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Grade 2 students' conceptions about working with sharing

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Abstract: The aim of this paper is to analyse and discuss grade 2 students' conceptions about working with six cases describing different sharing scenarios. We interviewed 25 students, ages 8–9 years. Five different themes emerged in the thematic analysis. The first theme was about the importance of how to make sharing fair and that the cases were realistic. The second theme was about solutions being valid. The third theme covered conceptions about the cases as context or as concrete materials versus working in a textbook, which was connected to the fourth theme: to discuss and think versus being quiet. The fifth theme was about working at your desk versus the whiteboard. The results signal that open problems can alter students' conceptions about mathematics but also challenges when creating spaces for such teaching.

Keywords: collective mathematical reasoning, conceptions, division, primary students, sharing

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1 Introduction

The different ways students engage with mathematics depend on their different affective disposition with the subject (Ingram, 2015). Different studies have focused on various aspects in affect to highlight the connection with affect and how mathematics is perceived and dealt with, such as the different motivation students' express (Nyman & Sumpter, 2019), or how different expectations function as a mediator for various choices students make when solving mathematical tasks (Sumpter, 2013). However, it is not the case that different affective constructs operate as singular entities. Several researchers stress how motivation, emotions, and beliefs are intertwined, with each other (e.g., Hannula, 2006) or internally such as different types of motivation being combined in one statement (Nyman & Sumpter, 2019). It means that it can be difficult to separate different constructs from each other, which provides various methodological challenges.

Looking at empirical studies, one worrying issue is that students' affective disposition in many countries decline with age (e.g., Sumpter & Sollerman, 2023; Wilkins & Ma, 2003). This is true for Sweden too, where grade 5 students have significant more negative view towards mathematics compared to grade 2 students (Blomqvist et al., 2012).

The measured decline generates different questions such as what causes it and what do keep the interest alive. In addition, recent reports stress that negative attitudes, emotions and so forth are established in early schooling (Larkin & Jorgensen, 2016). A study from Finland concluded that third grade students express conceptions that mathematics is either nice/ easy or dull/ difficult, but also that they as students can do mathematics (Pehkonen et al., 2011). It is therefore a more complex picture than just 'good vs bad'. When adding that few studies focus on young children's experiences with mathematics (Takeuchi et al., 2016), the conclusion is there are several gaps in the area of young children's affect in mathematics education. The present paper aims to offer information about what young students think about working with open problems compared to their traditional mathematics education, which here means working with textbook, quietly at your desk. The research question is: what different conceptions do students express with respect to problem solving and collective mathematical reasoning?

2 Background

Given that it can be difficult to separate between different affective constructs in data (e.g., Hannula, 2006; Nyman & Sumpter, 2019), and the aim here is to give a more general view of people experiences, the decision is to use the theoretical concept conceptions defined as an umbrella concept: "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p. 259). Hence, conceptions may have both affective and cognitive dimensions. 'Conceptions' can then be compared to other constructs aiming to capture similar phenomenon, such as mathematical view (Roesken et al., 2011). It means that we have no intention to try to separate between emotions, motivations, beliefs, or attitudes, but instead talk about students' conceptions as an umbrella concept.

There are several reports that students' affect can change depending on the mathematics teaching. One example is Higgings (1997) that describes how students that participated in a one-year long problem solving displayed more positive affect compared to the control group that had traditional mathematics instruction. Similar results have been reported with Swedish lower secondary students that got to experience a Japanese problem solving oriented lesson structure (Asami-Johansson, 2015). Hence, students affect including different socio-mathematical norms depends on teaching, such as what type of tasks one work with, how to work with them for instance, in random groups, and what type of support to expect from the teacher (e.g., Liljedahl, 2022).

Here, we are interested in collective mathematical reasoning (e.g., Sumpter, 2016), and empirical studies on such collaborative work where students face sharing tasks shows that there often is a tension between students advocating for different interpretations of what could be considered fair (Eriksson et al., 2023; Hedefalk et al, 2022). Previous studies show that when facing different types of tasks and working in a collective manner can causes some affective challenges such as feeling frustrated when your peers want something different (Ayalon et al., 2022). Other issues that were observed were negative emotions arising from peer critique, but also learning opportunities when correcting one's reasoning. In addition, such teaching means that one needs to formulate

arguments and listen to others. Then, one's first language also plays a role, especially with respect to communicating one's reasoning (Lee & Lee, 2019). Affective factors that may influence if students with another first language will communicate are self-confidence, speaking anxiety, and motivation (Mulyono & Saskia, 2021). Given that one of the norms and values in the Swedish school system is that the school should provide situations so that each student "can consciously determine and express ethical standpoints based on knowledge of human right and basic democratic values, as well as personal experiences" (Skolverket, 2018, p.10), means that teachers need to be aware of that affective aspects can function as an obstacle, for all students but especially for students that do not have Swedish as a first language.

