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Cognitive dissonances in mathematics teachers: A cross-case study

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Abstract: The presence of socio-affective variables is a constant in classroom interactions between teachers and students, and teachers need to understand how such affect and shape teaching and learning experiences. This research focuses on the search for cognitive dissonances that might occur in teachers who teach mathematics, identifying discrepancies between what they value and how they put it into practice in the classroom. A cross-case study research design is chosen to facilitate the comparison of similarities and differences between units of analysis or cases. The results show variations in the discrepancies obtained according to the selected study variables of gender, educational stage, and teaching experience.

Keywords: cognitive dissonance, mathematics teachers, affect system, discrepancies, socio-affective variables

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1 Introduction

Publications such as McLeod's (1992) work on affective domain in mathematics at the end of the last century sparked a growing interest in the subject accompanied by a huge scientific production associated with it (Schukajlow et al., 2023). One of the conclusions that can be drawn from a critical look at this evolution is that it is difficult to reach a consensus on how to understand and characterize the main construct of affective domain in mathematics, as well as the dimensions that comprise it and the relationships that are established between them (Grootenboer & Marshman, 2016). This has led some of the research in this field to focus its efforts either on the study of a single dimension or on the study of affect as a structure (Hannula, 2019). We decided in favour of the latter, which allows us to relate the main constructs to what Pepin and Roesken-Winter (2015) call the “dynamic affect system”. Moreover, these affective systems and the relationship between their constructs will not only influence the individual and his or her decisions (Schoenfeld, 2015) but also the group with which he or she is interacting. The presence of these affective processes is constant in classroom interactions between teachers and students, creating a system that feeds back on each other and influences the teaching and learning processes in mathematics. Hence the importance of teachers understanding how their own and their students' affective systems impact and shape both their own teaching, including the instructional choices they make, and the learning experiences (Hannula, 2015; Hargreaves, 1998). One of the problems for such understanding is that teachers are often not fully aware of their own emotions, the signals they receive or the responses they trigger. It is necessary, therefore, that we begin to pay more attention to the emotional life and conflicts that fill our unconscious (Fernández et al., 2009). It is based on reflection that we can be aware of and identify cognitive dissonances that might occur among teachers who teach mathematics. Therefore, the aim of this study is to capture potential cognitive dissonances through a questionnaire designed to bring to light discrepancies between what teachers consider to be relevant practices for learning mathematics and the attention they devote to such practices in the classroom as part of their teaching activity. In line with this we pose the following research questions: are there differences between what mathematics teachers and educators consider important for learning and what they actually foster in the classroom? Do such discrepancies, if any, vary according to gender, educational stage or teaching experience?

2 Theoretical framework

The emotional process known as cognitive dissonance arises when dealing with cognitions that do not match, where cognition is what a person knows, does, feels or the environment surrounding that person (Festinger, 1957). In the context of this research, we will refer to cognitive dissonance in terms of the tension that teachers experience when they encounter information or practices that conflict with their prior beliefs or knowledge about how to teach mathematics, that is, we are interested in dissonances as pairs of opposing forces that pull teachers in different directions having to choose one of them (Andrà et al., 2019). On the other hand, as dissonance generates psychological discomfort or distress, from the moment it appears the individual tries to eliminate or reduce it and avoid any type of information that may increase it (Festinger, 1957; Harmon-Jones & Mills, 2019). In the classroom and working with students, it is difficult to have absolute control over the interactions and events that occur, so that a new cognition can alter the system of consonance that teachers try to maintain. Any new information or situation may provoke unavoidable tensions that sometimes can be resolved and sometimes teachers will have to find ways to consonate them to be able to live with them even if they are not resolved (Liljedahl et al., 2023). The first step for teachers to learn how to manage cognitive dissonance is to be aware of the situations that may cause them discomfort, to think about the teaching model they are pursuing and the one they actually put into practice in the classroom. According to Festinger (1957) dissonance can be reduced or even eliminated by changing some of the individual's knowledge or behaviour, which also includes adding new consonant cognitions or diminishing the importance of dissonant or inconsistent cognitions. We emphasise here the idea not only of systems of thought but of affective systems whose elements are dynamically related, influencing the individual and his or her behaviour, which we will analyse using the “dynamic affect system lens” (Pepin & Roesken-Winter, 2015). In addition, mathematical identity is inextricably linked to mathematical experiences in the classroom and only by working more consciously on an affective level will mathematics teachers be able to question and challenge their identity as mathematics teachers (Grootenboer & Edwards-Groves, 2019). When teachers are in the classroom, they assess each of the situations that arise and when dissonant cognitions appear they will work to reduce them based on their own system of goals, norms, and attitudes. Knowing more about these processes will allow us to understand and shed light on how the affective domain works.

