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Contents

Guest editors

Markku S. Hannula

Markku S. Hannula is the leader of the MathTrack project. Markku is the professor for mathematics education at the University of Helsinki, Faculty of Educational Sciences and until summer 2019 visiting professor at Volda University College, Norway. He has been elected as the president for the International Group for the Psychology of Mathematics Education for 2019–2022.

Most of my research has focused on despair, delight, and desire of mathematics learners. Mathematics anxiety, boredom, and lack of self-confidence are the destiny of all too many students, sometimes leading to destruction of their self-concept as learners. On the other hand, problem solving that leads to flow and Aha!-experiences may bring joy and delight for students and lead to a desire to engage again with similar activities. I dream of designing teaching approaches that would let everyone engage with mathematics in a deeply satisfying and enjoyable way. Our MathTrack project aims to delve deeper into the area of collaborative engagement in mathematics learning and the role of student affect in student visual attention in the classroom.

Most of my publications have been around mathematics-related affect. Somewhat less known is my work in the area of problem solving. For the last couple of years, MathTrack has been the main area of my work. I am also leading the Finnish team in the international Lexicon project and – time permitting – leading the international Norba-TM study on mathematics teachers' beliefs. My network of collaborating is wide; I have joint research and/or publications with researchers from over 20 different countries.

I have supervised four PhD-theses: Emmanuel Adu-tutu Bofah, Laura Tuohilampi, Rauno Koskinen, and Jari Lakka. I currently supervise 11 doctoral students.

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Eeva Haataja is a doctoral student at Faculty of Educational sciences in the University of Helsinki in Finland. She works in in field of mathematics education in MathTrack research project. In her PhD project, Eeva explores teacher-student interaction from the viewpoint of visual attention and nonverbal communication in the context for collaborative mathematical problem solving. Prior to her position at the University, Eeva used to work as an elementary school teacher.

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Her main research areas are pupils' and pre-service teachers' mathematics-related affect, problem solving, knowledge and communication. She is especially interested in non-standard problem-solving and affective factors in learning. She is involved in several international research projects.

She teaches courses in mathematics education and supervises students doing their bachelor, master and PhD thesis. She has supervised about fifty bachelor and master thesis and three PhD thesis. She is interested in developing teaching and she was appointed to Teachers' Academy at the University of Helsinki based on her teaching merits.

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Editorial

Markku S. Hannula, Eeva Haataja, Anu Laine, Päivi Portaankorva-Koivisto and Enrique Garcia Moreno-Esteva

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This LUMAT special issue is a collection of selected papers from the 24th international conference of mathematical views that was held on August 20–22, 2018 in Helsinki, Finland. The conference was a wonderful opportunity to elaborate issues related to mathematics-related affect among colleagues interested in this area of research. The keynote at the conference was given by Reinhard Pekrun with a title: "Achievement emotions in mathematics". Out of the 25 conference presentations, 12 were submitted as a manuscript for peer review. We had one reviewer selected among MAVI 24 participants, and another reviewer was invited among mathematics affect researchers who were not at the conference. After the review and revisions, we ended up with eight articles that you can read in this special issue.

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1 Haataja

In the article "*Teacher-student eye contact during scaffolding collaborative mathematical problem-solving*" Eeva Haataja, Miika Toivanen, Anu Laine and Markku Hannula examine the role of eye-contact when the teacher scaffolds student non-routine problem solving. They used mobile gaze trackers in three lessons and their results showed that students often did not respond to the teacher's gaze. Moreover, they found out that dyadic eye-contact usually happened during affective or cognitive scaffolding and were typically initiated by the student and terminated by the teacher.

2 Hatisaru

In the article "*Lower secondary students' views about mathematicians depicted as mathematics teachers*", Vesife Hatisaru discusses what kind of views lower secondary students have about mathematicians when depicted as a mathematics teacher. The most common patterns that emerged in the drawings and associated writings were that mathematics teachers are predominantly female, viewed positively, lecture, explain and demonstrate, and use whiteboards and books as tools of the profession. Based on the findings she makes recommendations both for research and for mathematics education.

3 Lake

In the article "*Playing it safe' or 'throwing caution to the wind', risk-taking and emotions in a mathematics classroom*", Elizabeth Lake discusses risk-taking and related emotions as part of teacher profession. The article provides examples of teachers being playful and childish in order to keep teaching exciting and inspiring for themselves and their students. On the other hand, the paper also elaborates the teacher reflections for avoiding risk-taking. She concludes that the risk taking is most likely beneficial both for teachers and students, but only confident and experienced teachers tend to dare take risks.

4 Manderfeld

In the article "*Pre-service mathematics teachers' beliefs regarding topics of mathematics education*", Katharina Manderfeld and Hans-Stefan Siller examine the widely explored topic of beliefs of pre-service teachers from a novel perspective, that is, beliefs regarding didactics of mathematics. The beliefs on mathematics didactics differed from the general understanding in the research field, even though the students had participated in mathematics didactics education before the survey. The contents regarding curricular issues and the perspective of the learner in the didactics of mathematics were underrepresented, while the participants added contents of mathematics didactics that did not fit into the background theories.

5 Nyman

In their article "*The issue of 'proudlyness': Primary students' motivation towards mathematics*" Martin Nyman and Lovisa Sumpter discuss about year 2 and year 5 students' expressed motivations for doing mathematics. According to their results the children expressed both intrinsic motivation, that is cognitive-oriented emotional-oriented, normative and personal motivation, and extrinsic motivation like outward and compensation. Furthermore, they found out that motivational factors are intertwined and in relation to affective constructs. So, they suggest mathematics teaching cannot approach students' motivation in a one-dimensional way, but researchers and teachers need to reevaluate the role of motivation in mathematics education.

6 Portaankorva-Koivisto

Päivi Portaankorva-Koivisto and Barbro Grevholm examine 188 Finnish mathematics student teachers' metaphors for the teacher's role in their article: "*Prospective mathematics teachers' self-referential metaphors as indicators of the emerging professional identity*". About one third of the metaphors were classified as self-referential, focussing on the student teachers' personalities, their incompleteness as teachers, or new beginnings or eras. Most of the metaphors were dynamic in nature, describing the teacher as a live object, a person, or an animal. These metaphors show that while student teachers focus more on themselves and less on teaching than experienced teachers, they also see themselves embarking for a lifelong journey of learning through professional development.

7 Suriakumaran

Neruja Suriakumaran, Markku Sakari Hannula, and Maike Vollstedt compare Finnish and German students' personal meanings for mathematics. Their article "*Investigation of Finnish and German 9th grade students' personal meaning with relation to mathematics*" focuses on a comparison of personal meanings that students from Finland (n=256) and Germany (n=276) assign to (learning) mathematics. Indicators of educational system and curriculum could be found in students' responses to explain similarities and differences between the two samples. In both countries, social inclusion is meaningful for most of the students. In addition, Finnish students emphasize the importance of doing well in mathematics, which is likely related to the important selective application to secondary school after grade 9.

8 Wadanambi

In the article "*Exploring the influence of pre-service mathematics teachers' professed beliefs on their practices in the Sri Lankan context*", Gayanthi Wadanambi revisits the problematic relationship between teacher beliefs and their practices. The case study of two Sri Lankan teachers provides an account of how the teachers' professed beliefs encouraged them to adopt flexible practices. However, the influence of social expectations and contextual demands of their educational context, resulted in them adopting these beliefs to a different degree.

The articles in this special issue provide an illustration of the rich variation of theories, contexts, research questions, and methods of research in mathematics-related affect. The articles cover beliefs, emotions, and motivation - the three main areas of affect often identified in mathematics education. The contexts vary from elementary education to teacher education. Most studies use surveys or interviews that are the mainstream methods in this area, while others have analyzed observational data, metaphors, pictures, or eye contacts.

Teacher-student eye contact during scaffolding collaborative mathematical problem-solving

Eeva Haataja, Miika Toivanen, Anu Laine and Markku S. Hannula

University of Helsinki, Finland

Teacher's gaze communicates consciously and unconsciously her pedagogical priorities to the students. By creating and responding to eye contact initiatives, people can communicate both status and affection. This research explores the frequency of teacher-student eye contacts and their connection to teachers' scaffolding intentions. The data consisted of mobile gaze tracking recordings of two teachers and stationary classroom videos during three collaborative mathematical problem-solving lessons. The quantitative analysis showed that most of the teacher gazes on student faces did not lead to dyadic eye contacts and those gazes that did, occurred often during affective and cognitive scaffolding. These results offer us novel and important insight in the nonverbal part of scaffolding interaction.

Keywords
eye contact,
mathematical problem solving,
teacher gaze,
teacher-student interaction

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1 Introduction

Nonverbal interaction plays an important role in the classroom. In teacher-student nonverbal interaction, students learn by following teacher's gaze and through eye contact with the teacher (Csibra & Gergely, 2009; Shteynberg, 2015). The teacher directs her gaze towards targets that are of high priority to her and, by this direction, conveys these priorities to the students during instruction (Shteynberg, 2015). In the classroom, students are the most important attentional priority for the teachers (McIntyre, 2016). Especially their solution papers, faces, and hands are the most relevant gaze targets for the teacher during scaffolding collaborative mathematical problem-solving process (Haataja, Garcia Moreno-Esteva, Toivanen, & Hannula, 2018).

Dyadic (two-sided) eye contacts between two persons are one component in nonverbal communication and they can convey both immediacy and authority (Mehrabian, 1972). People tend to focus on other people's eyes to interpret their affects (Itier, 2015) and the information from the environment (Shteynberg, 2018). However, situational social structures and a person's intentions affect her willingness to form a dyadic eye contact with others, which highlights the importance of real-life contexts in data collection (Tatler & Land, 2015). Our data investigates this phenomenon from the teacher's viewpoint by combining classroom videos with mobile gaze tracking recordings. The unconscious nature of gaze direction (Tatler, Kirtley, Macdonald, Mitchell, & Savage, 2014) and the significance of the eye contact in human interaction and student learning underline the importance of examining this phenomenon and its relations to teachers' pedagogical intentions. In this study, we investigate the means of teacher-student eye contact interaction during collaborative mathematical problem solving. The results are reflected from the viewpoint of the teachers' scaffolding intentions.

2 Scaffolding interaction

The teacher's actions in supporting students' learning in problem-solving process towards the goals of their zone of proximal development are called scaffolding (Wood, Bruner, & Gail, 1976). Scaffolding is a contingent interactive process between the teacher and the students (Van de Pol, Volman, & Beishuizen, 2010). In scaffolding, the teacher facilitates students' learning by offering them guidance that she tailors for the student's needs in a certain phase of the problem-solving process (Hermkes, Mach, & Minnameier, 2018; Van de Pol et al., 2010). The teacher's intentions for scaffolding can be divided into three categories: cognitive, affective, and metacognitive (Van de Pol et al., 2010).

