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Perceived difficulty of a mathematical task: Do teachers and students have a common view?

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Abstract: This paper illustrates the first results of a qualitative research investigating the factors which influence students and teachers' perceived difficulty when approaching and solving a mathematical task. The factors that contribute to increasing or decreasing the perceived difficulty concerning a math task have been predominantly studied with respect to student's perceived difficulty. We started from factors that characterize students' perceived difficulty (identified in previous studies) and compared them to the ones expressed by teachers that we have collected, in order to highlight some of the teachers' beliefs connected to perceived difficulty in mathematics.

Keywords: perceived difficulty, teachers, students, mathematics education

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1 Introduction

The topic of difficulty in mathematics has been discussed and analysed thoroughly in research, describing a collection of factors influencing it, such as mathematical content (Radmehr & Drake, 2017), wording of the problem (Bolondi et al., 2018), text comprehension (Casalvieri et al., 2023; Spagnolo et al., 2021) and affective factors (Zan et al., 2006). All the aforementioned factors seem to have an influence on the difficulty of the students when solving various kinds of mathematical tasks. On the other hand, in Mathematics Education research very little can be found about the perception of difficulty and a definition of "perceived difficulty" has not been formulated yet.

Students' perceived difficulty has been analysed and some of the factors that seem to influence it have been qualitatively discussed (Saccoletto & Spagnolo, 2022). Teachers' perspective about the theme conversely has not been considered in depth. In fact teachers do not seem to be always aware of the reasons behind students' mistakes (Faggiano et al., 2023), but the factors that influence their perception of difficulty and consequently their beliefs about the issue have not been explained. This particular aspect of teachers' beliefs can be considered part of a wider topic, which is teachers' beliefs related to students' mathematical thinking (Philipp, 2007).

The aim of this study is to highlight factors that may influence teachers' perceived difficulty regarding a mathematical task. These factors do not always coincide with students' ones, and this allows us to highlight some of the teachers' beliefs concerning perceived

difficulty. In order to do that, the factors characterizing students' perceived difficulty will be compared with those of the teachers.

2 Theoretical background

At the moment, in Mathematics Education there is not a common definition of *perceived difficulty*, despite the fact that it appears to be an important factor influencing students' behaviour when approaching and solving tasks (Doz et al., 2023; Eccles & Wigfield, 2020; Saccoletto & Spagnolo, 2022). It is commonly known and accepted that the perceived difficulty is different from the difficulty of a task, as the latter is usually evaluated in retrospect, referring to the results achieved by students, while there is not a definition of the former.

In metacognition research, instead, the issue of subjective difficulty has been studied and investigated during the last 30 years, under different names (e.g., Doz et al., 2023; Eccles & Wigfield, 2020). Sometimes perceived difficulty has been considered a type of manifestation of self-efficacy, but that does not seem a rightful unification (Eccles & Wigfield, 2020). On the other hand, some close or overlapping concepts were defined, such as the "feeling of difficulty" (FOD), defined in different moments by Efklides between the last decade of the twentieth century and 2011. It has been defined as a "metacognitive experience that monitors cognitive processing as it takes place" (Efklides & Touroutoglou, 2010, p.172) and it is explicitly different from the perceived difficulty due to its "experiential nature".

Despite being conceptually different, feeling of difficulty and perceived difficulty are sometimes used as synonyms (Nuutila et al., 2021). Acknowledging their differences, in this paper we referred to some of the characteristics of perceived difficulty and of the feeling of difficulty, adapting them to the context of mathematical education research. From this point of view, the synthesis proposed in (Doz et al., 2023) was extremely useful and aligned with our perspective, stating that the nature of feeling of task difficulty is metacognitive as it comes from the monitoring activity of an ongoing task processing and the awareness of this process influences self-regulation, effort, affect and strategy use.

The factors that contribute to increasing or decreasing the perceived difficulty concerning a math task have been predominantly studied with respect to students' perceived difficulty (Saccoletto & Spagnolo, 2022). For our analysis we started from factors that characterize students' perceived difficulty (identified qualitatively in Saccoletto & Spagnolo, 2022) and compared them to the ones expressed by teachers that we have collected. Categories described in Saccoletto and Spagnolo (2022), which we are going to recall below, are supercategories progressively defined, used to group similar students' answers in relation to the factors that influence their perception of difficulty, as expressed in their answers. From this qualitative study characterizing students' perceived difficulty in mathematics, it emerges that perceived difficulty in mathematics can be classified into the following categories: *Resolution strategy, Capability and experience, Emotions, Task formulation* and *Personal consideration*. It is important to highlight that the categories are not mutually exclusive, so most of the answers contained, in fact, elements belonging to more than one of them.