3 Methodology

Being the main objective of the study focused on the identification of cognitive dissonances, based on the reflection of situations in which we find differences between what is valued and what is done, a quantitative approach was chosen and, within this framework, a cross-case study research design. This type of design is particularly suitable for the comparison of commonalities and differences between units of analysis which, in themselves, constitute a case and which, in this research, correspond to in-service teachers. In this study, the cases were defined by crossing three variables, namely: gender (male/female), educational stage (Primary/Secondary/University-Teaching Training/University-Mathematics) and teaching experience (Junior: 5 or less years/Senior: 6 or more years), thus giving rise to a total of 16 cases (2x4x2), each comprising a single participant. Why these three variables? First, because previous research on affect has found gender-related differences; second, since cognitive dissonance causes discomfort for the individual and forces them to manage it, we decided that using novice and more experienced teachers would give us an opportunity to study such management. Finally, we decided to analyse the differences between teachers at different levels of education, but in a way that somehow encompassed examples of different types of teachers who teach mathematics.

The cross-case study was organised in two stages, although here we will only discuss the results obtained in the first stage. The first stage aimed at an initial exploratory identification of dissonances based on apparent contradictions between the relevance given to certain practices for learning mathematics and the frequency with which certain practices are implemented in daily teaching activity. The second will be based on these contradictions and whether they generate or have generated dissonances in the participating teachers.

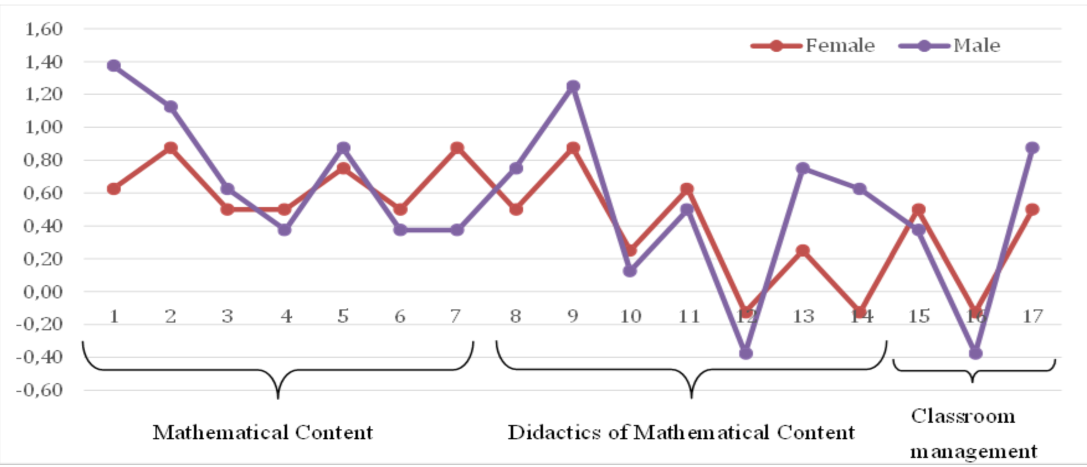
Data collection was done through an online questionnaire in which each case was first introduced to 17 good practices in mathematics education and asked to rate their relevance on a scale of 1-5 where 1 represented the option "Not at all relevant", 5 the option "Very relevant" and the rest intermediate values. Next, the questionnaire showed the same 17 practices and asked about the frequency with which they were used in the classroom, again with 5 levels, with 1 being the value "Never" and 5 the value "Very frequently", leaving the rest of the values for intermediate frequencies. The 17 selected “good” practices were taken from the POEMat.ES observation guideline, which is a validated instrument used to collect information on the performance of the mathematics teacher while teaching. The POEMat.ES observation guideline is organised into three dimensions (mathematical content, didactics of mathematical content and classroom management) and seventeen indicators, with each indicator graded into four levels (Joglar et al., 2021). Similarly, our questionnaire presents 7 questions oriented to actions related to mathematical content, another 7 related to didactics and the last 3 related to classroom management, making up the 17 practices already mentioned. We call them "good" practices because they are based on the highest level of performance that Joglar et al. (2021) claim is aligned with what the group considers to be the standards for mathematics teaching and learning in the classroom. The complete list of actions included in the questionnaire used in our research as well as how they were proposed is shown in Appendix A.