With *cognitive scaffolding*, the teacher structures and adapts the problem task to better correspond with students' competences (Van de Pol et al., 2010). To provide help of right quality and quantity, the teacher has to analyze carefully how students construct and try to solve the task (Hermkes et al., 2018). Elaborate introduction as well as tailored, activating questions on the strategies and contents of the problem advance students' cognitive learning during the process (Kojo, Laine, & Näveri, 2018).

The affective intention means teacher's acts to promote students' motivation and prevent frustration during the problem-solving process (Van de Pol et al., 2010). Problem solving is a teaching method that often includes multiple emotions (Pesonen & Hannula, 2014). Positive emotions enhance mathematical learning while negative emotions affect vice versa (Pekrun, Lichtenfeld, Marsh, Murayama, & Goetz, 2017).

Metacognitive scaffolding refers to guiding students' learning process by directing their attention and interaction towards relevant objects (Van de Pol et al., 2010). In collaborative problem solving, also the metacognitive processes are of social nature. Especially while solving demanding problem tasks, the successful learning process requires metacognitive negotiation between the members of the collaboration group (Iiskala, Vauras, Lehtinen, & Salonen, 2011).

3 Teacher-student eye contact

Shared attention conveys cognitive information and intensifies affective experiences such as emotional states (Shteynberg, 2018). Behaviors to produce the experiences of interpersonal closeness in other people are called nonverbal immediacy (Andersen, Andersen, & Jensen, 1979). Nonverbal immediacy behaviors, such as eye contact (Andersen et al., 1979; Mehrabian, 1972), are interdependent with teachers' communication skills (Bainbridge Frymier & Houser, 2000). Teacher's nonverbal immediacy increases the students' experiences of cognitive and affective learning and engagement in learning and studying mathematics (McCluskey, Dwyer, & Sherrod, 2017).

Eye contact transfers experiences of affiliation, positive attitude, and warmth towards the other person (Mehrabian, 1972). With eye contact, the teacher communicates to students that they are in the locus of her attention and the interaction or information implicates them (Adams, Nelson, & Purring, 2013; McIntyre, Mainhard, & Klassen, 2017). Teacher's gaze towards the students, while listening to them, increases their experience of close interpersonal relationship with the teacher (McIntyre et al., 2017). Teachers also see that eye contact with students is in connection to good teacher-student contact. The quality of the contact between the teacher and students form one link between teacher behavior and student learning (Korthagen, Attema-Noordewier, & Zwart, 2014).

Direct face-targeted gaze also includes information on person's social status and authority (Brey & Shutts, 2015; Mehrabian, 1972). With direct gaze, the teacher can address the role of the student in the instruction and the meaningfulness of the learning contents (Böckler, van der Wel, & Welsh, 2014). During teacher-centered instruction, students tend to relate teacher's eye contact with communication of authority and dominance (McIntyre, 2016). Mere immediacy cannot produce learning without clarity and relevance of the instruction (Mottet et al., 2008). Direct eye contact can increase the effectiveness of teaching, as the students interpret the nonverbal communication to form conceptions of the teacher and the instruction (Babad, 2009).

Doherty-Sneddon and Phelps (2007) have investigated teachers' interpretations of students' eye contact initiatives and found out that, during working on challenging tasks, teachers connected students' averted gaze with deep thinking. Thus, teachers interpreted students' avoidance of eye contact to correlate with contributing in the zone of proximal development and did not want to interrupt that cognitive process. During teachers' speech, student's eye contact initiative was understood as a sign of motivation and understanding (Doherty-Sneddon & Phelps, 2007).

Questionnaire and video research have shown the importance of teacher-student eye contact as a part of classroom communication from the perspective of student experiences on motivation (Zeki, 2009) and teacher immediacy (Babad, 2009) during instruction. However, the educational field lacks first-hand evidence on the occurrence and significance of this part of the nonverbal classroom communication. Our previous case study (Haataja et al., in press) found a relation between teacher's visual attention and the intentions of scaffolding on student collaboration. The teacher focused on students' faces more while scaffolding affective aspects than during cognitive or metacognitive scaffolding (Haataja et al., in press). The current study zooms into the characteristics and role of teachers' face targeted gazes in scaffolding interaction during collaborative mathematics lessons.

4 Research questions

To explore the relation between teacher-student eye contact and teachers' scaffolding intentions (cognitive, affective, and metacognitive), our research questions were:

1. "In what ways do teachers interact with students in terms of eye contact during collaborative mathematical problem solving?"
2. "How do the frequency and durations of teacher-student eye contacts differ across the teachers' scaffolding intentions?"

5 Methods

We collected the data for this research during three mathematics lessons in three Finnish lower secondary schools. We chose three qualified mathematics teachers with different levels of expertise. The data collection with teachers 1 (middle-career) and 2 (novice) took place in the spring of 2017 and with the teacher 3 (experienced) in the winter of 2018. The participating classes were situated in a large Southern Finnish city. The classes included ninth-grade students, 15-16 years of age. We received the permissions for the data collections from the school principals. The teachers and the classes volunteered for this research, students' parents were informed, and all students filled a written consent form. As an acknowledgement of their contribution, the classes received small donations to their school trip account and the target students, and the teachers received gift cards.

The mathematics teacher 1 was 39 years old and had 14 years of teaching experience. To secure her anonymity, we call her with pseudonym *Joanne*. Joanne had seven boys and twelve girls in her class. They were seated in pairs and, during the collaborative phase of the lesson, formed five collaboration groups of three to five students. The teacher 2 was 30 years old with three years of experience in teaching mathematics. This teacher is called with pseudonym *Fred*. Fred's class included eleven boys and eight girls. The students sat in pairs and worked in five collaboration groups of two to four students. One girl in Fred's class did not participate in the task of the lesson and had personal tasks. The teacher 3 was 56 years old and we call her *Lily*. She possessed 31 years of teaching experience and had six boys and three girls in her class. Lily's students were sitting in collaboration groups of four, three, and two members.

With all three classes, three stationary video cameras and several microphones recorded the actions and verbal communication during the lessons. Additionally, in every class four target students and the teacher wore mobile gaze tracking devices that recorded their visual attention during the lesson. The gaze tracking devices were self-made and consisted of two eye cameras, a scene camera, and simple electronics attached in plastic goggles (Toivanen, Lukander, & Puolamäki, 2017). A dedicated software computed the gaze point in the scene camera with average accuracy of approximately 1.5 degrees of visual angle.

To investigate the scaffolding interaction, we examined the collaborative problem-solving phases in the middle of the observed mathematics lessons and excluded the instructive phase in the beginning and presentation phase in the end of the lessons from the analysis. The collaborative phases lasted about 18 minutes in Joanne's, 16 minutes in Fred's, and 17 minutes in Lily's class. The task of the lessons was a geometry problem (Figure 1). The goal was to find the optimal way to connect four imaginary cities, located at the vertices of a square, with an electrical cable. To solve the problem, students were allowed to use paper, pencils, rulers, and calculators. In Lily's class, they also used laptop computers with Geogebra software.

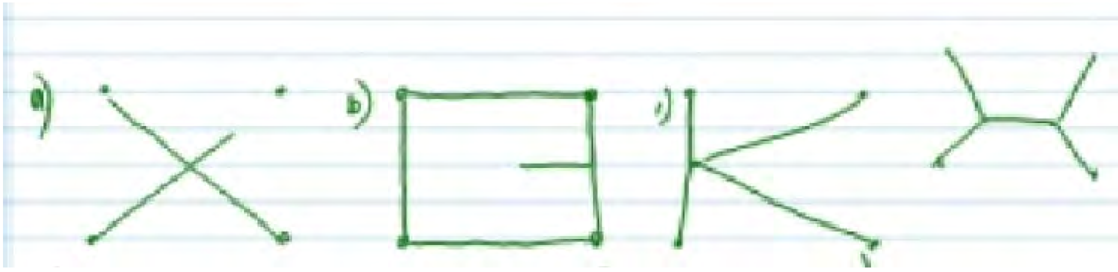

Figure 1. Four solution drafts to the problem task, the optimal solution furthest on the right. A capture from a Smartpen recording on a student in Joanne's class.

The researchers instructed the teachers to guide the students during the problem-solving process by supporting them and posing questions on the process but not giving hints on the optimal solution. During the session, all teachers roamed in the classroom and offered scaffolding to one group at a time. Joanne gave more whole-class instructions during the collaborative phase than Fred or Lily. Additionally, Joanne and Lily instructed their students to move in the class to observe the solutions of other collaboration groups.

To analyze the data, we coded teacher's gazes with ELAN software with a dwell time as a coding unit. Dwell time is a single gaze dwell on a specific target (e.g. face) that consists of one or more fixations (Holmqvist et al., 2011). Each dwell that was at least 80 milliseconds long was coded according to its target. As this analysis is done from the teachers' viewpoint, the gaze duration was defined as the dwell time of teacher gaze at a student's face. We separated one-sided student-face targeted dwells from those with dyadic eye contact and divided the dyadic eye contact gazes according to the initiative part of the gaze. Thus, we had three gaze categories: (1) *a teacher gaze on student face*, (2) *a teacher-started dyadic eye contact*, and (3) *a student-started dyadic eye contact*.

The scaffolding intentions were coded according to the classroom video and classified into five categories. Three scaffolding categories arose from the theory of Van de Pol et al. (2010): (1) *cognitive*, (2) *affective*, and (3) *metacognitive* scaffolding. These three intentions of scaffolding were defined by analyzing the verbal teacher-student interaction. During cognitive scaffolding, the teachers helped the students with the mathematical contents of the task, during affective scaffolding, they encouraged and motivated them, and during metacognitive scaffolding, the teachers directed the students' attention towards the task. To cover also the moments of scaffolding that did not include verbal interaction we added two more categories. These categories were (4) *monitoring* (teacher watching collaboration to get an idea of what stage the group is in the problem-solving process) and (5) *fading* (final scanning of the group before moving on to another group).

After the coding, we conducted quantitative analyses with IBM SPSS software. To form an overview of the research topic, we analyzed the teachers' gaze behavior during the collaborative problem-solving sessions with descriptive quantitative analysis. After that, the relation between the occurrence of the eye contact categories and teachers' momentary scaffolding intentions were analyzed with crosstabs and Pearson's Chi-square test using the Bonferroni adjusted pairwise comparisons with expected cell counts. To investigate the amounts of visual attention invested in the teacher-student eye contacts, the main effects of gaze durations for teacher persons, scaffolding intentions, and eye contact types were analyzed with two-way analysis of variance. The gaze durations were transformed into logarithmic scale to improve the validity of the ANOVA. However, in the results section we present them in the original, linear form for the clarity and readability of the report.

6 Results

The distribution of visual attention of Joanne, Fred, and Lily towards student-related targets were quite similar to each other. However, the total count of student-targeted gazes was twice as high in Joanne's (N = 1498) and Fred's (N = 1547) classes than in Lily's (N = 712) class. During Lily's lesson, some technical issues with student laptops stole her time, and she had to focus on non-pedagogical targets, such as the researchers and her own computer.

