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Research article

Comparing Likert and Q: The potential of Q-methodology to study pre-service teachers' beliefs of teaching and learning of mathematics

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Abstract: Different methods are used to study pre-service teachers' beliefs of teaching and learning mathematics. A common way is using Likert scale surveys to measure agreement or disagreement on items. In Q methodology, participants are asked to relate statements by ordering them in a Q-sort grid and explicate their sort. Then, the sorts of all participants are statistically analysed to detect different types of sorts (factors). In our study, $N = 25$ pre-service teachers filled out a Likert scale survey and then related the same items in a Q-sort. Our aim is to compare the scope of the two approaches to work out the specific insights of Q. Results from the Likert scale analyses show that pre-service teachers primarily hold constructivist views. In addition, the Q-analysis reveals typical sorts of a certain group of pre-service teachers. They can be described with respect to their subjective view based on distinguishing statements.

Keywords: Beliefs, Likert, Q-Methodology, Q-Sort, Pre-service teachers

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1 Introduction

Beliefs about mathematics teaching and learning play a central role as an affective component of mathematics teacher competence (Philipp, 2007). Typically, in empirical studies, beliefs are methodically investigated via Likert scale-based surveys, in which items are formulated in statements for which the degree of agreement is measured (e.g., Laschke & Blömeke, 2013; Voss, Kleickmann, Kunter, & Hachfeld, 2013). However, Likert scale surveys have been criticized in relation to the study of teachers' beliefs. Criticisms include the fact that items are answered independently of each other and that the situation-specific context of teaching is usually not part of the scales, which may obscure teachers' differentiated interpretation of the items (Safrudiannur & Rott, 2020b, 2021). Thus, an individual subjective interpretation of the items that relates to teachers' classroom experience cannot be expressed. However, these specific interpretations are particularly relevant to the study of teachers' beliefs about teaching and learning mathematics. Hence, there is a need for research methods in which items are related to each other so that teachers' subjective beliefs can be examined in more detail.

Accordingly, in this paper we consider how beliefs can be assessed using an alternative method that has the potential to bring out the subjective nature of beliefs: the so-called Q methodology. Within Q, participants relate items to each other so that researchers can explore in more detail the different 'points of view' that exist on a particular issue between groups of people (Coogan & Herrington, 2011). In this study, therefore, we explore the beliefs of pre-service teachers about the teaching and learning of mathematics using two different methods, Likert scale surveys and Q-sorting, in order to find out what potential the Q-sorting method has in comparison to the Likert scale approach.

2 Pre-service teachers' beliefs

Although the concept *beliefs* is widely used in mathematics education research, there is no commonly shared definition (Philipp, 2007). Philipp provides the following working definition:

> Beliefs—Psychologically held understandings, premises, or propositions about the world that are thought to be true. […] Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action. (2007, p. 259)

Like emotions and attitudes, beliefs are affective in nature, containing also cognitive components (Philipp, 2007). However, unlike professional knowledge, beliefs are not consensual but can be held to varying degrees (Beswick, 2005; Philipp, 2007). They are relatively stable constructs so that it is rather difficult to change them over time (Philipp, 2007; Reusser, Pauli & Elmer, 2011). It is also assumed that beliefs are organized into belief systems, which are grouped around an object in the sense of an overarching affect, and in which different and even contradicting beliefs can coexist (Philipp, 2007).

Despite the conceptual vagueness of the term *belief*, there is consensus that a corresponding intrapsychic construct, with varying degrees of affective and cognitive components, is central to teachers' professional competence and that is becomes relevant in the context of teaching (Schmotz, Felbrich & Kaiser, 2010; Schoenfeld, 1998). Yet, the current state of research with respect to the relationship between teachers' beliefs and their teaching practices is not clear: Some researchers have shown consistent beliefs and practices (e.g., Safrudiannur & Rott, 2017) whereas others have shown inconsistencies (e.g., Cross Francis, 2015; Li & Yu, 2010). One possible interpretation of these inconsistencies is that teachers may hold varying beliefs depending on the situation or context in question (Safrudiannur & Rott, 2020b, 2021).

Beliefs about the teaching and learning of mathematics (Kuntze, 2011; Staub & Stern, 2002) represent a significant dimension of beliefs. When measured empirically, *transmission-oriented beliefs*, in which students are viewed as passive recipients of knowledge (e.g., "Students learn mathematics best by attending to the teacher's explanations."), are often distinguished from *constructivist-influenced beliefs* that endorse the principles of constructive learning (e.g., "Teachers should encourage students to find their own solutions to mathematical problems even if they are inefficient.") (cf. Laschke & Blömeke, 2013; Staub & Stern, 2002).