3 Methods

To generate data (e.g., Mason, 2017) to answer the research question, we conducted interviews with 25 grade 2 students (aged around 8-9 years old) from two different classes from one school. The recording of one interview was not in a good condition and therefore discarded, leaving us with 24 interviews to transcribe and analyse. The research project, where this study is a part of, is situated in an area considered one of the most segregated areas of Sweden, meaning that none of the children have Swedish as their first language. The classes participated in a research design where three teachers have had the goal to, through didactic modelling, stimulate students' collective mathematical reasoning in three cycles. The teachers have worked with six cases describing six different sharing scenarios. One example is case 6, where the children are asked to share eight biscuits among two soft toys (here dogs). When the children have shared the biscuits, a third recipient with explicit need (hunger and sadness) is introduced. The children are now facing a more complex sharing scenario where there are different needs and where resources are scarce (see Sumpter & Hedefalk (2023) for a longer description of the cases). The first cycle of working with cases was in a class that was not filmed or interviewed. It functioned as a pilot for the teachers so they could try different didactical choices without having the camera in the classroom. Then, they moved to the two classes that were filmed and the children interviewed, one after another. After each teaching session, the teachers and one of the authors of the paper discussed the outcome and made appropriate changes in order to stimulate children's collective mathematical reasoning, so that the children got to give arguments but also listen to each other for the different choices that they made during the problem solving session. The children from the two classes that were filmed replied to a questionnaire in the start of the research project and then interviewed when the classes have worked with all the cases. It means that one class had their last interviews in the end of the fall semester (December 2022) whereas the second class had their interviews in early May 2023. Here, we will focus on the interview since it has information about how the children experienced working with problem solving.

The interview guide was a questionnaire developed for studying students' conceptions, including motivation and emotions, and has been tested in several studies (e.g., Blomqvist et al., 2012; Nyman & Sumpter, 2019). The questions in the survey aim to cap-

ture how students think about mathematics and mathematics lesson, especially in relation to Swedish lesson, but also general issues such as what you do in a mathematics lesson. In the second interview, we added the questions “Now you have worked with biscuits and dogs, if I say that is mathematics, what do you say?, “What was fun [when working with the cases]?, and “What was most difficult?”. The interviews were transcribed using the steps suggested by Mergenthaler and Stinson (1992). One of the steps that they recommend, given that the study is about conceptions, is that language has in some cases been changed so that the meaning of the children’s replies is in focus, not the exact wordings. The data was then analysed using an inductive approach of thematic analysis (e.g., Braun & Clarke, 2006). We, all four authors, started by going through the interviews together and discussed possible codes that could fall under different themes. One example of a code was ‘difficult to make it fair’ which was categorised as ‘Fair share is important and real’. When all four agreed on a saturation with respect to codes and themes, three of the authors analysed the interviews using the different codes and themes. Any unsure data points were discussed and agreed upon.

4 Results

The analysis resulted in five different themes: fair share is important and real, different solutions are valid, working with cases versus working in a textbook, to discuss or to be quiet, and working at your desk or working with whiteboards. The chosen quotes are selected to illustrate the themes. The replies are marked with a letter to indicate different classes (L and M, just to avoid A and B) and a number.

4.1 Fair share is important and real

The first theme is about working with the cases that several students described as difficult, important, and fun:

> Interviewer: What was it that was difficult?
> Student M4: That it isn’t…. it is three dogs and it is eight biscuits and then it is difficult. One can’t give it to them then.
> Student L14: When we should share eight biscuits. And [then] one more came. That was difficult. It never became fair.
> Student M1: No, [to make it] fair is difficult.

Several students talk about sharing as difficult, especially case 6. The conclusion made by Student M1 illustrates the struggle they were facing. However, making it fair was also fun and important:

> Student L6: Shared them and counted them and giving it [to] them. [To] make [it] fair.
> Student L7: Because I thought it was good and fun since we learnt a lot. And [to] think.