We start the results by presenting the overall distribution of teachers' gazes at student-related targets during collaborative mathematical problem-solving session (Figure 2). These gazes were recorded while students solved the problem task in collaboration groups and the teachers walked around the classrooms to scaffold the learning process.

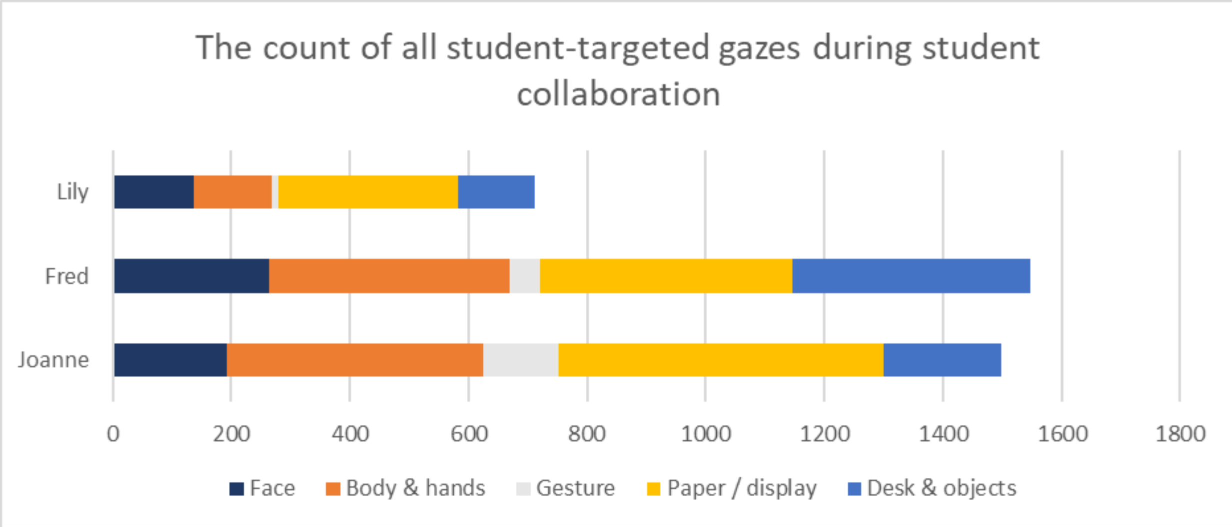

Figure 2. The count of teacher gazes towards student-related targets during collaborative phases of three separate mathematical problem-solving lessons. Fred's data has been previously presented in Haataja et al., in press.

While scaffolding the collaborative problem-solving session, the teachers focused most on students' solution papers to receive information on their cognitive process. Additionally, all the teachers payed attention to students' bodies and hands as well as desks and other objects while they monitored activities and working of the students. However, paper-targeted gazes were longer than body or desk targeted, and in the comparison of total dwell time, paper-targeted gazes formed the majority of teacher's visual attention. The teachers also directed attention to students' gestures when the students pointed at their solutions while explaining them.

In this study, we focus on teachers' student-face targeted gazes. Face-targeted gaze dwells covered 13 % of Joanne's visual attention by both count of gazes and the total dwell time. With Fred, students' faces captured 17 % of the gaze count and 18 % of total dwell time. With both these teachers, the mean durations of face-targeted gazes were close to the mean durations of all gazes. In Lily's class, the face-targeted gazes were shorter on average. By count, they cover 19 % of Lily's gazes but by total dwell time only 9 % of Lily's visual attention.

The teachers gazed at their students' faces frequently throughout the collaborative phase but the majority of teacher gazes at student faces did not include dyadic eye contact. Joanne gazed at her students' faces 191 times during the collaborative learning phase. The total dwell time of these gazes was 89 seconds and mean duration 0.46 seconds (*SD* = .632). In 40 % of Joanne's face-targeted gazes, a dyadic eye contact was formed between her and a student. Fred directed 264 gazes towards students' faces lasting 143 seconds in total and 0.54 seconds on average (*SD* = .633). Of these gazes, 30 % included a dyadic eye contact. Lily's 137 face-targeted gazes lasted 59 seconds altogether, with the average of 0.43 seconds (*SD* = .546). The proportion of dyadic eye contacts was 23 % of all Lily's gazes on student faces.

Figure 3 presents those gazes that included a dyadic eye contact between a teacher and a student. The gazes are categorized according to the initiative part of starting the contact. The distribution between teacher-started and student-started dyadic eye contacts was quite similar between the teachers.

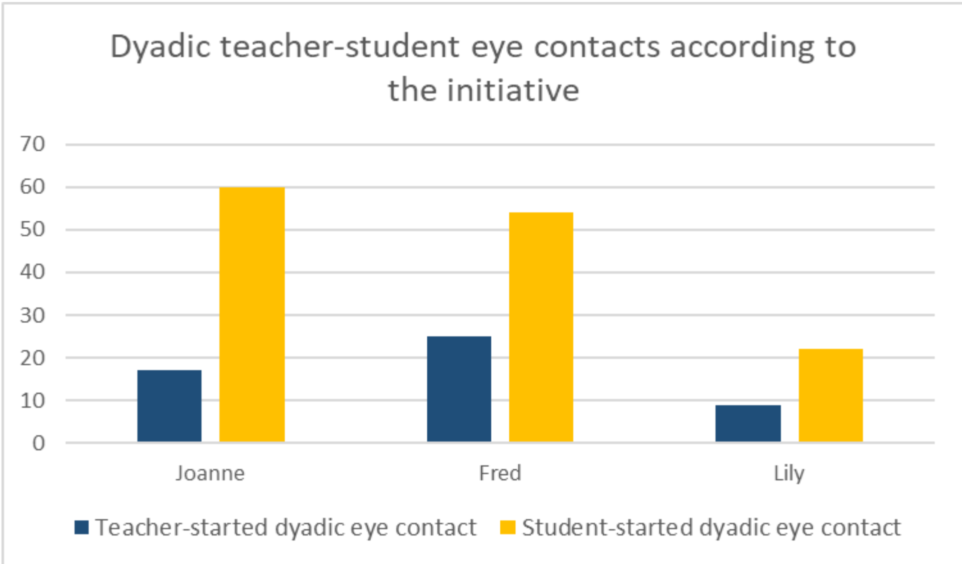

Figure 3. The count of dyadic teacher-student eye contacts according to the initiative part, three teachers compared.

Students started the eye contacts in 78 % of the cases in Joanne's, 68 % in Fred's, and 71 % in Lily's class. Joanne had a dyadic eye contact with a student 77 times, of which 60 were started by a student's initiative. After Joanne's response to a student's cue of shared attention, Joanne ended the eye contact more often (*n* = 38) than the student did. Eye contact gazes started by the teacher occurred less often. Joanne started 22 % of the dyadic eye contacts by looking the student in the face. Only three times Joanne continued looking at a student who had ended the eye contact that was started by Joanne.

In Fred's lesson, the distribution of initiatives of eye contact were relatively similar with Joanne. Fred had 79 eye contact gazes with students during the session. The most common type of eye contact was student first gazing Fred's face, Fred responding for a moment and then focusing to some other target (*n* = 25). In comparison to Joanne and Lily, Fred started dyadic eye contact gazes most often (32 %). With Fred, the students also ended teacher-started dyadic eye contacts more often (n = 9) than with the other teachers.

During the analyzed phase of Lily's lesson, 31 dyadic eye contact gazes were formed between her and her students. Similarly, to other two teachers, also with Lily, the students started, and she ended the dyadic eye contacts most often (n = 16). The nine (29 %) teacher-started dyadic eye contacts included only one student-ended eye contact and eight eye contacts that Lily ended.

To summarize, the dyadic eye contacts between the teachers and students were most often started by a student and ended by a teacher. The opposite category (teacher started, student ended) was the least frequent category. This can be interpreted to result from the general interactional roles of the teachers and the students in the classrooms, where the teacher is in a central position and in a leading role during interaction with the students. The following chapter examines, whether the teachers' momentary pedagogical intentions affected the occurrence of these gaze categories.

Teacher-student eye contacts during scaffolding interventions

The teachers tended to stop by one group at a time to receive information on their proceeding (monitoring and fading) and offer them cognitive (mathematical), affective (motivational and emotional), and metacognitive (attentional and procedural) scaffolding. During cognitive scaffolding, the teachers asked questions about the solutions the students had drawn on their papers and displays and gave the procedural advice on how to continue by drawing more solutions and comparing them by calculating or measuring the lengths of the lines. Affective scaffolding included encouraging and supporting the students when they were frustrated or bored. Scaffolding was coded as metacognitive, when the teachers asked the students to maintain or direct their attention towards the collaboration and share thoughts and talk about the problem task.

To explore the role of dyadic eye contacts in scaffolding interaction, we compared the occurrence of eye contacts with scaffolding intention categories using Pearson Chi-square test. This analysis showed significant differences in counts of eye contact categories between scaffolding intention groups (X^2 (8, 379) = 33.47, p < .001). The results of pairwise post hoc tests are presented in Table 1. The comparison of proportions revealed significant differences in the count of eye contacts during monitoring, cognitive scaffolding, and affective scaffolding.

Table 1. Eye contact categories (columns) during the scaffolding intentions (rows). The table shows the counts, the expected counts, and the percentages of each subcategory. The subsets of Eye contact categories with different subscript letters (a, b) differ significantly from each other in count versus expected count ratio at the *p* < .05 level.

Scaffolding intention		**Eye contact**			**Total**
		One-sided	Teacher-started	Student-started	
Monitoring	**Count**	**48_{a}**	**$1_{a,b}$**	**8_{b}**	**57**
	Expected Count	37.3	4.7	15.0	57
	% within group	84.2%	1.8%	14.0%	100%
Cognitive	**Count**	**54_{a}**	**$10_{a,b}$**	**41_{b}**	**105**
	Expected Count	68.7	8.6	27.7	105
	% within group	51.4%	9.5%	39.0%	100%
Affective	**Count**	**51_{a}**	**14_{b}**	**20_{a}**	**85**
	Expected Count	55.6	7.0	22.4	85
	% within group	60.0%	16.5%	23.5%	100%
Metacognitive	**Count**	**71_{a}**	**5_{a}**	**28_{a}**	**104**
	Expected Count	68.1	8.5	27.4	104
	% within group	68.3%	4.8%	26.9%	100%
Fading	**Count**	**24_{a}**	**1_{a}**	**3_{a}**	**28**
	Expected Count	18.3	2.3	7.4	28
	% within group	85.7%	3.6%	10.7%	100%
	Count	**248**	**31**	**100**	**379**
	Expected Count	248	31	100	379
	%	**65.4%**	**8.2%**	**26.4%**	**100%**

During monitoring and fading phases of teachers' scaffolding, dyadic eye contacts were less frequent and one-sided teacher gazes at student faces more frequent than the expected count assumed. This difference was statistically significant within the category of monitoring. Instead of dyadic scaffolding interaction, monitoring and fading reflected teachers' control on students as the teachers followed and scanned the work of small-groups before and after scaffolding interventions. Typically, the students did not disrupt their actions while gazed by their teachers. This led us to the interpretation that ninth-graders are already accustomed to being under teachers' visual attention and do not respond to teachers' face-targeted gazes without interactional purpose.