Empirical evidence from the German COACTIV study with in-service teachers has shown a correlation between constructivist-influenced beliefs about teaching and learning mathematics and the quality of teaching as well as student learning (Voss et al., 2013). In addition, further studies show that pre-service teachers' constructivist-influenced beliefs about the teaching and learning of mathematics increase over the course of teacher education, while transmission-oriented beliefs stagnate (Buchholtz & Kaiser, 2017). Although Voss et al. (2013) found strong negative correlation between them, there does not seem to be a fundamental contradiction between constructivist-influenced and transmission-oriented beliefs. The results of the COACTIV-study point to a crucial functional balance between different beliefs of teachers in varying teaching-learning contexts (Voss et al., 2013). Depending on the teaching situation, different beliefs may be relevant to teachers' professional actions, and beliefs therefore depend heavily on subjective judgements of the situation. Jaschke's (2018) findings support the assumption of subjective situation-specific beliefs. In his study with mathematics pre-service teachers, he finds two types of teachers, one of which shows a rather dichotomous orientation towards a constructivist-influenced belief and rejects transmissive-oriented beliefs, while the second shows agreement on both beliefs and shows a more mixed profile that integrates both constructivist-influenced and transmissive-oriented beliefs.

3 Different methods to investigate beliefs

Different methods can be used to study pre-service teachers' beliefs of teaching and learning of mathematics. A widespread way is the statistical evaluation of Likert scale surveys by exploratory and/or confirmatory factor analyses. These methods are usually applied in studies with larger sample sizes investigating teachers' professional competencies like TEDS-M (Laschke & Blömeke, 2013) and COACTIV (Kunter, Baumert, Blum, Klusmann, Krauss, & Neubrand, 2011). Likert scale surveys measure individuals' agreement or disagreement with a series of statements (items). Then, individuals are assigned a score based on their response pattern to each identified factor of the underlying construct, allowing comparisons to be made. This approach facilitates making comparisons of individuals on well-defined constructs and proves effective for investigating long-lasting beliefs or perspectives. However, the survey method may conceal contextual and subject-related expressions of beliefs. For instance, in traditional Likert scale surveys, teachers with different levels of expertise may rate items differently (Safrudiannur & Rott, 2020a). In addition, there is evidence that context-independent Likert based rating procedures favor socially desirable responses in terms of prioritizing constructivist beliefs (Aeschbacher & Wagner, 2016; Di Martino & Sabena, 2010; Safrudiannur & Rott, 2020b, 2021). Furthermore, Likert scale surveys require that the related statements are presented in the survey independently of context, so that individual statements do not need to be related to each other.

From the point of view of researching beliefs in different groups of expertise (e.g., pre-service teachers vs. practicing teachers) or with reference to different contexts, the question therefore arises of contextual and subject-related methods of surveying beliefs that go beyond previous approaches aiming at persisting constructs. Voss, Kleickmann, Kunter, and Hachfeld (2013) suggest person-centred approaches to meet this demand.

One such evaluation method that has already been used to investigate teacher beliefs (Jaschke, 2018) is the Q-method (Coogan & Herrington, 2011). It allows for the identification of the subjective nature of beliefs. Q-studies investigate correlations between individuals, not items. This methodology does not test or impose predetermined meanings on participants. Instead, participants are asked to determine what holds meaning and significance from their own perspective. They accomplish this through a process called Q-sorting, where they rank belief statements on a range. The data collected from multiple individuals is then subjected to factorial analysis, revealing groups of people who have ranked statements in a similar order. This yields a set of factors represented by all the presented statements configured in distinct and characteristic ways, rather than different subsets of the statements. The interpretation and significance of these configurations are attributed a posteriori through analysis, rather than predetermined assumptions. Different participants may interpret a statement differently, for example, while one person may perceive a transmission-oriented statement as negative, another may interpret it as a positive statement, particularly relevant e.g., for teaching students with learning difficulties (Coogan & Herrington, 2011).

The aim of this study is to compare the scope of the two methods, i.e., Likert scale surveys vs. Q-sorting, to study pre-service teachers' beliefs. In particular, we want to explore the potential of the Q method in this field of research. With our study, we want to answer the following research question: What specific insights does the Q-sort method bring when compared to Likert scale approach?

4 Research design and methodology

In our study, we investigated N = 25 pre-service teachers at the end of their studies (M.Ed. mathematics for primary or secondary school) at Universities of Bremen and Hamburg. All had several months of practical teaching experience at school so that they were able to relate the items with concrete teaching experience in the classroom.