The first category is *Resolution strategy,* and it contains answers that openly mention the strategy needed to solve the task, in students' opinion, or the fact that some elements were necessary to achieve a solution such as calculus or a reasoning.

The second category is *Capabilities and experience*, which is the broadest one. It includes the answers referring to the students' opinion of their competence and capabilities, but also different elements such as their previous experiences solving problems of a similar type and consequently their familiarity with the kind of task. In general, the answers that state that a problem is easy when it is similar to something already done or seen in the past belong to this category. Moreover, the category comprises also answers focusing on students' self-perception, doubts about their answer and issues faced solving the problem, also related to the time spent doing it.

The third category is named *Emotions,* and it regards the answers involving the emotions that, as the authors report, were very few.

The fourth category is *Task formulation*, which includes the answers mentioning the formulation of the task, especially its textual aspect.

The fifth category is *Personal consideration*, and it is understood as a student's personal reflection relative to his or her own success in mathematics.

3 Methodology

3.1 Sample description

Being this a first qualitative study, we involved six Italian high school teachers (who teach from grade 9 to grade 13 in Italy), who volunteered to participate in the research. The sample is obviously not statistically significative, anyway the teachers' characteristics are diverse, regarding type of school and curriculum in which they teach, years of experience and background. In particular, five teachers of our sample work in scientific high school (called "Liceo Scientifico" in Italy), in different curricula among which the sports one, while one works in a humanistic high school (called "Liceo delle Scienze Umane"). The data was analysed qualitatively.

3.2 Questionnaire description

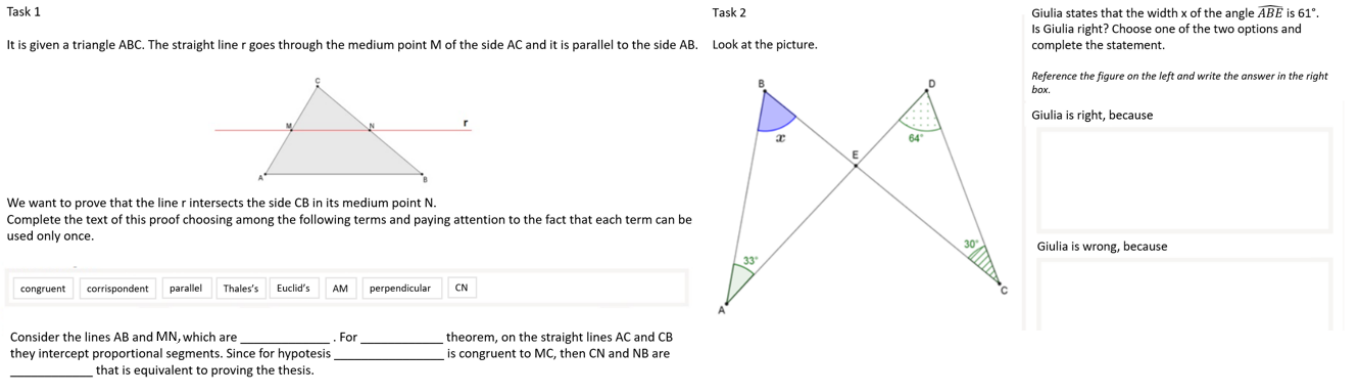
The teachers filled in an online questionnaire through Google Forms, containing two tasks, some specific questions related to each task and three general questions. Each teacher worked on the questionnaire on their own, without knowing the other participants' answers.

The questionnaire was structured in order to be similar to the one used in (Saccoletto & Spagnolo, 2022), as we considered the categories determined by them for students, as an initial reference for teachers' results. The tasks selected in this case, however, are different. Both tasks are geometrical and argumentative, since we meant also to highlight possible connections between teachers' solving strategies and their perception of difficulty, which could have been difficult to do with closed-ended questions. We choose tasks

from INVALSI tests for grade 10 students, because INVALSI (Istituto Nazionale per la Valutazione del Sistema Educativo di Istruzione e di Formazione), the Italian institution that evaluates students' skills in school subjects and many aspects of the Italian educational system, since 2007-2008 has been administering yearly tests to the students of selected grades and they are statistically validated (Lazarsfeld, 1958).