On the contrary, dyadic teacher- and student-started eye contacts were relatively frequent during cognitive and affective scaffolding. In cognitive scaffolding, the count of student-started dyadic eye contacts was significantly higher than the count of one-sided teacher gazes at student faces in relation to the expected count. During the cognitive scaffolding, teachers focused more on students' solution papers and less on student faces than during other intentions of scaffolding. However, the students tended to look at teachers' faces while listening to them. After making sense of students' written solutions, teachers often responded to students' initiatives of dyadic eye contacts to concentrate on students' questions and explanations, to explain the mathematical contents, or to ensure mutual understanding of the situation.

During affective scaffolding, the count of teacher-started dyadic eye contacts differed significantly from the other categories. During these phases, the teachers made more initiatives for teacher-started dyadic eye contact. The intention of the teachers in affective scaffolding directed their gaze to collect information on students' emotional stages. The interaction also included support and encouragement that probably created the feeling of immediacy and enhanced student responses to teachers' eye contact initiatives. The teacher-student interaction seemed to be more equal during affective than metacognitive or cognitive scaffolding. For instance, the students joked and argued with their teachers. In these situations, the dyadic eye contact seemed to form naturally between the participants of interaction.

In moments of metacognitive scaffolding, the teacher gazes at student faces were often one-sided. When dyadic eye contacts occurred, they were usually student-started. In metacognitive scaffolding, the teachers instructed students to collaborate and focus on the problem task and were more interested in their actions than their facial expressions. Students probably looked at them to understand better the instructions that often-concerned changing position or the patterns of peer interaction. In these moments, the teachers also often called the students by their names, which captured the students' attention.

To conclude, the one-sided teacher gazes at student faces were common during teachers' monitoring, fading, and metacognitive scaffolding. During these categories of interaction, the students were under teachers' supervision and not as active in the nonverbal communication as during cognitive and affective scaffolding. When scaffolding cognitive and affective aspects of the problem solving, the teachers and students were in reciprocal interaction, and created dyadic eye contacts to reach shared understanding on the momentary scaffolding intentions. However, also the durations of the gazes include information on the attentive choices of the participants, and these will be explored in the following.

Gaze durations

To form a broader picture on the amounts of teacher-student eye contact interaction, the durations of the gazes were also analyzed. The influences of three independent variables (teacher, scaffolding intention, eye contact) on the durations of teachers' face-targeted gazes were analyzed with two-way ANOVA. No significant interaction effects were found with the analysis. Additionally, the main effect of the teacher person was not significant ($F(2, 370) = 2.09$, $p = .125$), and thus we are able to examine the results of the three teachers together.

The main effect for eye contact category yielded an F ratio of $F(2, 370) = 18.83$, $p < .001$, indicating significant differences between all three categories of eye contact: one-sided teacher gaze at student face ($M = 409.69$ ms, $SD = 361.09$), teacher-started dyadic eye contacts ($M = 1001.84$ ms, $SD = 1256.12$), and student-started dyadic eye contacts ($M = 707.24$ ms, $SD = 811.49$). The pairwise Bonferroni adjusted post hoc comparison showed that teacher-started dyadic eye contacts were significantly longer than student-started dyadic eye contacts ($p = .001$) or one-sided teacher gazes at student faces ($p < .001$). Gazes with a student-started dyadic eye contact were also significantly longer than those without dyadic eye contact ($p = .019$).

The longest face-targeted gazes occurred during cognitive and affective scaffolding. The main effect for scaffolding intention yielded an F ratio of $F(4, 370) = 8.45$, $p < .001$. According to Bonferroni adjusted pairwise post hoc tests, the gazes during cognitive scaffolding ($M = 773.83$ ms, $SD = 1048.48$) were significantly longer than gazes during monitoring ($M = 282.25$ ms, $SD = 222.25$, $p < .001$), fading ($M = 334.36$ ms, $SD = 264.71$, $p = .007$), and metacognitive scaffolding ($M = 425.47$ ms, $SD = 401.80$, $p = .03$). Also, face-targeted gazes during affective scaffolding ($M = 737.20$ ms, $SD = 786.81$) were longer than those during monitoring ($p < .001$) and fading ($p = .04$) with statistical significance. Metacognitive scaffolding, monitoring, and fading did not differ significantly from each other.

The longest gazes occurred during cognitive and affective scaffolding when the teachers started the dyadic eye contacts. The student-started dyadic eye contacts were also relatively long. The one-sided teacher-gazes at student faces were generally short, as were the gazes during teachers' monitoring the student groups. To compare the gaze durations in the five scaffolding categories, we conducted a two-way analysis of variance.

7 Discussion and conclusion

This report examined eye contacts, one aspect of teacher-student nonverbal communication, in momentary scaffolding interaction. We compared the occurrence of teacher gazes at student faces and dyadic teacher-student eye contacts between the categories of teachers' scaffolding intentions. On all three collaborative problem-solving lessons, the dyadic eye contact gazes between teachers and students were less frequent than one-sided teacher gazes at student faces.

The one-sided teacher gazes were the most common during teacher monitoring and fading from the student group, that is, during moments with no verbal teacher-student interaction. Students seemed to be used to being under teachers' visual observation while working on their task. In these moments, teachers seemed to convey their pedagogical expectations and status in the classroom with mere gaze. From interactional perspective, students' aversion of gaze can be interpreted as both a sign of noticed teacher control (Böckler et al., 2014; Mehrabian, 1972) and a signal of well-proceeding problem-solving process (Doherty-Sneddon & Phelps, 2007). Beneficial instruction is a combination of immediacy and control (Mottet et al., 2008), and in these moments the teachers seemed to communicate the latter.

According to our research, a typical dyadic eye contact between a teacher and a student started from a student's initiative and ended by the teacher. The role of the teacher as the authority and leader in the classroom directs students' visual attention towards them (McIntyre, 2016). Additionally, students have the natural tendency to follow the instructing adult's attention to learn relevant contents (Csibra & Gergely, 2009). On one hand, the teachers may have distinct goals for the communication with students. For instance, during cognitive scaffolding, the teacher focuses mainly on students' solution papers (Haataja et al., in press). On the other hand, the teachers are responsible for scaffolding all the students, and thus may unconsciously shorten the gazes to distribute their attention evenly among the students.

The present study indicates that regardless of teachers' intentions of scaffolding, the students tend to start eye contacts with them. Probably the eye contacts ease concentrating on teacher's speech in a noisy class environment by looking at her face or try to predict teacher's emotional stage and demands from her eyes. However, this may lead to not noticing teacher's instructive gestures as she explains the task and solution procedures. In addition to sufficient scaffolding through the teacher's activating questions (Kojo et al., 2018), the students might benefit from explicit guidance on attention direction during scaffolding interaction. In future, it would be interesting to investigate these moments from students' perspective to interpret their understanding of teacher-student eye contacts and noticing of teacher gestures.

Teacher-started dyadic eye contacts were the least frequent gaze category. However, they consisted of the longest gazes in the analysis. In the comparison between scaffolding intentions, the longest gazes occurred during cognitive and affective scaffolding. This indicates that teachers also seek for information from dyadic eye contacts, especially during affective scaffolding. These gazes may strengthen the message of affective scaffolding by increasing the nonverbal immediacy between the teacher and the student (Korthagen et al., 2014; McIntyre et al., 2017).

This research is the first phase of our analysis on teacher-student eye contact. The small sample size can be seen as its limitation. However, our participant teachers represent different levels of teaching experience, genders, and school neighborhoods. These results on Joanne, Lily, and Fred's face-targeted gazes are in line with our previous case study on the distribution of the visual attention of Fred (Haataja et al., in press; Haataja et al., 2018). Additionally, the similarities of the participants' gaze behaviors despite the differences in their teaching experience, taught classes, and the course of the data collection lessons suggest that there are certain general patterns in teacher-student eye contact communication. This encourages us to continue in this path as well as with general theories of nonverbal interaction. In future, comparisons between a larger number of teachers and students from different backgrounds would be fascinating.

The reliability of the analysis is also important to be reflected. One researcher conducted the coding in an interaction with the rest of the research group. Some eye contacts were difficult to interpret from to teacher gaze data. Two researchers interpreted the unclear moments, and very fuzzy gazes were excluded from the analyses. Additionally, the highly accurate gaze data and the triangulation of data collection increase the reliability of the analysis. The stationary video recording and gaze data complemented each other during unclear moments.

This data collection method offers data on sensory perceptions that are partly unconscious for the participant. Thus, we have asked for an ethical review of the University of Helsinki Ethical Review Board in the Humanities and Social and Behavioral Sciences. We noticed that despite the inconveniences of the research setting, the participating teachers and students were interested in our method of data collection and excited about the mathematics lesson that included problem solving and collaboration.

To summarize the conclusion, students follow teachers' gaze throughout the lesson and teachers seem to use eye contact for at least two purposes. On one hand, they create immediacy by making initiatives for dyadic eye contacts to support affective scaffolding. On the other hand, they respond to students' initiatives during cognitive scaffolding to enhance the interaction. These both pedagogical goals alternate throughout collaborative learning, such as mathematical problem solving. In practice, teachers would benefit from understanding this phenomenon as they plan problem-solving lessons. With explicitly directing students' visual attention towards targets that are relevant for learning in a certain situation of scaffolding interaction, they may both foster success in solving the task and enhance the deeper learning on problem-solving procedures. In the future, we wish to explore these interactional processes and to form a more exhaustive conception on the teacher-student nonverbal interaction by analyzing multiple gaze data from these sessions.

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Lower secondary students' views about mathematicians depicted as mathematics teachers

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The present study examined lower secondary students' images of mathematics, comprised of stated attitudes to and perceived needs for mathematics, and views about mathematicians and their work. A group of 1284 lower secondary students drew a picture of mathematician at work and described their drawings. The students' drawings fell into two distinct groups: drawings that depicted their view of what a mathematician at work would look like, and drawings that depicted a mathematician who was clearly a mathematics teacher. This article presents the data regarding the latter group. Trends that emerged from the drawings in this sample included that mathematics teachers were: predominantly female; had a positive image; incorporated lectures, explanations, and demonstrations; and used whiteboards and books as tools of the profession. The article concludes with possible implications for practice and research.

