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'Nature of Mathematics weiterdenken': First findings of an empirical study among prospective teachers and mathematics students at the University of Vienna

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Abstract: The purpose of this paper is to present first findings, reflect on the chosen methodology and create hypotheses for the main project 'Nature of Mathematics weiterDenken', which is currently under development, examining and comparing mathematical world views and domain specific beliefs of prospective teachers and mathematics students. The analysis of the data collected by using an online questionnaire containing 93 closed items (five step Likert-scale) indicates that there is no statistically significant difference in mathematical world views among the surveyed subgroups (n = 75), either cross-sectionally (prospective teachers/mathematics students) or quasi-longitudinally (within each sample group at three different points of measurement).

Keywords: mathematical world views, teachers' beliefs, beliefs change, teacher education

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1 Introduction

What is the origin of mathematical knowledge? Does it exist as a result of discovery or creation? Does (school) mathematics benefit society and is it accessible to all? What is the role of creativity in mathematics?

There are a number of domain-specific (epistemological) belief research (e.g., Grigutsch & Törner, 1994; Blömeke et al., 2014) examining the relevance of beliefs to mathematical teaching and learning processes by evaluating the responses of students at various educational levels and teachers to these questions. Based on these findings, teachers' beliefs affect mathematical teaching and learning processes, which in turn influence students' beliefs (Weygandt, 2021). The findings indicate that the preparation of prospective teachers should not only focus on content, didactics, pedagogy, and methodology, but also on beliefs on, and about teaching and learning mathematics. "Nature of Mathematics weiterDenken" addresses these two last mentioned points and has the objective to replicate and validate previously elaborated factor-structures as well as to identify commonalities and differences among subject-groups (prospective teachers/mathematics students) and potential changes without interventions within a quasi-longitudinal design.

The aim of this paper is to present first findings, reflect on the used factors and to discuss the potential implications of these observations for the ongoing project.

2 Theoretical Background

Although there has been a long-standing and well-developed research tradition, there is no universally accepted definition of the term 'beliefs'. In fact, Pajares (1992) refers to it as a ‘messy construct’. The distinction between ‘general’ (interdisciplinary) and domain-specific beliefs by Urhahne and Hopf (2004) as well as the focus on epistemological beliefs, which are subjective beliefs regarding the structure of knowledge and the process of knowledge genesis within the respective disciplines, enables a narrowing of the research field.

One of the theoretical bases for the project 'Nature of Mathematics weiterDenken' is provided by Grigutsch et al. (1998) with their concept of 'mathematical world views' ('mathematische Weltbilder') (also refer to Grigutsch & Törner, 1994) which can be defined as 'structures of attitudes', characterized by the fact that they summarize individual attitudes as well as the relationships between them. According to the authors (Grigutsch et al., 1998), subjective, mathematical knowledge can be classified into the following categories of attitudes:

- Attitudes towards mathematics (e.g., concepts about the nature of mathematics)
- Attitudes towards learning and teaching mathematics
- An individual's attitude toward himself (or herself) as an operator in mathematics

Based on this theoretical foundation, Maaß's (2006) definition of 'beliefs' and ‘mathematical world views’ is used in the following sections:

> Beliefs are formulated from relatively lasting subjective knowledge about specific objects or issues, as well as their associated emotions and attitudes. As a whole, all beliefs about mathematics, mathematics education, and mathematics learning comprise the mathematical world view. It is possible for beliefs to be conscious or unconscious (Maaß, 2006, p. 119).

Further clarification and discussion of the terms 'beliefs', 'attitudes', 'emotions', ‘concepts’ and 'knowledge' can be found in Weygandt (2021, p. 98–116) and Philipp (2007, p. 259).

2.1 State of research

The study of (epistemological) beliefs and mathematical world views has a long tradition with a variety of research goals and methodologies. Beginning in the 1980s, beliefs research was primarily qualitative, but gradually became more quantitative to analyze shifts in beliefs related to research interests (Ebbelind, 2020). Currently, qualitative research is being re-emphasized, or rather qualitative and quantitative research are being combined (e.g., Eichler et al., 2023).

This section provides a summary of results of various research attempts and the motivation for implementing the (main) project ‘Nature of Mathematics weiterDenken’.

