LUMAT-B
International Journal on Math, Science and Technology Education

Conference paper

How is students' participation in a problem-solving project reflected in their drawings of a mathematics classroom?

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Abstract: The mathematics teaching young students encounter not only affects the mathematics they are given the opportunity to learn but also their view of what mathematics is, how mathematics is taught and how they view their ability to learn mathematics. In this paper, an Educational Design Research study on problem solving and problem posing with Swedish six-year-olds will serve as an example. We elaborate on how students' participation in education may affect their views of what it means to be taught and to learn mathematics. We do so by comparing drawings of mathematics classrooms made by students from eight classes, of which four attended the project. The results indicate a reform-oriented and more diverse view of content and form reflected in the drawings by the students participating in the intervention.

Keywords: problem solving, mathematical classroom, drawing, educational design research, primary mathematics

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1 Introduction and research question

The mathematics teaching young students encounter affects not only the mathematics they are given the opportunity to learn but also their view of what mathematics is and how mathematics is taught, and it affects how they view their ability to learn mathematics. The Swedish primary school curriculum highlights mathematics as an inherently creative, reflective, and problem-solving activity. Mathematics education in primary school aims to enable students to enhance their skills in posing and solving mathematical problems and evaluating different strategies and methods (Swedish National Agency for Education, 2018). This emphasis on problem solving and problem posing in the curriculum can be attributed to a nationwide assessment of mathematics teaching conducted in 2009. It revealed that individual counting dominated mathematics instruction in Sweden, with limited opportunities for students to develop problem-solving abilities (Swedish Schools Inspectorate, 2009). Thus, it has been concluded that most mathematics teaching is being done traditionally and not in line with reform-oriented teaching (traditional and reform-oriented teaching are briefly described below). Additionally, research indicates that young students who engage with challenging problem-solving tasks in school also exhibit a strong understanding of mathematical concepts (Hiebert & Grouws, 2007).

In this paper, an Educational Design Research study on problem solving and problem posing with Swedish six-year-old students will serve as an example. In Sweden, six-year-olds attend preschool class, which is the first year of the formal education system intended to provide a smooth transition between preschool and school. In the study, the students participated in several problem-solving and problem-posing activities for one year. In this paper, we use an art-based approach (Richardson & St. Pierre, 2018) to analyse drawings made by these young students. The question we pose is: How is young students' participation in problem-solving and problem-posing activities reflected in their drawings of a mathematics classroom?

2 The study: Problem solving in preschool class

Traditional teaching and reform-oriented teaching represent two contrasting approaches to mathematics education. While both approaches prioritise the overall goal of teaching mathematics, they differ in terms of what is considered learning and how it is assessed. Additionally, the roles and relationships between teachers and students vary, as does the importance placed on problem-solving and the structure of classroom activities. According to Sowder (2007), mathematics education researchers widely agree that primary mathematics education should align with the principles of the reform mathematics movement. This is because reform-oriented teaching advocates students' active involvement in exploratory and problem-solving tasks, enabling them to deeply understand important mathematical concepts and procedures (Skott et al., 2018).

Several international studies show that if students are to be successful in mathematics, problem solving should be integrated early into mathematics education (see, for example, Cai, 2010; Liljedahl, 2018). By engaging in problem solving and problem posing, young students can develop an understanding not only of problem solving but also of important mathematical ideas and content (Lesh & Zawojewski, 2007). Thus, taking problem solving and problem posing as a starting point for early mathematics education enables students to develop problem-solving skills and to learn different mathematical content. Also, problem solving and problem posing in early mathematics education have been shown to positively influence students' attitudes towards mathematics (Palmér, 2016; Palmér & van Bommel, 2018; van Bommel & Palmér, 2021).