4 Analysis of the results

In this section we present the results obtained after the application of the initial questionnaires to the 16 participants in our study. To carry out the analysis and cross-case comparison of the responses to the questionnaires, we have represented the discrepancies identified by means of frequency polygons in conjunction with radial diagrams that allow us to show more clearly the dimensions referred to in the previous section of mathematical content, didactics of mathematical content and classroom management. The value represented in each question comes from subtracting the value of the implementation of the corresponding action from its assigned value in terms of relevance. This will show the coherence between what each case considers important to do in the classroom and whether they practice it in their daily classroom life. The further the results are from zero, the greater the discrepancy. This is done individually for each of the cases and then grouped according to the variable under study, calculating the arithmetic mean, this being the value shown in the graphs below.

Firstly, if we focus on the results obtained grouped by gender, we observe a greater overall discrepancy in men than in women (Figure 1). Only in six of the seventeen questions there is a smaller difference between what they value and what they do in men than in women; and in five of these six the values are very close to each other.

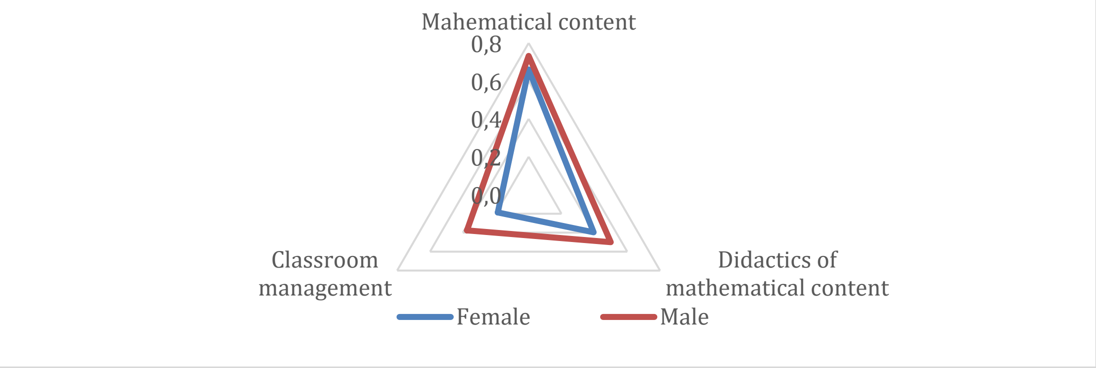
Figure 1. Discrepancies between importance and frequency of implementation by gender.



Note. The figure depicts the mean discrepancies for each questionnaire response for men and women.

If we look at the corresponding radial diagram, the greatest discrepancy is found in *Dimension 1*, related to mathematical content; both men and women show differences between the high relevance they attach to the practices being assessed and the low implementation of these practices in their own classes (Figure 2).

