LUMAT-B
International Journal on Math, Science and Technology Education

Conference paper

Educators' perspective on teachers' beliefs as key-factors for the implementation of mathematics learning activities designed from an enactive-embodied approach

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Abstract: If we are interested in the implementation of research findings, as teaching practices, within school settings, it becomes crucial to outline what beliefs should guide teachers in implementing these findings from the researchers' viewpoint. These beliefs essentially define the underlying philosophy behind these practices and offer valuable insights into how educators can facilitate teachers in bringing the research findings in schools. In our exploratory study, researchers from Italy and Australia hypothesized, specifically, potential beliefs of primary and secondary school mathematics teachers that can be related to the introduction and successful integration of mathematics learning activities designed from an enactive-embodied perspective. This constitutes a key preliminary step for exploring teacher beliefs by directly involving them in a survey.

Keywords: teachers' beliefs, mathematics teaching and learning, implementation research, embodied design research, enactive learning

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1 Introduction

One of the key challenges facing research in Mathematics Education is ensuring that research findings have a tangible impact on educational system, particularly within schools (Mariotti et al., 2019; Arzarello & Bussi, 1998). For reaching this goal, teachers assume a pivotal role. Indeed, they are potential users of research findings and are responsible for disseminating innovations schools, as well as implementing them (Century & Cassata, 2016).

In particular, teachers' beliefs are crucial in their orientation toward changes in education (Peterson, 2013; Coburn & Talbert, 2006), as well as in determining the use and transposition of research findings in teaching practices. Certainly, although it is deemed inaccurate to establish direct correlations and causal relationships between teachers' beliefs and their instructional practices (Schoenfeld, 1998), it is widely acknowledged that there exists a reciprocal influence between these two factors (Zhang & Morselli, 2016).

In the presented research, we have specifically focused on the implementation in schools of mathematics learning activities designed from an *enactive-embodied* perspective, i.e., activities in which students are actively engaged in exploring mathematical concepts throughout their perception and physical movement, using manipulatives, tools (virtual or physical), or simple hands or whole-body movements. Hereafter, we will refer to them using the acronym ABM activities, which stands for Active, Bodily experience Mathematics learning activities. The reason why we focus on this topic is briefly outlined in the following lines.

Indeed, the relevance of perception and bodily movement for the exploration and construction of mathematical concepts is a central topic in mathematics education, with long-time roots, that can be traced back to the early 1900s, to the Italian contributions of Maria Montessori and Emma Castelnuovo, as well as to Jean Piaget, John Dewey, and Jerome S. Bruner's well-known theoretical works. Moreover, this topic finds a renewed interest, in the last three decades, thanks to research findings in psychology and cognitive neuroscience (e.g., Goldin-Meadows, 2005; Nemirovsky, 2003; Seitz, 2000), that evidenced the relevance of perceptual-motor aspects in mathematical learning processes. Its relevance has been even more emphasized by theories from the cognitive psychology field of *embodied* and *embedded cognition*, as pioneered by Varela et al. (1991) and Lakoff and Núñez (2000). These research findings have found resonance in the field of Mathematics Education, where many theoretical works and experimentations in schools on these themes have been carried out by various research groups (Abrahamson et al., 2020). Some relevant examples can be traced by looking at the *enactivist pedagogy* (Abrahamson et al., 2022), the *inclusive materialism* (de Freitas & Sinclair, 2014), and studies on gestures and embodiment in mathematical thinking, teaching and learning from the *multimodal approach* perspective (e.g., Radford et al., 2017; Nemirovsky & Ferrara, 2009; Arzarello & Robutti, 2008).

Despite the growing interest in this research area, we don't have enough information to what extent these approaches have been translated into practice within school environments. This motivates the interest in investigating ABM activities implementation.

As stated at the outset, it's evident that teachers' beliefs can significantly impact the adoption and realization of teaching innovations. Consequently, an initial fundamental undertaking in this research involves identifying the specific beliefs that can be related to the introduction and successful integration of ABM activities into the classroom. Hence, the goal of the presented research is to identifying possible teachers' beliefs that can be linked to ABM activities implementation, drawing insights from the perspectives of researchers.