The two chosen tasks are different as regards both content and form. In fact the first one is a cloze aiming to test the knowledge and use of Thales's theorem to fill in a provided argumentation, while the second one involves the production of an original justification using properties of triangles and angles. The tasks are characterised by a different level of competence, as stated by INVALSI itself. In fact, being the levels of competence on a scale from 01 (lowest) to 05 (highest), Task 1 has a level of 03, whereas Task 2 has a level of 05, due to the fact that the first one only requires knowledge of theorems and geometrical properties while in the second one the ability to use the knowledge to create a proof is also required. In other words, this means that objectively Task 1 is easier than Task 2.

Figure 1. Both Task 1 and 2 have been administered to Grade 10 Italian students by INVALSI in 2018, www.gestinv.it



Each task was followed by the questions below, which are strictly connected to the task itself and its perceived difficulty:
(Q1) On a scale from 1 (very easy) to 10 (very difficult), how difficult was the task in your opinion?
(Q2) Why?
(Q3) Which aspects would you change to make the task easier?
(Q4) Which aspects would you change to make the task more difficult?
(Q5) What do you think is necessary to answer to this task?

In the end, the three general questions reported below were asked. These last questions had the scope of collecting elements related to the perceived difficulty in general according to teachers.
(Q6) According to you, which factors or aspects make a mathematical task difficult?
(Q7) According to you, which factors or aspects make a mathematical task easy?
(Q8) Do you remember a mathematical question or test that was particularly difficult for your students?

4 Results and discussion

4.1 Discussion of teachers' answers

The first aspect worth mentioning is that in general teachers almost always rated both the tasks as very easy or quite easy and in two cases they assigned the exact same rate to the two of them. One more teacher rated the first one as a 2 and the second one as a 3, so they considered the second task only slightly more difficult than the other. It is also significative that a teacher rated the first task as more difficult than the other (7 vs. 4), but "only for the meaning of the term correspondent". This is particularly interesting related to the fact that the first and the second task had been rated by INVALSI (in sense of absolute difficulty based on answer rates collected at national level) as having a level of competence respectively of 03 and 05, as stated in the previous paragraphs.

Nevertheless, teachers seem to be extremely aware of the importance of the terms and words used in the tasks; in particular, they value them as a support for the students in the first task (for example, a teacher states "expressions like *since for hypothesis* or *which is equivalent to proving* make answering very easy"). Also, in the answers to the general questions, some of the teachers highlighted that the wording of the question and consequently the students' comprehension are strongly connected to the perception of a task as difficult. It seems that even texts that appear not very clear or contain "hidden" data are perceived as difficult by teachers.

Another interesting remark is that in the reasons of their ratings, teachers attribute a great value to the type of question, meaning that the cloze is considered generally easier than an open-ended question, even with its "tricks" such as having more provided words than gaps in the text. What appears to affect the rating of the second task, making it neither high nor very different from the rating of the first one, is the fact that the task is "standard", and it requires "basic knowledge of Euclidean geometry".

In addition to that, teachers mentioned the figures and their use in many of their answers. Two of them agree that the presence of the figure makes Task 1 quite or very easy (e.g., "the presence of the reference picture helps choosing the right answer"; "picture is provided so this eases the interpretation of the initial text"). For Task 2 instead there is not such agreement, in fact a teacher considers the figure once again a perk of the task, whereas another one states that "the figure might distract from the concept of triangle", as in the picture the triangles are not presented in the traditional way.

Lastly, regarding what teachers think is necessary to solve the two tasks, we found meaningful that all of them considered only aspects related to the field of knowledge and to the specific geometrical notions required in the tasks. Anyway, two of them, in Task 1 mentioned also general structures of the mathematical reasoning as "the structure hypothesis-thesis-proof" or "forms of logical reasoning", instead none of these aspects was ever mentioned regarding Task 2, even though it required them even more because of its type (open-ended question asking to decide about the correctness of a geometrical statement and to produce a justification of it).

4.2 Comparison with students' answers and discussion

In the last phase of our research, we confronted the categories previously described for students' answers with teachers' answers, to analyse whether they fit also for teachers or not. In this case it was even more evident that some answers could be categorised referring to more than one set, being particularly articulated.