Figure 2. Discrepancies by gender grouped into dimensions.



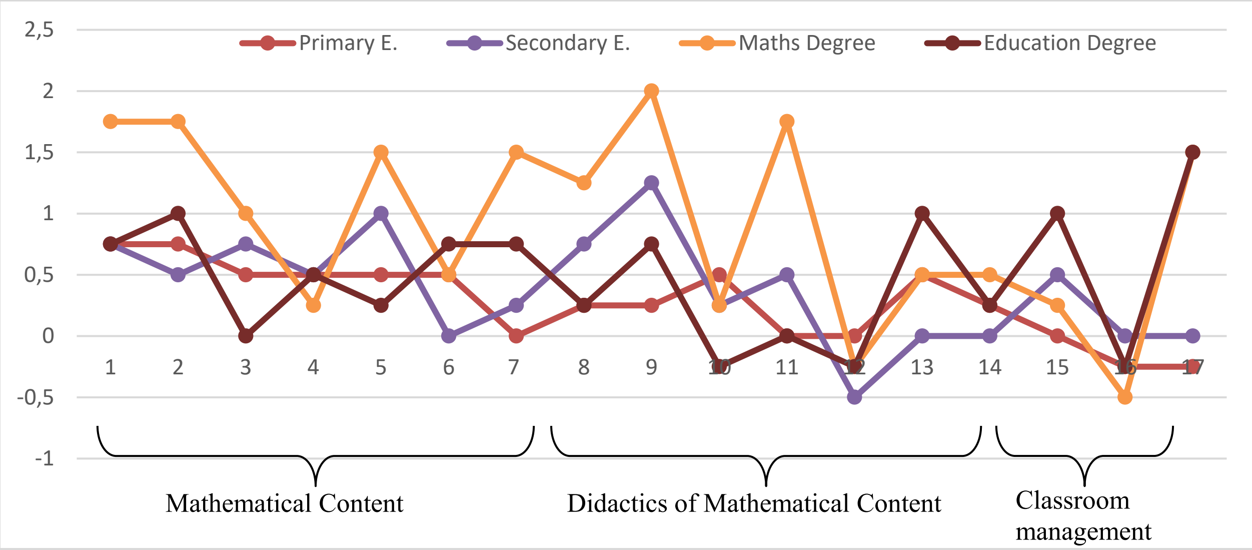
Note. The radial diagram groups the discrepancies by gender in the three dimensions in which the questionnaire is organised.

On the other hand, Dimension 3 is the one that shows the greatest difference by gender, where women barely reflect dissonance in the actions related to classroom management, although it should be noted that, as can be seen in Figure 1, in question 16 the result offers a negative value, so that the action is put into practice more than previously assessed, giving an overall computation close to zero.

These differences observed between men and women are in line with many other differences that research in recent decades has identified in both students and teachers in relation to affective aspects such as self-concept or anxiety, mainly (e.g., Relich, 1996; Szczygiel, 2020). In the specific case of the differences associated with cognitive dissonance, it will be the in-depth interview with the 16 cross-study cases that will be able to shed light on the origin of this gap.

In terms of the educational stage variable, university teachers who teach in the bachelor's degree in mathematics are the most dissonant and, at the other extreme, we find teachers at the Primary Education stage (Figure 3).