The framework provided by researchers could offer a comprehensive overview of the core beliefs that researchers consider crucial for guiding teachers as they engage with ABM activities in mathematics education. It represents a valuable resource for understanding the perspectives of experts in the field and can serve as a foundation for further exploration and implementation of these beliefs within educational contexts. Definitely, formulating these hypotheses represent a crucial preliminary step aimed at investigating the role

that these beliefs play in relation with ABM activities implementation, with a direct involvement of primary and secondary school mathematics teachers.

Since considering multiple contexts, with heterogeneous educational cultures, can reveal the presence of both features of contextual specificity, which may find reason in the particular cultures, and shared characteristics, which transcend cultural boundaries (Huang et al., 2020), the inquiry was conducted in Italy and Australia. Indeed, it leads us to be aware of the presence of latent and implicit characteristics, which might not emerge by conducting the research exclusively within one educational system (Boscolo, 2023).

2 Methodology

The research is an exploratory study, qualitative in nature, consisting in carrying out semi-structured online interviews with a selected group of Italian and Australian mathematics education researchers for identifying possible beliefs teachers should hold when implementing ABM activities.

The involved experts are mathematics education researchers with experience alongside teachers, expertise in implementing innovation at school, and research interests akin to the object under study. The selection process consisted in contacting researchers who possess the aforementioned characteristics; then, the researchers in the sample are the ones that decided to join the project voluntarily: six experts from Australia, all academics belonging to MERGA (Mathematics Education Research Group of Australasia), although three of them are also former secondary school teachers, and nine researchers from Italy, who are 7 academics and two teacher-researchers, and seven of them are members of the Italian national association of research in mathematics education (AIRDM).

To understand what, according to experts, might be teachers' beliefs linked to the implementation of ABM activities, both to ensure their dissemination and an implementation consistent with the philosophy adopted at the research level, they were asked to answer the following question: "*What are the beliefs that should guide teachers for proposing and when implementing ABM activities*?".

Their answers were transcript and, then, analysed according to the thematic content analysis methodology (Patton, 2002), organizing the narrative materials into units of meaning encoded in MAXQDA software. The framework resulting from the analysis was represented through a map, in which the overall and diverse opinions are presented, accompanied by a narrative, in which the emergent themes are reorganized. Further details on the data analysis process can be seen in Boscolo (2022).

3 Discussion of results

The analysis of interviews with researchers revealed a framework encompassing their perspectives on the key beliefs that should guide teachers in proposing and implementing ABM (Active, Bodily experience Mathematics learning) activities. This framework is

depicted in Figure 1, where each node in the map represents a code which represent a unit of meaning, i.e., a core theme, that emerges from the analysis of experts' answers.