Keywords
gender,
lower secondary students
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1 Introduction

Mathematics is "an enabling discipline for Science, Technology, Engineering, and Mathematics (STEM)-based university studies and related careers." (Forgasz, Leder, & Tan, 2014, p. 369). Over time, much research has been published on the images of mathematics (e.g., Martin & Gourley-Delaney, 2014) and mathematicians (e.g., Aguilar, Rosas, Zavaleta, & Romo-Vázquez, 2016). This body of research shows that many students find mathematics boring (Stiles, Adkisson, Sebben, & Tamashiro, 2008), difficult, and complicated (Markovits & Forgasz, 2017). Moreover, students appear to rely on some negative views about mathematicians (Picker & Berry, 2000), and have a lack of knowledge about the work of mathematicians (Latterell & Wilson, 2012; Picker & Berry, 2000; Rock & Shaw, 2000). Those student images significantly relate to their performance in (Wong, Marton, Wong, & Lam, 2002) and attitudes to mathematics (Aguilar et al., 2016). Furthermore, students who have negative images of mathematics (Latterell & Wilson, 2004) and know little about the work of mathematicians are less interested in pursuing mathematics (Latterell & Wilson, 2012; Piatek-Jimenez, 2008) or math-related careers (Piatek-Jimenez, Cribbs, & Gill, 2018).

The attainment of STEM degrees within the young population in Turkey is important for the progress of its economy (PwC, 2017). One of the fundamental concerns of business organizations is the need for an adequate number of qualified employers (e.g., The Turkish Industry and Business Association). On the other hand, the business sector needs a STEM-skilled workforce, "in order to stay in the race in the global economy, which is led by technology, innovation and digitalisation" (PwC, 2017, p. 9). On the other hand, between 2013 and 2016, the percentage of STEM graduates within the country was only approximately 17%. The performance of students in science and mathematics measured against international benchmarks has stalled or declined (Mullis, Martin, Foy, & Hooper, 2016) as has the participation in tertiary mathematics courses (Nesin, 2014) and interest in science related careers. A study comparing a total of 1,800 grade 3, grade 7, and grade 10 students' perceptions of scientists and doing science across Turkey, China, India, South Korea and the United States of America (USA) revealed that Turkish students held stronger stereotypical views of scientists than students in India, the USA and China, and about half of the Turkish sample indicated they would not want a career in science or science-related areas (Narayan, Park, Peker, & Suh, 2013). Unpacking students' images of mathematics and views about mathematicians is a useful first step in getting students to think about future careers in STEM (European Commission [EC], 2011).

A review of past studies indicated that relatively little research has been done on students' images of mathematics or mathematicians in Turkey (Ucar, Piskin, Akkas, & Tasci, 2010; Yazlik & Erdogan, 2016). Ucar et al. (2010) examined a group of nineteen elementary school student attending a supplementary school, while Yazlik and Erdogan (2016) focused on the perceptions of high school students. The aim of the study reported here was to determine the image of mathematics held by a large group of Turkish lower secondary students' (grades 6 to 8) through an examination of their drawings.

2 Framing the study

2.1 The image of mathematics

A review of the literature revealed that there is no universally-held view on the image of mathematics. Nevertheless, a consensus has been reached on the components that contribute to an individual's image of mathematics such as attitudes, beliefs, views and emotions regarding mathematics.

Synthesizing Roger's (1992) and Thompson's (1996) conceptions of image, Sam and Ernest (2000) defined the image of mathematics as "a mental representation or view of mathematics, presumably constructed as a result of social experiences, mediated through interactions at school, or the influence of parents, teachers, peers or mass media." (p.195). They operationalized this definition of the image of mathematics to include eleven components including stated attitudes; feelings; descriptions or metaphors for mathematics; views about mathematicians and their work; and beliefs about the nature of mathematics, mathematical ability, or sex differences in mathematical ability.

Wilson (2011) proposed an operational construct to define the factors that might influence individuals' engagement in mathematical activity which coincides with the image of mathematics construct. He used the term 'disposition' composing of beliefs/values/identities, affect/emotions, behavioral intent/motivation, and needs (relating to the Maslow's hierarchy). Wilson (2011) accepted needs as a contributory factor influencing an individual's disposition. That is, an individual might value learning mathematics despite a lack of interest or enjoyment because s/he found it useful. The needs element was not explicitly stated in Sam and Ernest's (2000) conception of the image of mathematics, but it could coincide with the factors that influence people's images in their model (Lane, Stynes, & O'Donoghue, 2014).

Combining the definitions of Wilson (2011) and Sam (1999) with other research in the affective domain such as Hannula (2007), Lane et al. (2014) defined the image of mathematics as "a mental representation or view of mathematics, presumably constructed as a result of past experiences, mediated through school, parents, peers or society." (p.881). According to Lane et al. (2014), the term 'image of mathematics' is composed of three domains: the affective domain (attitudes, emotions, and self-concepts relating to mathematics and mathematics learning experiences), the cognitive domain (beliefs relating to mathematics and mathematics learning experiences), and the conative domain (motivation relating to mathematics learning). Consistent with Sam and Ernest (2000), in Lane et al.'s (2014) study, students' images of mathematics were influenced by their mathematics teachers, past experiences of mathematics, parents, peers, and prior mathematics achievement. However, participant students cited their mathematics teachers as the main influence on their images of mathematics which aligns with Yazlik and Erdogan's (2016) findings.

This study examined lower secondary students' images of mathematics which were found to be comprised of various components. The images of mathematics provided were found to be well predicted by students' attitudes (Lane et al., 2014; Sam & Ernest, 2000; Wilson, 2011), perceived needs for mathematics (Wilson, 2011), and views about mathematicians and their work (Sam & Ernest, 2000). While in the larger study I researched these three components in relation to the image of mathematics, in the portion of the study presented here, I report on students' views about mathematicians and their work.

2.2 Drawings in exploring the views about mathematicians

In educational research, inquiring into students' own conceptions of their educational experiences is vital (Haney, Russell, & Bebell, 2004). One technique employed to document students' conceptions about their teaching and learning experiences is drawings (Gulek, 1999).

Historically, the research capturing individuals' views about mathematicians through drawings arose from Mead and Metraux's (1957) seminal work examining the perceptions students held about scientists. These authors asked approximately 35,000 high school students around the USA to write a short essay about their perspectives of science and scientists. Through the years the "Draw a Scientist Test (DAST)" (Chambers, 1983) was patterned from Goodenough's (1926) "Draw a Man Test" (see Thomas, Pedersen, and Finson (2001) for a comprehensive review). This effort opened the way for researchers such as Picker and Berry (2001) to consider having students draw a mathematician on a blank sheet of paper to describe the perceptions reflected in students' drawings of mathematicians, which was called "Draw a Mathematician Test (DAMT)". Over time, the use of drawings as a measure of the perceptions of young students was found to be a valid (Losh, Wilke, & Pop, 2008) and a less expensive alternative to systematic classroom observations (Haney et al., 2004), and a method which overcame students' reticence to completing a written questionnaire.

Below, I reported how existing studies provided evidence on students' views about mathematicians.

3 Views about mathematicians found in prior research

Picker and Berry (2000) investigated the perceptions of mathematicians held by lower secondary school students in the USA, the United Kingdom (UK), Finland, Sweden and Romania by using the DAMT, and compared students' images. The study revealed seven themes in the students' drawings including "mathematics as coercion", "the foolish mathematician" and "the mathematician with special powers" in which mathematicians are drawn respectively as large authority figures, as crazy men, or as people who have some special power. The results showed that when asked about their perceptions of mathematicians, 21.4% of the students portrayed a mathematics teacher. In some drawings, teachers appeared to be intimating, violent, or threatening, while in a few, the teacher was pictured pointing a gun at the student(s). The findings of Ucar et al. (2010) who used pictures and follow-up interviews to investigate the beliefs about mathematics and mathematicians of nineteen elementary school students attending a supplementary school, supported Picker and Berry's (2000) findings. Their results showed students mostly pictured mathematicians as lonely and unsocial, intelligent but weird people, while they viewed mathematics teachers as angry and unfriendly.

In their study of what students think about mathematicians, Rock and Shaw (2000) used three open-ended questions to survey students (kindergarten – grade 8) and invited them to draw a picture of mathematician at work. Their questions were: What do mathematicians do? What types of problems do they solve? and What tools do they use? A total of 215 students from kindergarten through eighth-grade participated in the survey and 132 students from kindergarten through fourth-grade drew a picture. The findings showed that students viewed mathematicians doing the same mathematics in their work as they themselves do in their mathematics classrooms. In most students' drawings mathematicians were shown in classrooms. Young respondents named tools with which they are familiar from their own classrooms (e.g., paper, pencils, whiteboards etc.) as tools of mathematicians, second and third grade respondents mentioned calculators, rulers, geometric shapes while fourth-grade and middle school students expanded their responses to include computers, calculators and protractors.

Grootenboer (2001) asked about forty pre-service primary teachers to draw a picture of their mathematics teachers. While some participants recalled particular teachers and drew them, others pictured caricatures to represent their memories of mathematics teachers. The author synthesized the participating teachers' descriptions and wrote three fictional characteristics: Mr Wilson, Mr Bates and Mr Dayman. The most common character was Mr Dayman who was depicted in several of the participants' pictures with horns and a pitch fork. He was a brilliant mathematician but did not have empathy for others who did not share this ability. Because of his belittling and sarcastic comments, some students were resentful of him; one student did not even talk to him for two terms due to being too scared. As Grootenboer (2001) summarized, some students "failed miserably, hated mathematics, and went on to avoid mathematics at all costs" by the end of their year with Mr Dayman (p. 15).

More recently, Aguilar et al. (2016) described the images of mathematicians held by a group of 63 high-achieving Mexican students, from their pictorial and written descriptions. They reported that students mostly imagined a mathematician as a mathematics teacher and predominantly pictured mathematicians as male, dressed casually or formally, intelligent people who enjoyed their work and were passionate about mathematics. The findings suggested that these high-achieving students held representations of mathematicians that were closer to reality. Yazlik and Erdogan (2016) investigated high school students' perceptions of mathematicians through drawings and written responses to open-ended questions. They found that out of 150 participant students, 146 pictured their mathematics teachers and other four a mathematics teacher. The authors reported that students mostly viewed mathematicians as male; wearing a suit or well-groomed; clever, hardworking, helpful, entertaining or asocial; working in a classroom or office; and using pencils, books and (in few drawings) computers as materials. Participant students perceived mathematics teachers as the main influences on their perceptions of mathematicians.

4 The study

For this study, I drew on DAMT which has been used around the world to research students' images of mathematics and/or views about mathematicians (e.g., Aguilar et al., 2016; Picker & Berry, 2000; Rock & Shaw, 2000; Stiles et al., 2008; Ucar et al., 2010). As previously described by Picker and Berry (2000), in this study students' drawings fell into two distinct groups: drawings that depicted their view of what a mathematician at work would look like (19.8%, n = 254), and drawings that depicted a mathematician who was clearly a mathematics teacher (70.5%, n = 905). In this article I present the data that emerged from the latter group in relation to the research question: *What views do lower secondary students have about mathematicians when depicted as a mathematics teacher?* Whether gender or grade level differences were evident in the students' views of mathematicians was also of interest.