In a longitudinal study that started in 2014, a problem-solving approach to mathematics was developed in close collaboration between researchers and preschool-class teachers (Ebbelind et al., 2023). During one school year, the preschool class teachers carry out at least six jointly chosen problem-solving and problem-posing activities in their classes. The mathematical content in the activities varies, including, for example combinatorics, algebra, and geometry. In general, one activity lasts for two lessons. In the first lesson, the students are to solve a problem-solving task, for example: We are going on a picnic, and you have to prepare some sandwiches. You have two types of bread and three different toppings. How many different sandwiches can you prepare (one slice of bread, one topping)? The students typically solve the problem-solving tasks in small groups, and the teacher walks around the classroom, talking with the groups and encouraging the groups to explain their thinking. At the end of the activity, the teacher collects different solutions, and selected solutions are discussed with the class as a whole. In the second lesson, the students are reminded of the original problem-solving task and asked to formulate a similar task. As the preschool class teachers have participated in the study for several years, they most often carry out several more problem-solving activities in their classes. Still, at least six activities each year are covered jointly by the participating classes.

3 Becoming a learner of mathematics

There are studies indicating that students' attitudes and emotions play a role in their learning of mathematics (Di Martino, 2019; Hannula, 2016) as well as their interest in the subject (Clements & Sarama, 2016). For example, research has shown that young students' achievement in science, technology, reading, and mathematics is influenced by their interest in and emotional disposition towards mathematics and science (Clements & Sarama, 2016). Further, correlations have been found between students' attitudes, emotions, and performance in mathematics, and factors such as mathematics anxiety and feelings towards mathematics and problem solving (Antognazza et al., 2015). Feelings such as frustration, anxiety, confidence, surprise, and curiosity have been shown in some studies to impact the process of solving non-routine mathematical tasks (Di Martino, 2019; Hannula, 2016), while other studies have found no such correlations (Dowker et al., 2012), and in some cases, the correlations found have been attributed to cultural differences in students' emotional responses towards mathematical problem solving (Dowker et al., 2019). Thus, the social and cultural contexts in which students learn mathematics may play a crucial role in shaping what they learn, their understanding of mathematics, and their perspectives on mathematics learning (Perry & Dockett, 2008).

Expectations in mathematics education are often implicitly expressed in classrooms, with socio-mathematical norms influencing the learning opportunities available to students (Yackel & Cobb, 1996). These socio-mathematical norms impact students' perceptions of mathematics, which, in turn, influence their behaviour and performance. Mathematics teaching approaches in Swedish primary schools exhibit significant variation, resulting in differences in students' experiences and perceptions of mathematics (Swedish Schools Inspectorate, 2009). Furthermore, socio-mathematical norms and child perceptions can vary across mathematical domains. Consequently, students' perceptions of problem solving can reflect their behaviour and performance when engaging in problem-solving tasks or activities they perceive as such. In a previous study focusing on the implementation of problem solving and entrepreneurship in preschool class education, interviews with students revealed that, while they recognised problem-solving tasks as having distinctive features, only a few demonstrated an awareness that such tasks could be approached and solved in various ways. Additionally, most of the younger students in the study failed to establish a connection between problem solving and mathematics (Palmér & Karlsson, 2016). Thus, changes in mathematics education may not always be noticed in the same way by teachers and students, and incorporating problem-solving lessons often necessitates a renegotiation and modification of socio-mathematical norms. Also, the existing socio-mathematical norms within a class significantly influence how teachers can successfully implement changes in mathematics teaching in the classroom (Wester, 2015).

4 Theory and methodology

Social semiotics is an approach that aims to understand how people communicate within specific social contexts, specifically, how individuals create signs within interpersonal and institutional settings (van Leeuwen, 2004). The perspective examines communicators' available semiotic choices and how they utilise them, such as when creating a drawing of a mathematics classroom. The meanings associated with these choices are related to previous social interactions in the classroom. Multimodality is an interdisciplinary approach recognising that communication and representation extend beyond language (Kress & van Leeuwen, 2001). In the research field of multimodality, the study of students' drawings considers the various communicative options available to them. This involves examining students' drawings and interpreting their meanings from a social-semiotic standpoint. Multimodality has emerged in the past decade to address critical questions regarding societal changes. Further, multimodality offers concepts, methodologies, and frameworks for collecting and analysing visual, embodied, and spatial aspects of interaction and environments and the relationships between them (van Leeuwen, 2004). This study utilises one of these frameworks, which will be further described below.