By analyzing the data from 310 in-practice teachers (middle and high school) using a questionnaire with a five-step Likert scale, Grigutsch et al. (1998) investigate the concept of the mathematical world view. Based on their used factor analysis, four factors (Table 1), which have already been confirmed in several similar studies (e.g., Weygandt, 2021) describe the mathematical world view. The authors conclude that the results of their study are ‘positive’ because teachers' views on mathematics tend to be characterized by process- and application-, (mathematics as a dynamic activity), rather than schematic- or formalistic-orientation (mathematics as a static, abstract, and already completed subject). Based on this framework, it is possible to interpret the findings of Diedrich et al. (2002). Stronger process- and application-aspects from teachers are associated with constructivistic teaching- and learning strategies and a higher degree of student engagement. Additionally, there are negative correlations between students' process-oriented attitudes and their receptive opinions regarding mathematics learning and teaching. Grigutsch and Törner (1994) demonstrate that students with a strong process orientation, a passion for mathematics and a positive self-perception are better equipped to handle frustration during their first semesters. In addition, Rott (2019) indicates that the number of students who are capable of responding ‘sophisticated’ to mathematical epistemological questions is extremely low (even within studies with a high degree of specialization) and does not significantly change during the course of their studies unless lectures specifically address epistemological topics. Weygandt (2021) empirically evaluates such a lecture named ‘EnProMA’ for prospective teachers, confirms the four latent factors of Grigutsch et al. (1998) and identifies five additional ones, as can be seen in Table 1. These nine elaborated factors and their inherent items form the basis for this project.

This overview shows the opportunity of projects such as ‘Nature of Mathematics weiterDenken’ since (epistemological) beliefs and mathematical world views are critical for successful learning in school and university, as well as for those who teach. Rott (2019) notes that, unlike in the context of identity research, little is known about the evolution of beliefs during the study of mathematics. It is the intent of the still ongoing project to gain further insight into this research gap.

2.2 Implementation of ‘Nature of Mathematics weiterDenken’

The previous sections provide a theoretical overview of beliefs-research and selected findings so that the still ongoing project ‘Nature of Mathematics weiterDenken’ can be positioned appropriately in the ‘messy construct’ of beliefs research. Consequently, it can be characterized as a mathematical, (epistemological) beliefs research, following Törner (2002) as having the focus on ‘identifying existing beliefs’. Furthermore, it is based on the already mentioned concept and analyzed factors of Grigutsch et al. (1998) and Weygandt (2021), as shown in Table 1.

3 Methodology

Attitudes that describe the mathematical worldview are latent variables that cannot be measured directly. Therefore, we use observable variables in the form of items to gain information about them. To operate the mathematical world view of participants, we use a five-step Likert scale (1 = 'strongly disagree'; 5 = 'strongly agree') to assess the state of agreement to all items outlined in the elaborated four factors of Grigutsch et al. (1998) and five factors of Weygandt (2021). A high factor value indicates a high level of agreement with the respective factor. Low factor values indicate rejection.

Due to the large number of participants required for representative data, the Likert scale methodology was chosen, although social desirability is a known limitation of this approach (e. g., Kreitchmann et al., 2019). To conduct further, more in-depth surveys with smaller sample groups, additional methods (such as Q-studies) will be used.

The still ongoing project uses confirmatory factor analysis to test the validation of the mentioned factors. Additionally, the evaluation of correlation between factor values provides a deeper insight into the structural characteristics of the mathematical worldview. Through the combination of cross-sectional and quasi-longitudinal mean value comparisons, differences in agreement values can be identified. This methodology allows us to evaluate differences in respective factors between sample groups as well as between different measurement points within the sample groups.

One purpose of this paper is to analyse the first results regarding the internal consistency (Cronbach's Alpha values), as well as to compare mean values in a quasi-longitudinal and cross-sectional design (Shapiro-Wilk-, and Kolmogorov-Smirnov-test for distribution; Levene-test for homogeneity of variance; t- test and Mann-Whitney-U-test for comparison of mean values using SPSS28). Based on the results of this first analysis of data, hypotheses are generated for the main study.

Table 1. Confirmed and extracted factors and comparison of Cronbach's Alpha values (α = Alpha values in this study; $\alpha_{Weygandt}$ = Weygandt's (2021) Alpha values.