4.1 Instrument and data collection

The study was primarily qualitative in which the DAMT (see Appendix A) developed by Picker and Berry (2001) was used (with permission) to collect data, by a research team leaded by the author. Combining drawings with written responses, DAMT consisted of a task and two open-ended items, and a section related to students' demographic information (namely grade level, age, and gender). Students were asked to draw a mathematician at work and then explain their drawing. The purpose of the descriptive narrative was to clarify or expand the information contained in the drawings and to help coding. Item 1 questioned students' views about why we would need mathematics and mathematicians aiming to seek students' views about the work of mathematicians and perceived needs for mathematics. Item 2 aimed to examine students' attitudes to mathematics by presenting the following stem and asking students to complete it: "To me, mathematics is ...". In this study both the drawings and associated written descriptions were analyzed.

We piloted the DAMT with 130 grade 6-8 level students at three schools not participating in the actual study to ensure the clarity of the instrument and determine the time necessary for completing it. After the pilot, the DAMT was sent to schools through the respective district Directorate of National Education, to maximize the response rate. In schools, teachers other than the mathematics teachers provided directions to and collected data from the students. Students were surveyed in classes other than mathematics to eliminate a possible mathematics teacher effect. The DAMT took students approximately thirty minutes to complete and schools returned the completed instrument in a sealed envelope to protect participant confidentiality.

4.2 Participants

The study focused on lower secondary students (aged 12 to 15 years) in order to tap into students' views about mathematicians at the beginning of their secondary education, prior to being influenced by their secondary coursework and the national university entrance exam. A convenience sample of 1284 students (grades 6 to 8) in Ankara, enrolled in twenty different lower secondary schools, with a mix of private and public, participated in data collection under the auspices of the Ministry of National Education (MoNE). The schools were co-educational metropolitan schools located in the center of the city, with a relatively middle or high socioeconomic population based on family income. Boys and girls were almost equally represented across the cohort, but the number of grade 6 and grade 7 students were more than the number of grade 8 students (see Table 1).

Table 1. Distribution of participants by grade level and gender.

Grade Level	Girls	Boys	N/R	N
6	213	204	3	420
7	148	157	-	305
8	82	96	1	179
No response	-	-	1	1
Total	443	457	5	905

4.3 Data analysis

The data was analyzed using content analysis (Creswell, 2007) by utilizing a “descriptive and interpretive” approach (O’Toole & Beckett, 2010, p. 43). Instead of seeking the meaning behind each of the drawings, data analysis focused on identifying patterns in the drawings (Haney et al., 2004) documented using excel spreadsheets. A Chi-square test was used to compare perceived teacher gender difference by student gender and grade level.

Coding drawings: The coding began with reviewing a random subsample of drawings and recording the various aspects present in the drawings, and simultaneously reviewing past research to create a comprehensive list of the elements of the views about mathematicians pictured as mathematics teachers in student drawings. In particular I drew upon the work of Blake, Lesser and Scipio (2004), Aguilar et al. (2016) and Losh et al. (2008). To score students’ drawings of mathematicians, Blake et al. (2004) included gender (male, female, no gender), actions of main figure (e.g., teaching, working on problem, performing work), mathematics tools (e.g., books, blackboard, calculator, computer), equations (mathematical symbols, equation), and words (e.g., explaining action, mathematical words). Aguilar et al. (2016) used the following elements in the analysis of mathematician depictions produced by students: kind of clothing (formal, sporty, casual, and laboratory), gender (feminine, masculine), hairstyle (extravagant, formal, and bold), mood (smiling, serious, and angry), eyeglasses (with or without eyeglasses), settings (e.g., classroom, office, laboratory), mathematical symbols (e.g., Sets, Algebra, Geometry), and instruments (e.g., desks, blackboard, sheets, pen, calculator). Among others, Losh et al. (2008) coded occupational details (e.g., beakers, lab coats, or animals for scientists or veterinarians, and chalkboards, books, or pencils for teachers) and attractiveness of the figure in drawings, i.e. whether the figure smiled or whether the drawing was of a fantasy figure (e.g., a monster). Thomas et al.'s (2001) Draw a Science Teacher Test Checklist consisted of three sections: Teacher (the teacher's activity such as demonstrating or lecturing and the teacher's position such as head of the classroom), Students (students' activity such as passive information receiver, responding to the teacher, and the position of the students such as seated in rows), and Environment (elements typically found inside classroom such as desks in rows, symbols of teaching like whiteboards and materials).

By focusing on the elements that emerged in the students' drawings particular to this study and drawing on prior research, I focused on six elements in the analysis of the drawings and associated written words: (1) the gender of figure, (2) the physical environment, (3) the activity of the figure, (4) the content area, (5) the tools of the profession, and (6) attractiveness feature. Each of these elements comprised several codes as shown in Table 2. After defining the elements and associated codes, I developed statements about what constitutes those codes. The narrative descriptions of students assisted me in coding and allowed me to confirm or reconsider my interpretations.

Table 2. The codes of the elements in the student depictions.

Gender	Physical Environment	Activity	Mathematics content areas	The tools of the profession	Attractiveness
Female Male Undefined	Classroom Office No indication	Teaching Working Researching Reading No indication	Algebra Numbers and operations Geometry Undefined No indication	Whiteboard Books Concrete materials Exam papers Pinboard Technological tools No indication	Smiling Serious Thinking Angry Silly Bored Undefined

I coded the gender of figure as female or male considering the physical appearance of the figure (e.g., hair, clothes, mustache) and/or student writing (e.g., the figure's name indicated by the student). If the drawing or writing was not clear enough to decide the figure's gender (e.g., the stick figure had no clothes or details), I created an undefined code.

The physical environment element corresponded to the setting or context in where the figure was depicted. I used the classroom code when the drawing consisted of elements typically found inside classrooms such as a whiteboard, desks, and/or students and the figure was teaching; whereas I used the office code when the figure was depicted at a table, working/studying alone, and the typical classroom elements were not included. When there was no indication to a context, I used a no indication code.

The activity element corresponded to the figure's action. The dominant activity was teaching or instructing, followed by working which included studying mathematics and solving questions. I coded the drawing as teaching when the figure was depicted in classroom, at the whiteboard or desk, instructing, demonstrating or explaining the content area to the students, and coded it as working when the figure was in an office environment alone and doing teaching related work (e.g., marking), studying, or solving questions. When the drawing or writing included no reference to the action of the figure, I used the no indication code.

The mathematical expressions and symbols that appeared in the drawings corresponded to the mathematics content areas element. Notably three areas appeared in the drawings: Numbers and operations, Algebra, and Geometry, but in some drawings, there was no indication to a content area, or the content area was ambiguous (e.g., doodles on a piece of paper). When there was ambiguity, I used the undefined code.

The tools element was occupational materials represented on drawings such as whiteboards, books, and concrete materials used in mathematics teaching. Similarly, on some drawings there was no indication to occupational materials. For those cases, I used the no indication code.

I coded the attractiveness feature through the perceived image of the figure. I coded whether the student's drawing appeared to be some kind of positive rendering such as a "smiling" figure, neutral rendering such as a "serious" or "thinking" figure, and negative rendering such as an "angry" or a "frustrated" figure. When the figure was turned away thereby obscuring its face and there was no any other evidence either in the drawing or writing, I used the undefined code.

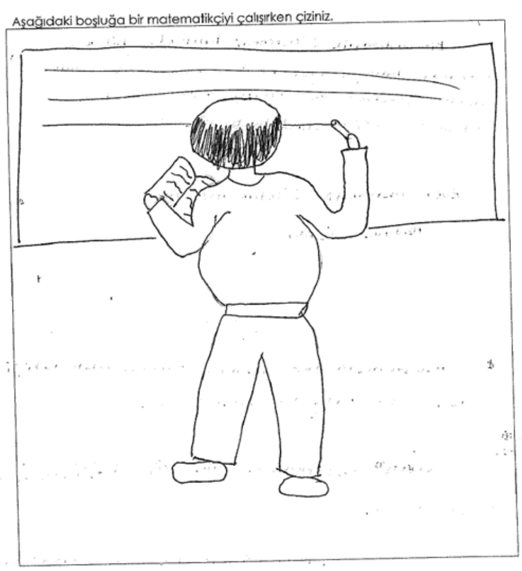
In Figure 1 and Figure 2, I present typical examples of student drawings and descriptions to illustrate these codes.

[I like math.] [Can't think a life without math!]

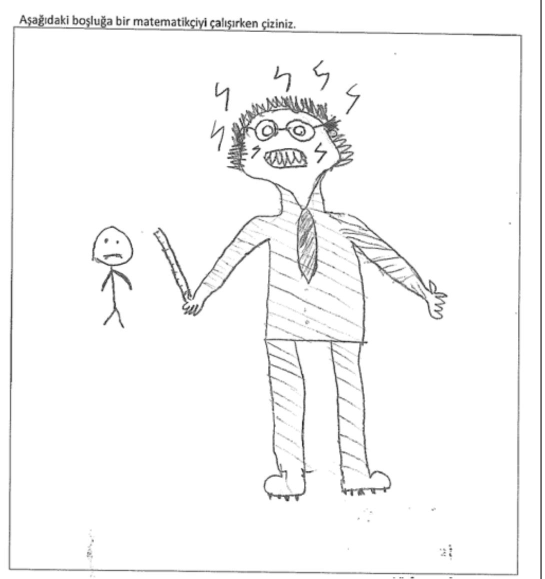
"I depicted Mehmet teacher [a pseudonym male name] in this picture. He likes math a lot. Likes solving questions very much. Likes thinking very much. I depicted these in this picture." (Grade 8, girl)

"The teacher teaches and students listen in the classroom. The people in the picture are the teacher and students." (Grade 6, girl)

Figure 1. Examples of student drawings and descriptions illustrating the following codes in Table 2, respectively: (on the left) male; office; working (solving questions); Algebra; books, pinboard, and concrete materials; neutral (serious, thinking); and (on the right) female; classroom; teaching; Numbers and operations; whiteboard; smiling.



"A math teacher; he writes the things that we can't understand from the book to the board and lectures, and gives them homework. Mehmet Sari [a pseudonym male name]." (Grade 7, girl)



"The math teacher in this picture is very angry but very clever, he carries a stick." (Grade 6, boy)

Figure 2. Examples of student drawings and descriptions including illustrating the following codes in Table 2, respectively: (on the left) male; classroom; teaching; undefined; book and whiteboard; undefined; and (on the right) male; no indication; no indication; no indication; no indication; negative (angry).