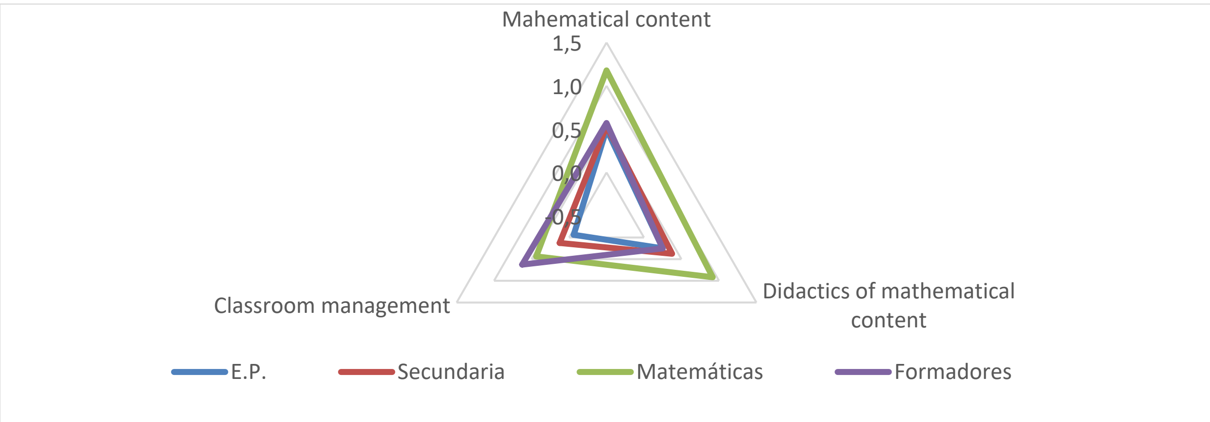
Figure 3. Discrepancies between importance and implementation by educational level.



Note. The figure depicts the mean discrepancies for each questionnaire response for each group of teachers studied, depending on the level and grade.

In Dimension 1 and Dimension 2, related to mathematical content and the didactics of mathematical content, respectively, less dissonance is observed in both Primary and Secondary school teachers and in university teacher educators, although the latter show a notable increase in discrepancy in Dimension 3, classroom management. Teachers at secondary education level, in turn, show greater dissonance than those at primary level in the second and third dimensions (Figure 4).

Figure 4. Discrepancies by educational level grouped into dimensions.

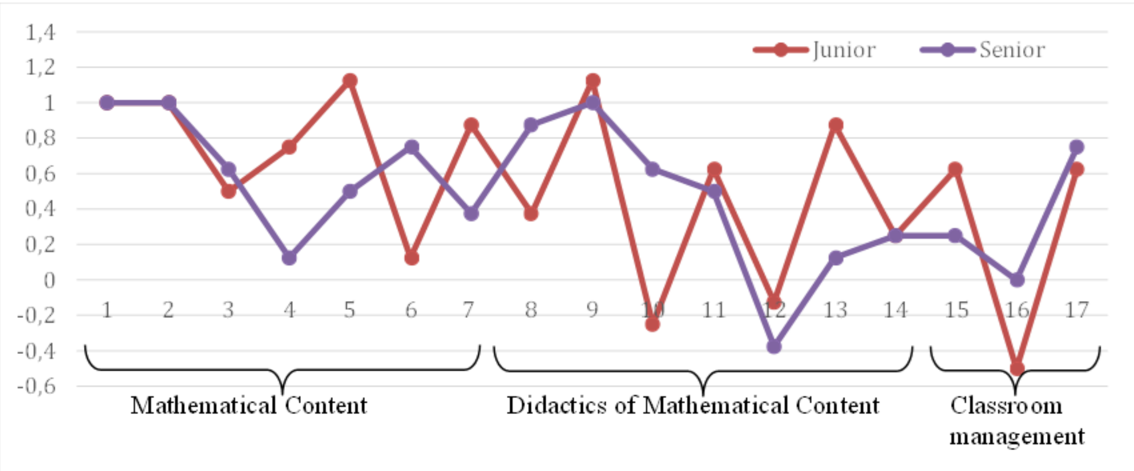


Note. The radial diagram groups the discrepancies by educational level in the three dimensions in which the questionnaire is organised.