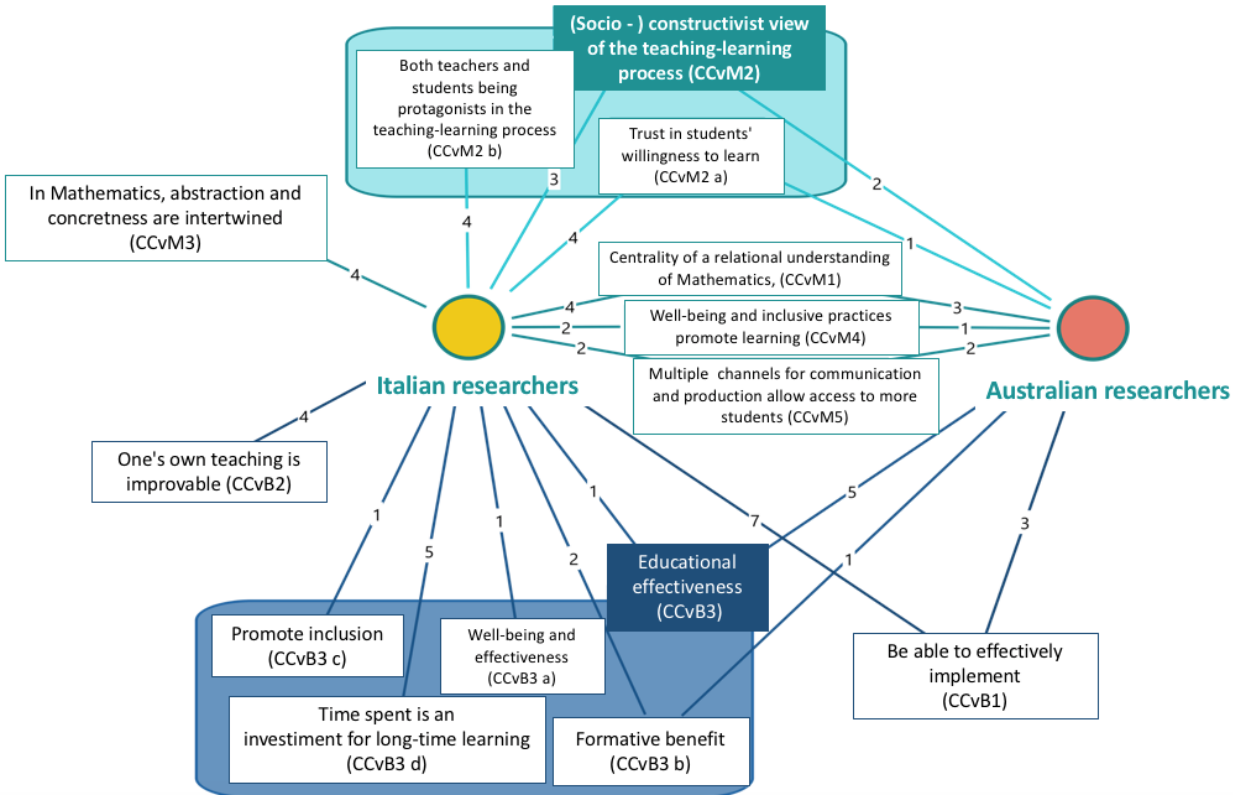
To conduct the analysis, the transcribed narrative material was structured into two overarching narratives. The first narrative compiles the contributions of the Italian researchers, while the second narrative encompasses the responses provided by the Australian researchers. In the map, you'll observe numerical values assigned to each link. These values indicate the cumulative frequency with which the core theme (represented by the code on the node) recurs in the narratives of the respective group of experts. In other words, it illustrates how many times Italian / Australian researchers mentioned this theme, even considering multiple mentions by the same researcher if the theme reoccurs through their interviews.

In the following, for citing direct quotes from the researchers, we will adopt a standardized format. We will use "Au.R." to denote an Australian researcher, followed by a numerical identifier representing their person within the group. Similarly, we will use "It.R." for Italian researchers, along with their respective numerical identifier. We will also include a reference to the specific paragraph within the interview transcription in MAXQDA. Direct quotes from Italian researchers have been translated into English by the author.

The beliefs mentioned by researchers are of two types:

1. Beliefs related to the nature and teaching-learning of mathematics (in light blue, located in the upper half of the map). These beliefs pertain to fundamental beliefs about what is mathematics and how it is best taught and learned. They provide insights into the foundational principles that underpin the use of ABM activities in mathematics education.
2. Specific beliefs concerning the necessary conditions for, and the potential benefits derived from, the implementation of these activities (in dark blue, situated in the lower half of the map). These beliefs are centred around both the advantages and potential outcomes that can be achieved by incorporating ABM activities into mathematics instruction and beliefs regarding one's ability to implement them. They shed light on the perceived advantages and positive impacts associated with this pedagogical approach.

Figure 1. The overall perspective of Italian and Australian researchers on the beliefs that should guide teachers in proposing and when implementing ABM activities.



Note. Overall map (XMap two-case model MAXQDA Analytics Pro) of the Italian and Australian researchers' answers at the question: What are the beliefs that should guide teachers for proposing and when implementing ABM activities?