The participants were first presented with a Likert scale survey with 14 statements on beliefs about teaching and learning mathematics from the TEDS-M study as a paper and pencil test, where all items are visible at the same time but are answered independently (Laschke & Blömeke, 2013). The instrument has already been tried and tested in previous empirical studies with larger samples of pre-service teachers, and within the set of items two scales were previously identified by factor analysis: On the one hand a transmission-oriented scale on "learning mathematics through teacher instruction" (eight statements) and, on the other hand, a constructivist-influenced scale on "learning mathematics through active learning" (six statements) (Laschke & Blömeke, 2013; Blömeke, Kaiser, & Lehmann, 2010; Buchholtz & Kaiser, 2017). The statements were to be answered on a 6-point Likert scale (1 = "strongly disagree" to 6 = "strongly agree").

Data collection for the Q method took place immediately after the Likert scale paper-and-pencil test. The participating students placed the same statements in a corresponding Q-sort grid (-3, -2, -1, -1, -1, 0, 0, 0, 1, 1, 2, 3; see Figure 1), with one statement at position -3 being assigned the lowest level of agreement and the one at position 3 the highest. The sorts of all participants were then statistically analysed to detect different types of sorts (factors). The evaluation of the Q-sorts was done using the software KADE v1.2.1 (Banasick, 2019). In a first step, a Q correlation matrix was calculated, representing the relationships between two Q-sorts. This matrix was then subjected to principal component analysis followed by varimax rotation.

Table 1. Empty Q-sort grid.

-3	-2	-1	0	1	2	3

5 Results

5.1 Likert Scale Survey

As mentioned above, the items used in this study were intensively used in other studies with larger samples of comparable groups of pre-service teachers (e.g., Buchholtz & Kaiser, 2017; Laschke & Blömeke, 2013), and form two reliable scales: the transmission-oriented scale on "learning mathematics through teacher direction" vs. the constructivist-influenced scale on "learning mathematics through active learning" (Laschke & Blömeke, 2013). To confirm this structure, due to the small sample in our survey, only exploratory factor analysis could be calculated with the data. We performed a principal component analysis (PCA) to extract the most important independent factors. Kaiser-Guttman criterion showed six factors with eigenvalues >1, and four factors showed an explanation of variance over 10%, furthermore, several items showed multiple loads on factors, thus the structure could not be confirmed with our sample. However, to meet the theoretical assumptions of the two belief scales, and for the purpose of this study, ultimately only the two factors with eigenvalues ≥ 2 were considered, but they accounted for only 35% of the total variance. We therefore only report descriptive statistical data of the participants' sum scores on the two scales (Table 1).

Table 2. Sample means and range of participants' agreement to Likert scale items.

Scale	Items	Mean	Min.	Max.	SD
Learning mathematics through teacher direction (transmission-oriented)	8	2.17	1.25	2.75	.42
Learning mathematics through active learning (constructivist-influenced)	6	5.18	3.83	6.00	.52

Results show that our pre-service teachers primarily hold strong constructivist beliefs and rather disagree with transmission-oriented items. However, possibly due to the small sample size, no significant manifest correlations between the two scales were found and the results need to be interpreted with caution.

5.2 Q-Sort

The analysis of the Q-sort revealed three different factors, i.e., typical sorts of a certain group of pre-service teachers. The factors explained 77 % of variance in the sorts. In contrast to conventional factor analysis, in Q-factor analysis people (or, more generally, sorts) load on factors with factor loadings. In analogy, statements have *z*-scores on factors that indicate how much they represent a particular factor. Figure 2 shows an example of a composite Q-sort (exemplary for Type 2, see below) calculated in KADE (Banasick, 2019). The statements are sorted in ascending order of their *z*-values into the slots of an empty Q-sort from -3 to +3 (see Figure 1).

Figure 1. Composite Q-Sort of Type 2; * distinguishing statement at p < 0.05; ** distinguishing statement at p < 0.01; ► z-score for the statement is higher than in all other factors; ◄ z-score for the statement is lower than in all other factors.

-3	-2	-1	0	1	2	3
** ◄ Teachers should allow pupils to figure out their own ways to solve mathematical problems.	** ◄ Time used to investigate why a solution to a mathematical problem works is time well spent.	Pupils learn mathematics best by attending to the teacher's explanations.	Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.	** ► Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.	It is helpful for pupils to discuss different ways to solve particular problems.	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.
		It doesn't really matter if you understand a mathematical problem, if you can get the right answer.	Pupils need to be taught exact procedures for solving mathematical problems.	** ► Hands-on mathematics experiences aren't worth the time and expense.		
		When pupils are working on mathematical problems, more emphasis should be put on getting the correct answer than on the process	The best way to do well in mathematics is to memorize all the formulas.	Pupils can figure out a way to solve mathematical problems without a teacher's help.		
			To be good in mathematics you must be able to solve problems quickly.			