The results show smaller differences between the assessment and implementation in the classroom between university teachers of education and teachers of primary and secondary education. As we have seen in the interviews, these teachers have received training in mathematics didactics or have felt the need to train in it on their own or in a self-taught way to face the problematic situations and characteristics of their classrooms. Their own experience in the mathematics classroom influences the development of their mathematical identity (Grootenboer & Edwards-Groves, 2019), which shapes their knowledge and beliefs in such a way that they value and need the implementation of the actions reflected in the questionnaire. Similarly, university lecturers who teach on the Bachelor's Degree in Mathematics are exposed to other types of experiences and this may influence their belief system and the didactic design of the classes they intend to teach (Erens & Eichler, 2019), which may mean that in their belief system these types of actions are less important, which would allow them to reduce the weight of these actions, even if they are dissonant or inconsistent with what they value.

Finally, if we look in detail at the discrepancies per question, we find a greater overall discrepancy among teachers with less experience (Figure 5). The biggest difference, however, is in Dimension 1, which corresponds to the actions related to mathematical content (first seven questions), with the greatest dissonance being shown by teachers with less teaching experience. As we move forward in the representation, we can see that the differences decrease in both groups, although it is still lower in teachers with more teaching experience.

Figure 5. Discrepancies between importance and implementation by teaching experience.



Note. The figure depicts the mean discrepancies for each questionnaire response for less experienced teachers, junior teachers, and more experienced teachers, senior teachers.

As already mentioned, we do not include the results obtained in the semi-structured interviews due both to the space limitations that this document must respect and to the fact that they are currently still being carried out and analysed. In any case, it can be anticipated that a first result from the interviews shows, in relation to teaching experience, that the most experienced teachers state that they have learned to manage dissonance, although in some cases they do so by accepting the reality of the classroom and its situations and incorporating cognitions related to the impossibility of change due to the inertia of the system, the characteristics of the students or the constraints of the institution.

Accepting the reality of classroom situations would explain a management of the differences between what they should do in the classroom and what they actually do, that is, fewer discrepancies in the questionnaire responses. According to Erens and Eichler (2019), teachers' beliefs may change depending on the stage of their professional development, influenced by their experiences and reflections in the search for greater adaptability to conceptions of mathematics teaching.

5 Final reflections

After studying the results obtained in the questionnaire, we can highlight some of the characteristics of this first exploratory analysis of cognitive dissonance present in mathematics teachers. Thus, firstly, although it was not part of the objective of this research, we observe how the teachers, without differences of gender, experience or educational stage of their professional activity, value very positively all the practices described in the questionnaire in terms of their relevance for the teaching and learning of mathematics. Given that, in this case, we start from a validated observation guideline in which experts in mathematics education have selected actions that come from models of mathematics teachers' specialised knowledge and other studies with a focus on mathematics teaching (Joglar et al., 2021), we can say that teachers recognise the importance of using and implementing these types of actions. However, the responses do point to possible dissonances when analysing how they incorporate these practices into their professional activity, with differences appearing, some of them very notable, between what they think and what they do. These discrepancies serve as a starting point both for the teachers' reflection on their teaching practice and for the approach to the emergence and management of dissonance in the semi-structured interviews, especially bearing in mind that the differences seem to be sensitive both to gender and, above all, to teaching experience and educational stage. The first could be a legacy of the gender gap that is observed from the early stages of education in relation to issues such as mathematical self-concept or mathematical anxiety (Relich, 1996; Szczygiel, 2020). With regard to the educational levels, we suspect that the results obtained point to a different way of experiencing beliefs towards mathematics teaching. Thus, teachers who are more closely linked to educational stages where the presence of didactics of mathematics is more obvious, show a lower level of overall dissonance towards the actions proposed, than those who work in stages more focused on the mathematical content itself.