To ensure reliability of the results, I worked with a second researcher in the research team to code the drawings. Through frequent discussions, we decided the elements in drawings that the data analysis would focus on and described the indicators for each element. We independently coded a random subsample of drawings (n = 50) and calculated a high (96%) independent coder agreement. To settle disagreements, we had a re-coding session for the 4% discrepancies. This led to us resolving inconsistencies and reaching consensus about our interpretations. Once finalized, I continued coding for the remainder of the drawings. However, throughout the coding process, I consistently discussed issues that required attention or needed resolution with the second researcher. Wherever possible, I used students' own words in my analysis and reporting.

5 Results

A summary of the elements and respective associated codes that emerged in the depictions that students created to describe the mathematician depicted as a mathematics teacher are presented in Table 3. It is important to note that some responses were coded in more than one category (e.g., across the Algebra and Geometry categories), and therefore the responses did not align perfectly. Below, I present the results in four sections around the elements and use students' own words to illustrate them. Except for the perceived teacher gender, the results did not vary across grade level or student gender, therefore the results have been presented for the whole group.

Table 3. A summary of results classified by the elements in depictions (N = 905).

% Gender	% Physical environments	% Activity	% Mathematics content areas	% The tools of the profession	% Attractiveness
Female 57.1 Male 39.9 Undefined 3.0	Classroom 78.7 Office 14.4 No indication 7.0	Teaching 76.5 Working 20.1 Researching 0.3 Reading 0.1 No indication 3.0	Numbers and operations 41.8 Algebra 20.3 Geometry 14.7 Undefined 18.6 No indication 4.5	Whiteboard 73.3 Books 33.5 Concrete materials 7.3 Exam papers 1.5 Pin board 0.7 Technological tools 0.4 No indication 5.5	Smiling 52.4 Serious 16.7 Thinking 7.1 Angry 6.4 Silly 0.5 Bored 0.7 Undefined 20.0
Total 905	905	905	963	1106	939

5.1 Gender

The analysis revealed that students tended to view the mathematics teacher as female. More than half of the students (58.6%) depicted the mathematics teacher as female (see Table 4). Compared to boys, girls showed statistically significant tendency to depict a female mathematics teacher [$\chi^2_{(1)} = 75.27$, $p < 0,05$]. That is, 73.1% of girls depicted a female and 26.9% of a male mathematics teacher, whereas 44.1% of boys pictured a female and 55.9% of a male mathematics teacher.

Table 4. Depicted teacher gender difference in student gender.

		Depicted teacher gender		
Student gender		Female	Male	**Total**
Girls	n	320	118	438
	%	73.1	26.9	100
Boys	n	192	243	435
	%	44.1	55.9	100
Total	n	512	361	873
	%	58.6	41.4	100

c2 = 75.27 sd = 1 p = 0.000

The tendency of picturing the teacher as a female became less strong by grade level. The higher the grade level, the number who depicted the mathematics teachers as female decreased. Less grade 8 students depicted teachers as female (49.1%) (see Table 5) compared to grade 6 (58.9%) and grade 7 (63.8%). In grade 8, the percentage of students who depicted teachers as male or female was almost equal. Depicted teacher gender difference in grade level difference was found significant [$\chi^2_{(2)} = 9.65$, $p < 0,05$].

Table 5. Depicted teacher gender difference in grade level.

		Depicted teacher gender		
Grade level		Female	Male	**Total**
6	n	241	168	409
	%	58.9	41.1	100
7	n	187	106	293
	%	63.8	36.2	100
8	n	84	87	171
	%	49.1	50.9	100
Total	n	512	361	873
	%	58.6	41.4	**100**

$c^2 = 9.65$ sd = 2 p = 0.008

5.2 Activity

The teacher was commonly depicted in the classroom (78.7%) and at the whiteboard. In about 15.0% of depictions, the teacher was in an office or a room, but even in some of these depictions they were portrayed at the whiteboard or at the desk.

Key aspects of the teacher's activity in the classroom were lecturing, explaining, solving exercises, or disciplining. In 76.5% of depictions, the teacher pictured when teaching. The mathematical content included in the drawings was generally simple calculations and basic arithmetic. The expressions and symbols that appeared in the drawings corresponded to the primary or lower secondary subject areas such as Numbers and operations (41.8%), Algebra (20.3%), and Geometry (14.7%). In some, the subject being taught was vague. In two drawings, the teacher was seen to be explaining the importance of mathematics. Some verbal expressions on the drawings provided more detail about the teacher's pedagogies and focus, for example:

> "Who would like to do this calculation?" (Grade 6, boy).
> "Ok, class! We will study Integers today." (Grade 7, girl).
> "Come on, let's do the questions." (Grade 7, girl).
> "Solve this very simple questions!" (Grade 7, boy).
> "If there is anyone who has not understood, I will repeat." (Grade 7, girl).
> "If 5x + 1 is 10, what is x? Whoever solves get 100 points." (Grade 8, boy).

In 20.1% of drawings, the teacher was depicted as working, or the students implied that the teacher was working. In 41 of these depictions, students stated that the teacher's work was related to class teaching such as preparing for the class, marking, or writing exam questions. In the remainder the teacher was shown to be studying mathematics or solving questions, as these student quotations exemplify:

> "The teacher is studying with great ambition." (Grade 7, boy).
> "He is studying and trying to solve the challenging question." (Grade 7, girl).
> "Thinking [5 x 1005 = 5025]." (Grade 7, boy).
> "Doing the calculations [2 + 2 = 4; 5 + 6 = 11]." (Grade 8, boy).
> "Solves a million of problems." (Grade 6, girl).
> "Studying the Fractions." (Grade 6, boy).

In three drawings, the teacher was depicted as being busy with research, but their research activity could not be defined. While in another the teacher was reading, there was no indication of the teacher's activity in the remaining scripts.

5.3 Attractiveness

In general, the references in drawings and/or writings reflected a positive mathematics teacher image. In 52.4% of drawings, the students' rendering of the teacher was positive. Within this group, in most depictions the teacher was pictured as smiling while in four of them the teacher was portrayed as an angel. One drawing showed that the teacher was upset because "She could not solve a million questions." (Grade 6, boy), and in another, because "The students did not do well [in the exam]." (Grade 8, girl), indicating that the students perceived that their teachers wanted to be good at mathematics and wanted their students to do well in mathematics. In their descriptions, students were observed to have positive views toward their teachers.

In 23.8% of drawings, the student's rendering appeared to be neutral. In this group, the teacher was sometimes portrayed as serious (16.7%) and/or in mathematical contemplation (7.1%) through either thought-balloons drawn next to the teacher's head or in their statements.

Although more commonly there was evidence of love for the teacher, negative views or even loathing of the teacher were presented in the students' depictions (7.6%). For example, the teacher was angry or annoyed in 6.4% of the drawings, because, according to the students, the teacher was cross and scolding them. Some expressions incorporated into the drawings showing the teacher was angry included:

> "You girl! Why did you not do your homework? Copy the board down into your notebook, come on!" (Grade 6, boy).

> "Are you dumb or something? Study, don't do it like that, sit, shut up. Didn't you get it again? Hey son, do you have a problem? Be a man. Behave yourself!" (Grade 7, boy).

In nine of these drawings the teacher was portrayed as evil, like a monster, alien, or a devil, and as explained in three drawings, due to the teacher asking difficult questions, for example:

> "The teacher asks questions at his own level [beyond the students' level]." (Grade 6, boy).
> "The teacher asks about the subjects that are not in the curriculum." (Grade 8, girl).

What was manifested in such drawings was a lack of love towards the teacher and even hatred. Depictions that showed hatred for the teacher were evident in drawings from twelve schools, yet they were intense particularly in six schools (6-13 mentions). The frequency of mentions in the other six schools was lower (1-3 mentions). An important observation in twenty-one depictions was that the loathing of teachers resulted in unhappiness in mathematics classes and loathing of mathematics, as exemplified in these quotations:

> "Horrible with this teacher!" (Grade 8, girl).
> "Important, but we have come to hate it, thanks to our teacher." (Grade 6, boy).
> "The thing I dreaded the most in life, because I don't like my teacher." (Grade 7, girl).

The teacher was bored and quite overwhelmed in about 0.7% of the depictions, with students explaining the causes of this boredom as:

> "The teacher is bored with his own class." (Grade 6, boy).
> "The teacher is writing some boring things on the board; he is having difficulty comprehending them." (Grade 7, girl).

In 0.5% of the depictions, some of which were in the form of caricature, the teacher appeared confused while in the others (20.0%), the teacher's attitude could not be defined.

5.4 The tools of the profession

The occupational tools that appeared most frequently in the drawings were whiteboard (73.3%) and/or book(s) (33.5%). In 7.3% of the drawings, concrete materials such as a ruler (46 mentions), geometric objects (6 mentions), a compass, a protractor, or a miter (13 mentions), and test tubes (1 mention) were depicted. Technological tools appeared the least often (0.4%) in these drawings and included a smart board (1 mention) and computer (3 mentions).

6 Concluding words and recommendations

The use of the drawings to tap into lower secondary students' (grades 6 to 8) views about mathematicians was informative. The most common patterns that emerged in the drawings and associated writings were that mathematics teachers: are predominantly female; are viewed positively; do lecture, explain and demonstrate; and use whiteboards and books as tools of the profession.

The student views on the gender of mathematics teachers might be influenced by their personal experiences and/or by the society as a whole. In Turkey, it is not uncommon to have more female than male teachers in schools, including mathematics teachers. Moreover, although teaching is also a sought-after career path for males, the school teaching profession is viewed more as a female profession in Turkish society. These data complement the findings of the literature on students' images of teachers that showed that a typical teacher is more likely to be viewed as a female (Kestere, Wolhuter, & Lozano, 2013), and in a significant portion of girls' drawings (Weber & Mitchell, 2003). Losh et al. (2014) explained this as an occupational sex or same-sex preference. In this study it was found that as the grade level increased, the portrayal of female teachers decreased. It seems that as the grade level increases, students' possible occupational sex or same-sex preference becomes less strong, and they might view teaching as a profession equally valid for males and females. However, more research is needed.

Overall, the drawings and descriptions rendered positive images of mathematics teachers. When these drawings are considered together with the drawings rendering negative images of mathematics teachers, it would appear that students' stated attitudes about mathematics itself are influenced by their perceived images of mathematics teachers. In some drawings, it was observed that the perceived negative image of the mathematics teacher resulted in unhappiness in mathematics classes and a loathing of mathematics. While such a connection has been widely reported in prior research (e.g., Boaler, 2006; Grootenboer, 2001), less of the earlier research is based in evidence. The next step of this research would be focusing on the connection between students' views about mathematics teachers and their stated attitudes in Item 2 ("To me, mathematics is:", please see Appendix A).

Based on the findings from this study, I would make four recommendations. First, mathematics education research would benefit from explicit and cohesive definitions, in particular in relation to the image of mathematics and views about mathematicians as mathematics teachers. Secondly, given that some of the students' drawings and descriptions in this study cited mathematics teachers as the main influence on their relationship with mathematics (see also Lane et al., 2014), it is increasingly important that teachers are aware of student views and emotions in mathematics classes (Picker & Berry, 2000, 2001). Teachers can use drawings to access and become aware of student views and emotions about mathematics and mathematics learning (Stiles et al., 2008), and use such understanding as a basis for reflecting on their own views. As a result, teachers may focus on alleviating negative emotions and replace loathing with loving (Darragh, 2018). However, this would require teachers to be critically reflective of their own practice, have a trusting relationship with their students, and mindful of the power differential between the teacher and students in classroom environments.