4.1 Data

The empirical material used in this study is from eight different classes at four different schools, and the ethical regulations for research in Sweden were followed. Guardians and students approved the students' participation (Swedish Research Council, 2017). Two classes at each school participated, one attending the project and one not attending it. The non-attending classes were chosen because they are parallel classes. This might be considered a limitation since these classrooms are not secured as a specific type of classroom. We are also aware that the teachers attending the project might have influenced their colleagues in the parallel classes. All eight classes were given the same task: to draw a mathematical classroom. Hannula (2007), Hatisaru (2020) and Dahlgren, Johansson and Sumpter (2010) point out that younger students may have difficulties communicating their conceptions orally, and therefore using pictures as a tool provides additional information that the other research methods might not cover. If a student wanted it, the teacher helped them write comments on their drawing. Altogether, 108 drawings were analysed, of which 58 were from students in classes attending the project.

4.2 The analytical tool

The analytical tool (Table 1) is inspired by a tool developed by Danielsson and Selander (2016). It was originally developed for the purpose of analysing textbooks but has lately been used for other purposes as well. For this study, the analytical tool is slightly modified to fit the context of students' drawings, but the use is similar. The analysis is done with a multimodal focus, and the interpretations are done in relation to classroom focus (see the result section).

Table 1. Analytical tool.

	Multimodal focus	Classroom focus
General structure	What objects are presented? What is the picture about?	What classroom views can be interpreted?
Interaction between textual parts	How are objects and entities arranged?	Reflection about how different resources interact and what appears to be central.
Figurative language	What kind of teaching is represented?	What does this tell us about teaching?
Values	Explicit and implicit values	What are the implications for their views of mathematics?

By analysing the general structure of the drawings, we notice how students position themselves in certain activities concerning mathematical objects. Here we start by asking what the drawing is about, how it is arranged, and what objects are present. At this stage, the drawings are examined on a relatively general level regarding layout and content. This is an appropriate starting point for meta-textual discussions of the classroom views. What resources stand out, what roles do different types of entities (persons and artefacts) seem to play, and is there an expected classroom view? An essential aspect of the multimodal analysis is the relationship between the different objects, in this case persons and artefacts, and the various ways they are positioned. We ask to what extent the different drawings give the same, overlapping, or additional/complementary information. We then look for any traces of teaching strategies that can inform us about how teaching may be conducted in these classrooms. Finally, we analyse how the drawings express the values of a mathematics classroom to consider implications for the students' views. One drawing from the non-attending classes could not be classified in this respect.

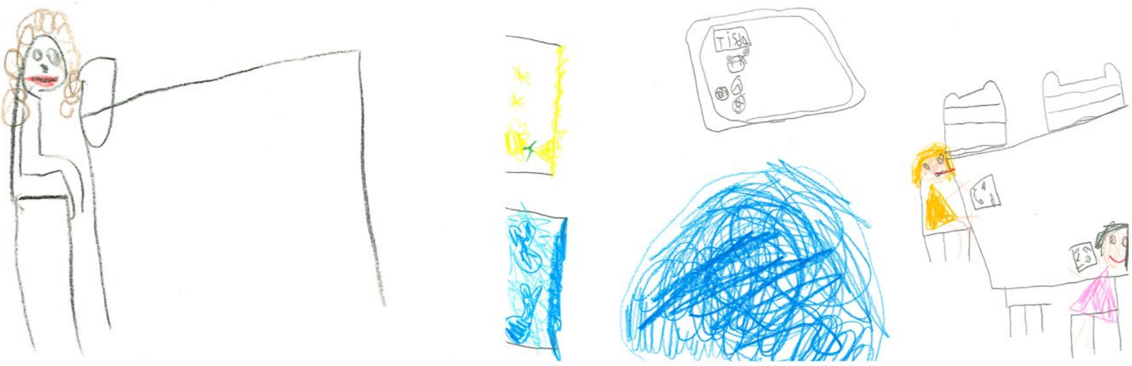
5 Result

The result section will be presented in the way Table 1 is conveyed, beginning with the general structure, and ending with values. The two groups are named PG (project group) and CG (comparison group).