Factor (Weygandt, 2021)	Factors ('mathematical world views') (Grigutsch et al., 1998)
Linkage/Structure of mathematical knowledge (L) mathematics as a precise system of multiple interconnected terms 10 Items (α = 0.68/ $\alpha_{Weygandt}$ = 0.78) *The concept of mathematical knowledge can be thought of as a system of interconnected terms.*	**Formalism-aspect (F)** formal-deductive, strictly logical and objective view on mathematics 10 Items (α = 0.79/ $\alpha_{Weygandt}$ = 0.75) *A fundamental component of mathematics is the application of logical rigor and precision, also known as objective thinking.*
Efficacy of results (E) an efficient and determined approach to teaching mathematics, the process of discovering and exploring is viewed as inefficient 10 Items (α = 0.45/ $\alpha_{Weygandt}$ = 0.69) *Approaches that do not lead to the intended result hinder mathematical learning.*	**Scheme-orientation (S)** mathematics as a collection of procedures/rules detailing how to accomplish a particular task 8 Items (α = 0.71/ $\alpha_{Weygandt}$ = 0.7) *Mathematics consists of learning, understanding, and applying.*
Platonism/Universalism of mathematical knowledge (PU) mathematics as a collection of knowledge inherent in the universe that has been discovered by humans 6 Items (α = 0.70/ $\alpha_{Weygandt}$ = 0.65) *Mathematical statements behave like natural laws, which means that they can be discovered by humans, but cannot be altered.*	**Application-character (A)** application-oriented view on mathematics 10 Items (α = 0.72/ $\alpha_{Weygandt}$ = 0.77) *Mathematical knowledge is essential for students' future success.*
Discretionary power in mathematical phrasing (D) formulation and communication of mathematical knowledge are subject to arbitrary limits 7 Items (α = 0.61/ $\alpha_{Weygandt}$ = 0.68) *Mathematics can be formulated according to one's own judgment.*	**Process-character (P)** constructivist view on mathematics 9 Items (α = 0.75/ $\alpha_{Weygandt}$ = 0.61) *One can discover and test many things in mathematics on their own.*
Creativity (C) like art, mathematics is the result of creativity three subcategories; overall 23 Items (α = 0.78/ $\alpha_{Weygandt}$ = 0.75) *The ability to think and to imagine widely is often more critical in mathematics classes than the ability to memorize and learn.*	

3.1 Sample

According to the study plan, prospective teachers and mathematics students will be surveyed at three times in a quasi-longitudinal design during the summer and winter semesters of 2023 and 2024 (at the start and end of the BSc/BEd program and once during the master's program). Using the collected data from mathematics students, it is possible to compare and reflect upon the development of mathematical world views within two different study paths.

In this paper, we focus on the students which participated in the first of four rounds of evaluation. Due to an orientation to the recommended study path, the sample group of prospective teachers at the beginning of their studies could not yet be evaluated. Furthermore, only three mathematics-master students participated during the first round of evaluation. Both sample groups are excluded from the analysis in this paper.

Table 2. Participants of the initial collection of data in each sample group.

Sample group	**n**
BEd_Advanced	26
MEd	29
BSc_Beginners	8
BSc_Advanced	12
Σ	**75**

4 Results and Conclusion

Cronbach's Alpha values (for measuring internal consistency) are acceptable ($\alpha > 0.7$) for all four factors of Grigutsch et al. (1998). Comparing the alpha values of this study with Weygandt's (2021), we see similar but mainly questionable ($\alpha < 0.7$) values (concerning his factors) (see Table 1). 'Efficacy of results' has an unacceptable alpha value ($\alpha < 0.5$).

In the grouped boxplots, it can be seen that students' responses to the factors ranged widely. It is also noteworthy that several world views or factors may be internalized simultaneously. Additionally, all sample groups show a high average (median) state of agreement in terms of the formalism-aspect compared to other studies (e.g., Grigutsch et al., 1998).

According to Shapiro Wilk- and Kolmogorov-Smirnov-tests, the distribution of all factors is normally distributed ($p > 0.05$), and there are no extreme outliers within all sample groups. Using Levene's test, homogeneity of variance could be assumed within all analyzed sample groups and factors ($p > 0.05$), except within "BSc_Beginners" and "BSc_Advanced" for the factor 'C'. As a result, the Welch-test is used to determine whether there are any differences in the mean values of these groups within this factor. Considering the controversy regarding whether Likert scales should be evaluated using parametric or non-parametric methods (see De Winter & Dodou, 2010), we use both the t-test and Mann-Whitney-U test.

Figure 1. Grouped boxplots – four factors based on Grigutsch et al. (1998).