> Student L3:To… that we got to learn and it was teddies. It was real.

The quotes illustrate a conception that working with the cases was not just straightforward – it was difficult, especially when cases were complex. But at the same time, many of the students found the work rewarding.

4.2 Different solutions are valid

Another aspect of working with cases was that all solutions were valid:

> Student L3:The solutions were always right. You answered [using] the biscuits every time. How far [one] wish to go.

The students identified a difference between the teacher knowing the right answer and the students providing a valid solution. In addition, all students got to present their reasoning:

> Student M10:We used the white-board and [we] got to stand [next to] it all the time. We all [got to] tell.

One student compared how mathematics teaching normally looks like:

> Student L4:Yes, but we have only shared to the dogs. And, the teacher has written on the white board and said that all [solutions] were right. Otherwise, the teacher writes on the white board and has all [the] right [answers]. One figure is always twice the area of the other.

The student stresses the difference on who has the agency to determine what is a correct solution, and when the power shifts. The shift of power was positive:

> Student M6:We all got to tell. It was fun.

Most of the replies talk about the joy of presenting their solutions to the other students.

4.3 Working with cases or with textbook

Several students talked about the differences between working with the cases and working with the textbook:

> Student L11: That… one gets to write not so much and in math one has to write a lot. You have to write almost 100 pages. […] One gets tired quicker at math [lesson]. It is not that you get to think and stuff and get to share [things] out.

The students talk about the traditional mathematics lessons. In such a lesson, they have to sit still and write, talking about pencil and paper work. The work with the cases also meant that the concrete materials were different:

> Student M11: The biscuits. We got the biscuits.
> Student M8: What is that? When [it] is dogs and biscuits, we have dogs and biscuits. Other in math class, [the] only thing is to write.

It is not just that they have different materials, it is also what you are get to do with the materials:

> Student L14: When we have other math, we do not have the dogs and the biscuits.
> Interviewer: So, what do you have then, when you work with other maths?
> Student L14: Then we do not have the dogs. Then we have to write and think.

Here, in the students' quotes, it is not clear what 'to think' means. It appears to be connected to being quiet:

> Interviewer: If we compare work with the biscuits with normal math…
> Student L1: It is different to work with biscuits and books. You draw and write and count.
> [talk about thinking]
> Interviewer: Do you think when you work with the biscuits as well?
> Student L1: Yes, but more when doing maths. Math is math. You think, then draws and then teacher correct your answers. But [when working with] biscuits [you] draw. [When working] with a big biscuit, a lot.

It appears that 'thinking' represents different things for different students. Some students also brought forward what is the same:

> Student L3: The difference is that one work with bears and you work with your hands. […] What is the same is addition, subtraction, multiplication and division.

Student L3 points out that the difference is not in the mathematical content but in the concrete materials – that you got to work with your hands. The replies that were coded in this category were often connected to the next two categories: to discuss or to be quiet, and only to work at your desk or to be able to present your work using whiteboards.

4.4 To discuss or to be quiet

A lot of the replies dealt with the differences with working with the textbook at your desk being quiet whereas when working with the cases, the students were both encouraged to work together:

> Student M5:Like, we get to work alone and stuff.
> Student L13:If we work well, we get good. And we have to listen to the teacher. We have to think.

Many students describe a situation where they sit still at their desks and work in silence. The theme cover also replies to how the children got to present their solutions:

> Student M6:We all got to tell. It [was] fun.
>
> Student L5:Tedious/ long. And listen. All.

The chosen quotes illustrate that there are mixed feelings about presenting your reasoning. Some students talk about how difficult it was to listen to all solutions; others stress the importance of listening to each other.

4.5 Desk or whiteboard

Several students stress the joy to go up in front of the class and present their solutions in front of the whole class:

> Student M6:We all got to tell. It [was] fun.
> Student M10: We used the whiteboards [small ones that the students have at their desks] and we were at [the big] whiteboard all the time. We all [got to] tell.

Although most students found presenting at the whiteboard was fun, one student recall one incident:

> Interviewer:You are marking the sad face. Why are you feeling in such a way?
> Student L10:Because… because… some [other students] laughed when we were there. When we were at the whiteboard and the teacher had to tell them that they should not [laugh], but then they already [have laughed].

The student continued that it did not matter that the teacher told the students off – the damage was already done. The quote illustrates how difficult it is to create a safe space when working with collective mathematical reasoning.