Based on the insights provided by both Italian and Australian researchers, there are several key beliefs that teachers should hold to effectively implement ABM activities in their mathematics instruction.

Firstly, for implementing ABM activities, teachers should embrace general beliefs about teaching and learning mathematics such as a constructivist or socio-constructivist educational paradigm (CCvM2), e.g. "I think they need to have beliefs that are constructivist and support an inquiry-based approach" (Au.R1, p.30), placing students as the protagonists of their learning process, e.g. "I think that they would need to [..] a kind of general constructivist, socio-constructivist understanding of learners needing to actively construct the knowledge themselves" (Au.R6, p.37). Additionally, they should recognize the teachers' central role in guiding and facilitating this process (CCvM2 b), having confidence in students' willingness and capacity to reconstruct deep mathematical understanding (CCvM2 a).

> [Teachers need to have] the confidence that students want to learn things all the way through. And that they learn things by doing, they learn by experiencing and, also, by questioning, perhaps in a way that is less formalized at first and then may become more formalized. However, the confidence that

> they want to learn and they desire to understand things thoroughly. This, I think, is the fundamental attitude of the teacher. (It.R7, p.23)

Furthermore, they have to believe that the construction of mathematical meanings and deep conceptual and relational (Skemp, 1976) understanding is the ultimate goal (CCvM1):

> They have to believe that developing a conceptual understanding is the ultimate goal and then they should want all students to do that. 'Cause if they don't really want them to understand, [but] they just want them to complete worksheets for them, they might not see the value of the material. [...] Probably we have to believe that there's a purpose sort, that they're not compromising the learning or that they are really focused on the conceptual understanding, and that is the goal that they [the teachers] should want for all students. (Au.R5, p.30)

Teachers should prioritize the development of conceptual understanding and they must genuinely desire all students to achieve this understanding, conceiving it as the primary objective.

This aligns with the idea that students benefit from engaging with diverse modes of information and expression, which ties in strongly with the belief that learning is optimized in inclusive and well-being-oriented contexts. Indeed, researchers indicates that teachers must believe in the value of proposing activities that enhance the use of multiple channels of information access and production in the learning process (CCvM5), and emphasized creating environments that support the diverse needs an well-being of all students (CCvM4): "The beliefs that could potentially lead the teacher to embrace this alternative view of mathematical practice, which is, indeed, a rather different view, revolve around the idea that these activities can genuinely promote well-being, if, let's say, well designed, benefiting both the teachers and the students." (It.R8, p.42)

> Then, in my opinion, the belief that - and maybe the experience is the thing that gives it to you the best - that giving the opportunity for students to communicate with you, and receive, let's say, information from you, and give information back to you in multiple different ways - simplifying, those of typically of the 4 channels of access and production of information, but, if you want, there are more - so, not only to communicate with written or verbal language, but also, for example, kinaesthetic, and informal verbal, does it count. Therefore, to accept a multi-level communication - multimodal if they happen together, and if not, through multiple channels - to open the proposal on multiple channels, and to have the belief that this actually facilitates more students to follow you, to come with you in the construction of knowledge, is fundamental. [...] To be convinced that really having more channels is better, because I take more students with multimodal activities, so where I use not just one but more of these ways of interacting with teachers, but also with your peers, and, also, through the manipulation of the artifact. So, let's say: there's the level with the artifact, there's the level with the classmates, there's the level with the teacher, and, yet, I can use all these ways that I've

> constructed, including physical. "I'll show you": I have to accept that as an answer, and then from there I build the discourse. (It.R4, p.32)

Specifically, the Italian researchers also emphasize that teachers should not consider Mathematics as a purely mental discipline, relegated to the world of abstraction. Instead, they should acknowledge it's, at least, dual nature, where aspects of concreteness and abstraction are intertwined (CCvM3):