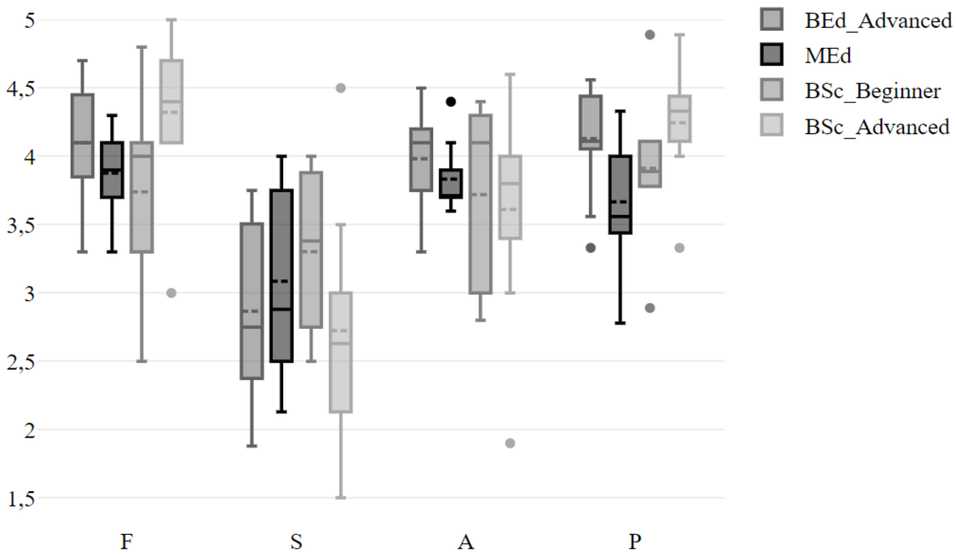
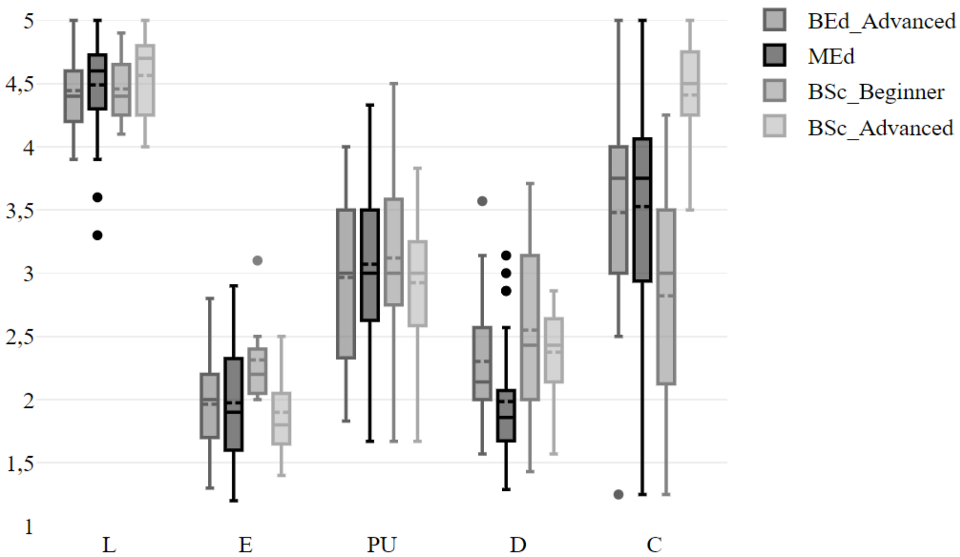


Figure 2. Grouped boxplots – five factors based on Weygandt (2021).



Regarding the results within the quasi-longitudinal evaluation, there are statistically significant differences between 'BEd_Advanced' and 'MEd' students regarding the factor ‘D’ ($t(52) = 2.51$, $p = 0.08$, Cohen's $d = 0.68$ / $U = 207$, $Z = 2.710$, $p = 0.006$), as well as within 'BSc_Beginners' and BSc_Advanced' regarding the factors ‘E’ ($t(17) = 2.60$, $p = 0.019$, $d = 1.24$ / $U = 15$, $Z = -2.296$, $p = 0.02$), and ‘C’ ($t(18) = -4.89$, $p = 0.002$, $d = -2.23$ / $U = 6$, $Z = -3.285$, $p < 0.001$). The cross-sectional comparison of mean values between 'BEd_Advanced' and 'BSc_Advanced' shows a statistically significant difference concerning the factor ‘C’ ($t(35) = -3.90$, $p < 0.001$, $d = -1.37$ / $U = 41$, $Z = -3.559$, $p < 0.001$). There are no statistically significant differences ($p > 0.05$) in all other quasi-longitudinal and cross-sectional comparisons of mean values.

Accordingly - keeping the limitations of this paper in mind - the following hypothesis can be derived for the main study:

- Mathematical world views (measured with factors of Grigutsch et al., 1998) of prospective mathematics teachers and mathematics students do not change significantly without specific interventions over the course of their studies.
- Mathematical world views (measured with factors by Grigutsch et al., 1998) of prospective mathematics teachers and mathematics students do not differ significantly at the same level of study-experience.
- Beliefs of mathematics students and prospective teachers about single factors by Weygandt (2021) differ significantly between, as well as within (quasi-longitudinal) these sample groups.

A confirmation or rejection of these hypotheses in the main study, along with additional insights due to further analysis into students' mathematical world views, will serve as the foundation for (possible) future interventions.

Research ethics

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

As a result of privacy restrictions, data is not available.

Conflicts of Interest

The author declares no conflicts of interest.

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