The study has limitations associated with the research design itself and the way in which it has been conducted. On the one hand, the cases were selected with the purpose of acting as instrumental cases, but there is always a risk that this might not be the case and that some cases might have been included that could be considered more intrinsic. A replication of the study with new cases could help in this respect. At the same time, it should be taken into account that each case interprets the content of the questionnaire on the basis of its own previous training and knowledge, both mathematical and pedagogical, which may lead to answers that do not correspond to what is requested but to what each participant considers to be asked and which, in some cases, may be very different from the initial purpose. In the same way, the value that each case gives to the questionnaire labels on frequency, for example, when considering "Very often" in the context of their teaching activity, one participant may consider that once a month fits this category and a second participant that it only fits if it is put into practice every week. These limitations include the problem of participants answering questions in the way they consider most socially acceptable. Although this might seem a limitation in this first reading of the results, it serves its purpose for the continuation of the research. What matters to us is that they reflect on these discrepancies to see whether cognitive dissonance or as Andrà et al. (2021) say tensions arising between different views. We try to minimise these limitations in the interview by asking each participant for information on what they have interpreted and how they have responded to the questions in the questionnaire.

The interview will allow us to gather more information on whether these discrepancies can generate or have generated cognitive dissonance in the teacher. In this way we will try to differentiate responses related to social desirability and be more specific in related situations that may have generated discomfort in the individual.

As for future lines of work, the main one is to continue exploring the existence of cognitive dissonances by replicating the study with new cross-case studies and incorporating complementary studies of an intrinsic nature as well as systematic observation processes in the classroom to observe from the outside possible dissonances that teachers themselves are not able to identify. In addition, the semi-structured interviews will enable us to collect information on how participants deal with such dissonances: by changing reality, changing their beliefs or changing some of the cognitions related to the way they perceive such actions.

Research ethics

Author contributions

“M.M.: conceptualization, investigation, formal analysis, validation, visualization, data curation, writing—original draft preparation, writing—review and editing.
J.M.M.: investigation, methodology, formal analysis, validation, visualization, data curation, project administration, writing—review and editing.
A.M.: formal analysis, validation, visualization, writing—review and editing.

All authors have read and agreed to the published version of the manuscript.”

Informed consent statement

Informed consent was obtained from all research participants.

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Conflicts of Interest

The authors declare no conflicts of interest.

Appendix A

Questions for the identification of possible dissonances concerning actions taken from the observation guideline called POEMat.es.

DIM.	How relevant for learning do you find the following actions? / How frequently you perform the following actions in your mathematics classrooms? *
1	To use of different registers of representation in the presentation and development of mathematical activities/tasks
	To display parallel conversions showing the task at the same time in more than one representational register in addition to the natural language register
	To state or describe (through examples and counterexamples) properties of the object to be defined which are understood as necessary and sufficient conditions for subsequently institutionalizing the definition
	To state or describe (through examples and counterexamples) properties of the object to be defined which are understood as necessary and sufficient conditions for subsequently institutionalizing the definition
	To solve the same mathematical task using different reasoning or solving strategies
	To establish connections of the mathematical concept being worked on with other concepts within the same topic and with other mathematical topics to which it is related
	To work on or from mathematical errors in class
2	To use manipulative material so that students reflect and work on the mathematical content seen
	To pose open-ended questions that involve students in research tasks and allow them to construct new knowledge
	To contextualize the mathematical content with every day or professional situations close to the students
	To promote situations in which students are the protagonists of their own learning and in which your role as a teacher goes unnoticed
	To use mathematical language with rigor, irrespective of the level at which students are
	To take advantage of student interventions to mobilize and reflect on knowledge
	To manage time so that it is mostly dedicated to students working autonomously
3	To use different resources for the exposition of information that can be clearly visualized by all students
	To rely on the use of written material in the classroom
	To react to student behaviour that prevents the normal development of the class
*First, participants evaluate the 17 actions in terms of their relevance for the learning of mathematics and then they are asked to consider again the 17 actions according to how frequently they execute them in their usual mathematics classrooms.	

DIM.1	MATHEMATICAL CONTENT
DIM.2	DIDACTICS OF MATHEMATICAL CONTENT
DIM.3	CLASSROOM MANAGEMENT

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