5.1 General structure

We first identified the actors that were present by analysing the pictures' general structure. Different actors participated in the activities drawn by the students: students working, peers attending, teachers attending, and whole-class activities. Of the PG drawings, 36% contained actors compared to 22% in the CG. However, what stood out in the analysis was that the CG drawings showed only one actor (Figure 1: left) whereas eight out of twenty-one PG drawings had multiple actors (Figure 1: right).

Figure 1. Example of one actor (left) and several actors (right).



Objects in the drawings include the mathematics textbook, the workspace, mathematical symbols, whiteboard, gathering place, and laboratory materials. In the CG drawings, the mathematics textbook is present in 38% of the drawings, often together with a workspace and a student working (Figure 2: left). Of the PG, 19% also have a mathematics textbook together with a workspace. The drawings contain a lot of mathematics, numbers, geometry, patterns, small and large, and categorising. However, a detailed analysis revealed that 48% of the CG drawings only represent numbers, geometry, patterns, small and large items, and categorising (Figure 2: middle). In the PG drawings, that number was 14%. In most PG drawings, mathematics is related to actors and other objects to a larger extent (Figure 2: right). A big difference in the drawings is that students in the CG did not draw any common gathering sites or laboratory materials. Students attending the PG drew gathering sites (21%) and laboratory materials that can be used when 'doing' mathematics (40%).

Figure 2. Objects in the drawings.

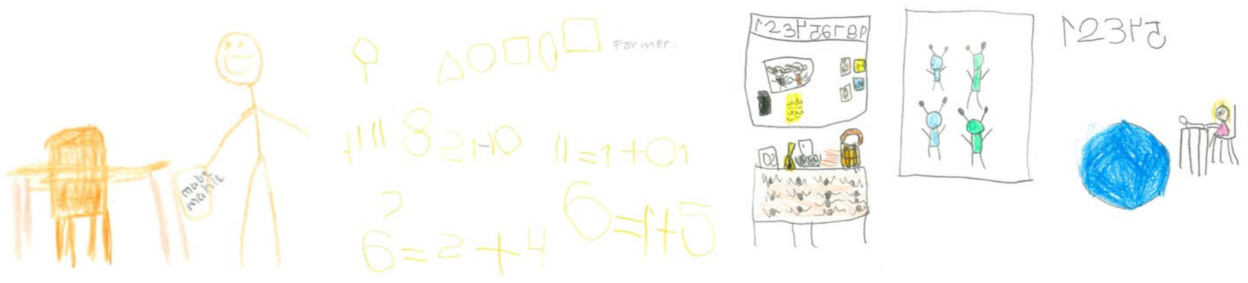


Table 2. Objects in CG and PG drawings (rounded percentages).

	Actors	Textbook	W-space	Maths	W-board	G-Place	C-material
CG	22%	38%	24%	60%	4%	-	-
PG	36%	19%	34%	62%	14%	21%	40%

During this first stage of the analytical process, we also tried to identify what each picture was about. Six different categories were developed when interpreting the pictures: (1) Student sitting and working with the mathematics textbook, (2) Student sitting down working with mathematics (not the textbook), (3) The mathematics textbook, (4) Mathematical symbols, (5) Objects in the classroom, and (6) Working in pairs or in groups.

Table 3. Categories of content of picture (rounded percentages).