5 Discussion

The five themes that were the results from the analysis were that fair share is important and real, different solutions are valid, there is a difference between working with cases compared to working in a textbook, often connected to whether you are allow to discuss or to be quiet, and working at your desk or working with whiteboards. The last three categories combined present traditional mathematics as to be quiet, working alone with the

textbook at your desk which can be compared to the findings from Pehkonen and colleagues (2011). On the other hand, working with the sharing cases ('dogs and biscuits') means that you work together and discuss your solutions, often using concrete materials, and then present your solution to the other students in the class including listening to their solutions. The conceptions reflect a mathematics education as a problem solving teaching (e.g., Asami-Johansson, 2015). Several students explicitly said that presenting and listening was good but also difficult. Here, 'difficult' appears to be a proxy for different things, similar to when the grade 2 students in Nyman and Sumpter (2019) used boredom as a stand in for a handful of negative emotions and motivations. For instance, difficult could be tedious since you have to listen to several presentations, but also frustrating when wanting different things. These are affective challenges similar to those that have been reported in earlier studies (e.g., Ayalon et al., 2022). Results like these are of interest in the light of the norms and values of Swedish school: to foster future members of the society that can listen to others and are able to formulate one's own reasoning (Skolverket, 2018). One conclusion is that working with collective mathematical reasoning (e.g., Sumpter, 2016) is not a straightforward and easy goal for schools, but an important one if to fulfil the overarching goal of the Swedish school system. The implication is that if teachers want to work with this, they need to be aware of that creating such a norm where students are willing to listen to each other might take time.

Overall, most of the students thought it was fun to discuss and present different solutions, albeit it could be tedious. One student (L10) brought up one incident that was connected to negative emotions. Given that research has pointed out different affective aspects that may influence students with a different language background and their willingness to speak in a classroom (Lee & Lee, 2019; Mulyono & Saskia, 2021), the student's reply illustrates such findings with respect to mathematical problem solving and collective mathematical reasoning (e.g., Sumpter, 2016). Since creating a collective mathematical reasoning space is not easy, our results suggest that doing it in a non-first language classroom adds more challenges. One possible implication is that teachers, especially in multi-lingual classrooms, need to be aware of how to create a thinking classroom (e.g., Liljedahl, 2022) where everyone feel secure to present their solutions (Ayalon, et al., 2022).

The view of mathematics being working in a textbook, quiet and alone at your desk as different as working together, with concrete material, discussing and then presenting your solution signal that students do form socio-mathematical norms or mathematical views rather quickly (e.g., Roesken et al., 2011) and at an early age Blomqvist et al., 2012; Larkin & Jorgensen, 2016). It appears that they have different affective dispositions towards traditional mathematics compared to problem solving (e.g., Ingram, 2015) which is similar results as previous studies (e.g., Asami-Johansson, 2015; Higgings, 1997). The students signal it is a positive shift, including a shift of power of who is right. Here, the focus has not been on measuring a change but more to describe students' conceptions after working with six sharing scenarios. Therefore, we cannot say if there has been a change on the basis of the results of the present study. We therefore suggest a quantitative study as a follow up study to see if the students maintain a positive disposition towards mathematics (e.g., Blomqvist et al., 2012). Such as study will provide information

if a small intervention – to work with six mathematical problems – can influence how children experience mathematics as a subject. Also, given that there are few studies focusing on young children's experience of different mathematical teaching (Takeuchi et al., 2016), such a study would increase our knowledge how different teaching is received.

Research ethics

Author contributions

L.S.: conceptualization, investigation, methodology, project administration, validation, visualization, writing—original draft preparation, writing—review and editing

H.E.: data collection and curation, formal analysis, writing—review and editing

M.H: formal analysis, writing—review and editing

P.M.: initial analysis, writing—review and editing

All authors have read and agreed to the published version of the manuscript.

Artificial intelligence

Grammarly has checked the spelling, but no artificial intelligence has been used for the analysis of data or the writing of the paper.

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Institutional review board statement

The project has been checked according to the standards of research ethics through the system provided by Stockholm University, Sweden.

Informed consent statement

Informed consent was obtained from all research participants and their parents.

Data availability statement

Transcribed data can be shared upon request. Other data is unavailable due to privacy or ethical restrictions.

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Conflicts of Interest

The authors declare no conflicts of interest.

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