Premising once again that this is an initial qualitative and explorative study, we can affirm that the five categories seem to be coherent and appropriate also to analyse teachers' answers about the perceived difficulty. An interesting aspect to mention is the fact that even though both teachers' and students' answers can be classified according to the categories, the factors emerging by them are differently distributed in proportion among the two groups.

The first aspect that we immediately noticed is that none of the teachers' answers could be categorised in the third and fifth set (*Emotions* and *Personal consideration*). Emotions was an underrepresented category also in students' answers, so its complete absence might have been caused by the smallness of our pool, but we should also consider the possibility that teachers do not consider emotions a factor that may influence the difficulty of a question. Apart from two answers, which contained many aspects from different categories, we were able to assign every answer to a category and each of them was almost equally represented.

The teachers' answers belonging to the first category, *Resolution strategy*, were similar to the students' ones. Students said that a task was easy because "the calculus was easy" while teachers state that Task 2 was easy because "basic notions of Euclidean geometry are used" or "basic notions about triangles and angles".

The second category, *Capabilities and experience*, was the most widespread for students and it included diverse kind of answers. In teachers' answers we could find the reference to the familiarity with the task: [Task 2's difficulty is 2 because] "it is a *standard* task proposed by textbooks"; however, students tended to always define difficult a task never seen before, while it is not the case for the teachers. In general, for teachers this did not seem to be a crucial factor. Nevertheless, they appear to be aware of the fact that for their students the "exercises that are not a mere application of a formula" or "not immediately linked to a standard model" are difficult, as they explained answering the last general questions. We classified in this category also answers like [Task 1's difficulty is 2 because] "It has been almost natural to fill the gaps with the right choices", very close to the ones given by students such as [the task was easy because]"I figured it out right away".

The fourth category, *Task formulation*, is very present in teachers' answers, while in the students' case it was really small. Teachers highlighted what, for students, emerged only during the focus groups, which is the impact that the type of question has on the perceived difficulty of it. Specifically, a teacher wrote that [Task 1's difficulty is 2 because] "it is a cloze with options of answers" and others mentioned the relevance of the figure, as discussed in the previous paragraph.

From a certain point of view, the relevance of the figure might be a new category itself, but on the other hand this aspect is part of the task formulation, so we reserve to consider it in further researches on the theme. From teachers' answers, however, we observed the

necessity to include in the category of *Task formulation* also the aspects explicitly related to the wording of the task, aspects of which teachers seem to be very conscious of.

As we expected, teachers demonstrated awareness about the mathematical aspects that might be perceived difficult by students and they sometimes were able to identify precisely the core of students' difficulty, when they were not able to do it themselves. On the other hand, some issues emerged from students' answers were not even mentioned by teachers, such as the emotional dimension. This confirms that the factors influencing students' and teachers' perceived difficulty for the same task are not always identical, as the two groups seem to focus on different aspects to determine whether a task is difficult or not. This might be a reason why teachers sometimes do not grasp the causes behind students' mistakes (Faggiano et al., 2023).

5 Conclusion

Despite a very small pool sample, this qualitative study gives a first insight into the factors that influence teachers' perceived difficulty, comparing them to the ones influencing students' perceived difficulty.

It shows that the categories defined in (Saccoletto & Spagnolo, 2022) are useful also for the analysis of teachers perceptions and beliefs about the issue, demonstrating that the factors influencing teachers and students' perceived difficulty are almost the same, varying in proportions among the two groups. Generally speaking, teachers seem to be aware of the reasons behind students' perceived difficulty but they sometimes underestimate it and, above all, they seem to not take into account the emotional aspect linked to the difficulty.

Further studies, possibly with larger samples, could allow us to explore the issue more in depth. Firstly, it would be interesting to collect more elements to build an initial definition of perceived difficulty in Mathematics Education, considering the factors emerged until now. Secondly, a reasonable development is to carry on a direct comparison between students and teachers' perceived difficulty when confronting the exact same task, to determine whether the same factors emerge or not.

Research ethics

Author contributions

CS: Conceptualization, Formal Analysis, Investigation, Methodology, Project administration, Resources, Supervision, Writing – original draft, Writing – review & editing, Validation.

BN: Data curation, Formal Analysis, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization.

All authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

The data presented in this study are available on request from the corresponding author. The data are not publicly available due to privacy.

Conflicts of Interest

The authors declare no conflicts of interest.

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