> The idea that mathematics is not a purely mental discipline, instead it encompasses a double component of mathematics, at least a dual nature, let's say, which refers to activities of a physical, manipulative nature, and then to reflections from the point of view... - aided by language and other tools, to reconstruct and understand, let's say, what are the mathematical meanings. (It.R3, p. 40)

Concerning the second typology of beliefs, the specific beliefs concerning the ABM activities, according to both groups of experts, foremost the teacher must believe in the value of the activity. It means that they should be convinced of their educational effectiveness (CCvB3): in particular, that these activity does not undermine learning and rather promotes the development of a deep conceptual understanding of the curricular content (CCvB3 b,d): "[...] they need to be convinced that this approach can somehow assist them in attaining educational objectives that are otherwise challenging to reach or may not even be pursued" (It.R1, p.73).

Moreover, they should have confidence in their ability to implement ABM activities effectively (CCvB1): e.g., "I may have certain positive beliefs about mathematics, but feel unable to teach in that way for a wide lot of reasons" (Au.R4, p.32-40). It's not just about believing in the value of these activities but also feeling capable of integrating them into their teaching practices: "The teacher must somehow be convinced or convinced that them can do it, that them can handle that thing. Then, there is a level concerning formative aspects and a level related to beliefs about one's abilities, in a certain sense." (It.R1, p.29)

Finally, an additional belief, highlighted only by the Italian researchers, concerns an even more primitive belief necessary for having the predisposition to embrace a teaching innovation, including ABM activities: belief in the potential for improvement. Definitely, teachers must be convinced that their teaching can be improved (CCvB2).

4 Concluding remarks

In summary, the research provides a comprehensive framework based on th perspectives of researchers regarding the general and specific beliefs that a teacher should embrace for and when implementing ABM activities. These beliefs encompass pedagogical philosophies, goals for mathematics education, and specific beliefs related to the value and

feasibility of incorporating ABM activities into the teaching practice.

These beliefs reflect a commitment to student-centred, (socio-) constructivist learning experiences that prioritize conceptual understanding and well-being in mathematics education for all students, and for teacher, as well as the confidence in both their essential role in facilitating students learning and in the students' willingness for a deep learning, giving the opportunity to all of them to access to the mathematical concepts.

Furthermore, it's essential for teachers to recognize the manifold aspects of mathematics that lie beneath its abstraction. They also have to be convinced that they can improve their teaching, being able to effectively implementing ABM activities, and, certainly, that students learning will be enhanced by implementing them.

These core beliefs, as identified by both Italian and Australian researchers, with no substantial difference, provide a foundation for teachers to successfully implement ABM activities in their mathematics instruction. They encompass pedagogical, conceptual, and practical dimensions, highlighting the importance of aligning beliefs with effective teaching practices.

5 Limitation and further steps

Indeed, for the explorative nature of our research, we could only hypothesize potential beliefs relevant to and for the implementation of ABM activities. By including, combining and comparing the viewpoints of diverse groups of experts, e.g., from other different countries, the resulting framework may highlight different or additional hypotheses of beliefs that should be guiding principle for teachers in implementing ABM activities. Therefore, it might be valuable to replicate this study with different cohorts of experts.

The natural further step of the research is to directly investigate the beliefs of mathematics primary and secondary mathematics school teachers, linked to ABM activities implementation, building upon the hypotheses generated from the presented preliminary research.

Moreover, the presented research approach, which involves elucidating hypothetical beliefs that should guide teachers for and when implementing an innovation in school, from the perspective of research innovation developers, could be seen as a model which can be also adapted to other object of study, different from ABM activities. Particularly, it can assist teacher educators in effectively facilitating the realization of innovations into school settings.

Research ethics

The research underwent a review process by the Scientific Research Ethics Committee (CERS) of the Free University Maria SS. Assunta, Protocol No. 14/2021, and the Human Research Ethics Committee of the Australian Catholic University (ACU HREC), Protocol No.2021-199E, receiving positive feedbacks from both.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

Data is unavailable due to ethical restrictions.

Conflicts of Interest

The authors declare no conflicts of interest.

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