Of the 25 participants, 20 could be clearly assigned to one type. Five people had strong secondary loadings and could not be clearly identified. Eleven people in the sample load on Factor 1 ($p < 0.05$), which we a posteriori call Type 1 “understanding-oriented constructivist”. People of this type prioritize the statement “In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.” in the highest position. This statement is followed by various constructivist-influenced statements emphasizing the importance of giving students enough time in class to develop and explore their own solutions. A one-sided focus on memorizing formulae is least important to these people.

A small group of two people load on Factor 2 ($p < 0.05$), which we a posteriori call Type 2 “understanding-oriented demonstrator” (see Figure 2). Like people of Type 1, people of Type 2 also rank the above statement focusing on understanding highest.

However, Type 2 people are strongly opposed to allowing students to find their own solutions. They prefer that students are shown different ways of solving problems, but only if these are standardized and can be recorded in a time-efficient manner.

Seven other people load clearly on Factor 3 ($p < 0.05$), which we a posteriori call Type 3 "time-conscious constructivist". This type emphasizes awareness of the sensible use of time in learning and teaching mathematics. People of this type think that it is important to emphasize the importance of different ways of solving a task, to encourage students to develop their own solutions, to reflect on ways of solving problems and to avoid rigid procedures. At the same time, they emphasize that understanding different ways to solve a problem and deep engagement with mathematical tasks are more important than quick results.

6 Discussion

With regard to the Likert scale surveys, we reported the means for the entire sample, which indicate that students tend to hold constructivist-influenced beliefs and reject transmission-oriented beliefs. These results are in line with previous similar analyses of German pre-service teachers, such as shown by Buchholtz and Kaiser (2017). As our sample consists of pre-service teachers at the end of their studies, our results also support previous results that constructivist-oriented beliefs about the teaching and learning of mathematics increase over the course of teacher education, while transmission-oriented beliefs stagnate (Buchholtz & Kaiser, 2017; Voss et al., 2011). Yet, we have no information about our participants' beliefs at the beginning of their studies so that we cannot make any statement about their longitudinal development.

In the analysis of the Q-sorts, three factors could be identified, which we have called a posteriori as Type 1 "understanding-oriented constructivist", Type 2 "understanding-oriented demonstrator" and Type 3 "time-conscious constructivist". When analysing the distribution of the transmission-oriented and constructivist-influenced items in the composite Q-sorts of the different types, a clear pattern emerges: In Types 1 and 3, there is a clear distinction between constructivist-influenced and transmission-oriented items in that all transmission-oriented statements are on the left side (i.e., tend to disagree), whereas all constructivist-influenced statements are on the right side (i.e., tend to agree). Type 2 has a mixed form in that all transmissive-oriented statements are in the middle, while all but one of the constructivist-influenced statements are spread either on the side of highest or lowest agreement. Thus, Types 1 and 3 are both constructivist in nature, while Type 2 is mixed. Our types are therefore consistent with Jaschke's (2018) constructivist and mixed types. Yet, going beyond Jaschke's types, we were able to describe more fine-grained groups of pre-service teachers, because we were also able to distinguish between a constructivist-oriented group, for whom deep understanding of content is also important, and a group that is especially concerned with a conscious use of time. Thus, the distinguishing statements provides us with information about underlying frames of reference that were important for the pre-service teachers when looking at all the items together.

These distinctions reflect the subjective views of different groups of people about the statements and their corresponding relative prioritization, which emerge from relating the individual statements in the context of a Q-sort. In addition, they provide us with additional information about the frames of reference that are seemingly important for the pre-service teachers when sorting the statements (focus on understanding vs. time-consciousness). Such a clear interpretation is not possible on the basis of Likert scale surveys. The composite Q-sorts, which represent prototypical Q-sorts of people of the different types, allowed us to describe the subjective view of people in more detail.

The results presented here are based on the Q-sorts only. In addition to the data from the Q-sort, we also collected the participants' explanation of their sort as well as a subsequent interview about the differences between their sort and the answers in their survey. Accordingly, further in-depth interpretations and insights into teachers' beliefs were possible when these data were also analysed. Taking into account the explanation and interview would offer, for instance, the possibility to get more information about subject-specific frames of reference for the interpretation of the items, which cannot be gained from the Q-sort alone. This would also provide an opportunity to revise the items for Likert scale surveys on beliefs about teaching and learning mathematics to make them more situation specific. So, there is even more potential in the Q-methodology than what we have explored within this study. To conclude, while Likert scale surveys offer the possibility to get average results on cross-contextual item scales about large sample sizes, Q methodology offers a promising approach to describe the subjectivity of groups of individual pre-service teachers' beliefs in more detail.

Research ethics

Author contributions

Nils F. Buchholtz: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, reviewing & editing

Maike Vollstedt: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, reviewing & editing

All authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all research participants.

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Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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