livraria Press.

Philipp, R. A., Ambrose, R., Lamb, L. L., Sowder, J. T., Schappelle, B. P., Sowder, L., ... & Chauvot, J. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38*(5), 438–476.

Roth, W. M. (2003). *Toward an anthropology of graphing: Semiotic and activity-theoretic perspectives*. Springer Science & Business Media.

Roth, W. M., McGinn, M. K., & Bowen, G. M. (1996). Applications of science and technology studies: Effecting change in science education. *Science, Technology, & Human Values, 21*(4), 454–484.

Skott, J. (2015). Towards a participatory approach to ‘beliefs’ in mathematics education. From beliefs to dynamic affect systems in mathematics education: Exploring a mosaic of relationships and interactions, 3–23.

Thielsch, M. T., Scharfen, J., Masoudi, E., & Reuter, M. (2019). Visual aesthetics and performance: A first meta-analysis. In: *Proceedings of Mensch und Computer 2019* (pp. 199–210).

Zingarelli, N. (2020). *Italian dictionary*. Edition Zingarelli.