	1	2	3	4	5	6
CG	16%	-	18%	60%	4%	-
PG	4%	10%	5 %	26%	33%	22%

For the CG, the drawings can be classified into two groups. Those in the first group contain the textbook (Figure 2: left) in some way, categories 1, 3, and 5 (Table 3). The other 60% focus on mathematical symbols (Figure 2: middle). If we look more closely at the PG drawings, it gets more diverse. The mathematics textbook is still represented (categories 1, 3, and 5 – Table 3), but as one object amongst other objects. A closer look at category 5 reveals that 17% contain manipulatives that can be used when doing mathematics. Category 6 is visible only in the PG drawings, where 22% indicate some kind of collaborative work.

5.2 Interaction between textual parts

When analysing persons and artefacts and the various ways they are positioned in the drawings, we see a big difference between the two groups. On a general level, the PG students used a lot more objects in their drawings than the CG students. They not only drew more objects, but they also drew a greater variety of objects. Their drawings are richer and provide more information. Relating to how the classroom was arranged, one can conclude that all drawings of a workspace in the CG can be interpreted to be a teacher-fronted classroom, while this is not the case in the PG drawings.

5.3 Figurative language and values

Table 4. Frequency of different teaching strategies.

	CG	PG
Objects representing mathematics	40 out of 50	36 out of 58
Actor working with textbook	8 out of 50	3 out of 58
Actor working with mathematics	1 out of 50	11 out of 58
Group working with mathematics	0 out of 50	8 out of 58

Finally, we interpret whether any traces of teaching strategies can inform us about how teaching might be conducted in these classrooms. If mathematics is to produce correct answers, students are less likely to place value on engaging in discussions on alternative solution strategies. A traditional view of mathematics may constrain students' participation in meaningful ways and inhibit them from engaging in problem-solving activities. Reform-oriented perspectives suggest that, for mathematics to be meaningful, students should make conjectures, explain, and share their reasoning with others, and discuss and question their thinking. Therefore, the mathematics portrayed in the reform documents requires students to think differently. We can conclude that students in the PG do seem to think differently. As such, we conclude that the mathematics students encounter seem to influence not only the mathematics they are given the opportunity to learn but also their view of what mathematics is.

6 Discussion

In this paper, we asked how students' participation in problem-solving and problem-posing activities are reflected in their drawings of a mathematics classroom. In the drawings we can see two contrasting classrooms with two distinct approaches to mathematics education: traditional teaching and reform-oriented teaching. Our analysis of the students' drawings suggests that the roles and relationships between teachers and students in these classrooms differ from those in other classrooms, as does the emphasis placed on the structure of classroom activities. On a general level, we can conclude that students' involvement in the project seems to influence not only the mathematics they are allowed to learn but also their view of what mathematics is and how mathematics is taught. These results are interesting because, as highlighted in the methodology section, one limitation of the study is that the PG teachers might influence their colleagues in the GC group. Despite this possible influence, we can conclude that the two groups' differences are striking.

We want to highlight two observations of interest that will be focused on in future studies. We observed the role of actors: Students within the PG related more to group work, and more actors are present in their drawings. Another observation is that drawings from the CG, to a large extent, include mathematical symbols. In contrast, the PG drew manipulatives that can be used when doing mathematics, and we can speculate that students in the PG have more available semiotic choices. Therefore, they have more opportunities to use manipulatives and thereby to develop representational competence. We conclude that students in the PG have a richer view of a mathematics classroom than those in the CG.

In line with Hannula (2007), Hatisaru (2020) and Dahlgren and Sumpter (2010), we agree that using drawings provided insights that other research methods might not. We are therefore encouraged to keep trying out innovative ways of analysing young students' views of mathematics using an art-based approach (Richardson & St. Pierre, 2018).

Research ethics

Author contributions

A.E.: investigation, methodology, analysis, writing, review, and editing.

H.P.: investigation, methodology, writing, review, and editing.

J.v.B.: investigation, methodology, writing, review, and editing.

All authors have read and agreed to the published version of the manuscript.

Informed consent statement

Informed consent was obtained from all research participants.

Conflicts of Interest

The authors declare no conflicts of interest.

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