My third recommendation emerges from my observation that students' views of the activity and tools of the mathematics teacher might mirror their classroom experiences. Most students pictured their actual mathematics teachers and classrooms, and some expressed their teaching and learning practices. The classroom environment that emerged from these depictions, in terms of the activity and role of the teacher (instructing, demonstrating, or explaining and general knowledge giver), the tools of the profession (mostly whiteboard and books), and instructional practices (generally direct teaching methods), are worrying because these teacher-centered approaches negatively impact students' attitudes (Hasni & Potvin, 2015), making it difficult for students to remain engaged in STEM subjects (EC, 2007). Such trends have longer term implications for students' mathematics learning, and it is my recommendation that this area be a focus of further research. For instance, it would be worthwhile to investigate both student-perceived and actual mathematics classroom environments and determine their relationship to student outcomes such as attitude to and views about mathematics, and their interest in STEM careers.

Finally, researchers must take care when using drawings (with descriptive stems) and short answer questions to interrogate student perceptions. As Losh et al. (2008) found, students in this study took the drawing task seriously and put some considerable amount of effort and thought into completing it. While drawings provide a valid alternative measurement of perceptions/understandings in mathematics education, including classroom environment research, extreme caution is needed in both implementing the task and analyzing the data. Notably, when students are asked about mathematics or mathematicians, it is their mathematics teachers that immediately come to their minds. It is important, therefore, for researchers who aim to interrogate student images of mathematics or mathematicians take this into account in the prompts they provide. Prompting students to describe their depiction is necessary for interpreting their perceived views but no matter how thoroughly a prompt is given, unanticipated interpretations of what is expected in the task may arise. For example, in this study students were asked to look back at their drawing and describe it so that anyone looking at it would understand what it meant. Some students simply wrote that if their classmates looked at the drawing, they would understand it! This meant the student drew their mathematics classroom, but without further explanation made interpretation of the drawing difficult (e.g., activity of the figure).

7 Limitations and further research

The data for this study was collected from students at twenty schools in a region within Turkey. The sample might not be representative of the entire population of lower secondary students within Turkey or in other countries. An interesting study would be to see how lower secondary students' perceptions about mathematicians change throughout their secondary education and if this varies due to their branches, i.e. language, literature and art; social sciences, and mathematics and natural sciences.

Another limitation of this study is that interviews which allowed students to reflect on their thinking and to explain and clarify their drawings and answers to the open-ended items were not used. A future study would be to collect interview data from a subsample of participants to enable data triangulation and help overcome this limitation of this study, but also to tease out any possible differences that may exist between the drawing and interview data.

Finally, students' (DAMT) drawings in this study fell into two distinct groups with the present article only providing the data regarding the drawings that clearly represented a mathematics teacher. The results are therefore being regarded in isolation from the remaining data from the overall study and should be interpreted with caution. The study demonstrated, however, that we need to develop measurements that assess the views of mathematics teachers or mathematics teaching held by young students. These measurements may help identify possible reasons (e.g., approaches to teaching mathematics at schools, the teaching resources used) for the low levels of enjoyment of mathematics that have been found among students. Further use of drawing tasks with students from different countries might also highlight the efficacy of the use of drawings in revealing views about mathematics teachers or mathematics teaching.

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Appendix A: Draw a Mathematician Test

Dear Students;

This survey aims to explore your images of mathematics. You are invited to draw a mathematician at work on the first page; and then describe your picture and answer the two questions on the second page. Please read each item carefully before answering them. Your drawings and responses will just be used for this study and keep confidential. Thank you for your participation.

SIDE 1: PLEASE COMPLETE THIS SIDE BEFORE SIDE 2

In the space below draw a mathematician at work.

SIDE 2: PLEASE COMPLETE AFTER SIDE 1

Grade: Grade 6 ☐ Grade 7 ☐ Grade 8 ☐

Age:

Gender: Female ☐ Male ☐

Look back at the drawing you made of a mathematician at work and write an explanation of the drawing so that anyone looking at it will understand what your drawing means and who the persons are in it.

1. If you have a leaky faucet, you need to hire a plumber; if you break your leg, you need the services of a doctor. With this view in mind, to you;

   (a) Why would you need mathematics?

   (b) Why would you need mathematicians?

2. Please complete this sentence:

   To me, mathematics is:

'Playing it safe' or 'throwing caution to the wind': Risk-taking and emotions in a mathematics classroom

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This paper attends to teacher intellectual risk-taking when attached to expression of positive emotions, in order to explore some of the reasons why teacher risk-taking may not appear in mathematics lessons. We know that risk-taking can be beneficial, but research has not really examined what form this might take in a classroom. In recent research, I investigated how positive emotions are discussed and used by experienced mathematics teachers. In particular how to examine the 'in-the-moment' emotions of the teacher, and what the modelling of experienced teachers tells us about the role of affect in mathematics teaching. This paper examines some affect episodes for elements of teacher risk-taking. The evidence suggests that teacher risk-taking enables the use of emotions, and vice versa, is integral to 'good' teaching, and that, in Bandura's Social Learning Theory terms, modelling such behaviours appears beneficial to student learning and should be encouraged.

Keywords
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1 THE 'R' in F.R.E.S.H; Examining the emotions of experienced teachers

The aim of this paper is to present examples from the practices of experienced teachers which connect emotions and teacher risk-taking. The paper addresses the question of how emotions and risk are connected within the practices of experienced mathematics teachers. Beginning with a brief discussion of the method used in this research for exploring affect in context, this leads into a contextual definition of risk, how creating an emotionally supportive climate seems integral to encouraging risk, which leads into the examples from mathematics classrooms. The examples are intended to highlight how risk and emotions combine in practice. The subsequent discussion is structured around modelling as constructed in Bandura's (1971) social learning theory and includes some implications for teachers who are interested in developing their practices, as there is inevitably risk in such an endeavour.

Bibby (2011) calls teaching an 'impossible profession' because it is fraught with contradictions, risk, tensions, and subjectivities that can rarely be reconciled. This is particularly the case for researching affect as it is never possible to isolate this dimension from the complex context of a mathematics classroom. The research from which this paper is drawn (Lake, 2018a) attempts to build a model that acknowledges such social complexity from a socio-cultural perspective; that learning is primarily through teacher modelling (Bandura, 1971). This implies that the degree of risk taking modelled by a teacher will have an effect on the degree of risk taking by students.

One way to consider positive emotional expression is as a mechanism to support student approach behaviours (Linnenbrink & Pintrich, 2004), a definition which draws attention to the differing and unique intentions of the teacher, and recognises that emotions only have transitory existence through social interaction, exist in order to meet a desired goal, and are confined by a patterned and repetitive place (such as a classroom) which provides limiting conditions for the appropriateness of an emotion (Mottet & Beebe, 2002).

The determination of risk by a teacher is one reason why positive emotions may not always be utilised fully whilst teaching, a point rarely explored in relation to teacher affect. Yet, in context,

> "Schools have traditionally been intellectually stifling, controlling environments that are highly resistant to risk taking and change. [...] It is within these environments that teachers and administrators are asked (and often mandated) to risk changing classroom and school practices." (Ponticell, 2003, p.6).

Risk-taking is defined here as the degree of willingness to engage in an activity when the outcome is uncertain, which inevitably has an emotional dimension. Behaviours are considered risky when there is a chance of undesirable consequences. Assessing risk is an exercise of judgement, conscious or intuitive, which forms a subjective assessment based on context, willingness and predisposition, drawing from options defined by experience and an assessment of likely outcome (cost/benefit). But for teachers, they also assess for self and students simultaneously. According to Clifford (1991), these risks constitute a special class called intellectual risk-taking (IRT) which is engaging in adaptive learning behaviours (sharing tentative ideas, asking questions, attempting to do and learn new things). Although Clifford considers IRT for students, the model also applies to teachers. Engagement in IRT places the learner, or in this case the teacher, at risk of making mistakes or appearing less competent than others. The definition implies that teachers would define risk-taking in teaching as trying out, most likely spontaneously, something new or unfamiliar, potentially out of their usual comfort zone, and at least different.

There is research that suggests a teacher taking risks whilst teaching is essential to develop 'good' teaching, and further, that modelling of this form of risk-taking is essential for learning. For example, Dweck (2000) suggests that encouraging children to enjoy challenges, which frequently involves risk, could increase their persistence and learning abilities. Palmer, Johansson, and Karlsson (2016) when looking at competencies that support entrepreneurship identify six competencies; creativity, the ability to take responsibility, the ability to take initiative, tolerance for ambiguity, courage and the ability to collaborate. They suggest that adapting teaching to support developing these competencies in students requires teachers to take risks.

Psychology researchers suggest there are three affective elements essential to assessing the degree of risk. These are potential loss, the significance of the loss, and uncertainty (Yates & Stone, 1992). Although Ponticell (2003) suggests that this model is inadequate, that "Constructs of emotion and gain, which appeared to be embedded in loss and significance of loss, need further identification and study" (p. 5). Yet risk-taking in a social context, such as a classroom, is different again, as status then becomes significant, especially for a teacher within the classroom power structure. The risk potentially becomes greater as it is for both self and others. However, emotions can be used to manage risk-taking appearing in the form of vulnerability as 'a state subject to emotions' (Kelchtermans, 2005). Emotions can address perceived threats, and effect resistance or subversion if required. This emotional response may be more apparent when constant reconstruction is, in Zembylas (2005) terms, more contingent and fragile. Indeed, seeking risk-taking is itself a motivator. Bullough (2005) when discussing management of risk within the vulnerabilities of a teaching role, suggests, "Some teachers seek to make themselves invulnerable, immune to the possibility of failing, whilst others seem to enjoy risking self" (p.23). Kelchtermans (2005) adds that because vulnerability enables a pedagogical relationship, then it enables joy too, and should be embraced, not contained.

There is much literature about creating an appropriate climate for learning. For example, Sharma (2015), when writing of promoting risk-taking for students, comments that, "Indeed, to learn and grow people must take risks, but most people will not take risks in an emotionally unsafe environment" (p.290), and a climate that supports risk-taking appears to be crucial for a social-constructivist approach. However, a teacher modelling risk-taking to the students must be a powerful driver for developing students to take intellectual risks in their learning. This paper reports on the outcome of revisiting the study observation and interview data to examine what risks the participant teachers took, and what implications can be drawn. This builds on previous work exploring how an experienced teacher models and manages error (Lake, 2017). For example, if criticism is perceived as a likely outcome of error, then students may become risk averse. This suggests that how a teacher views risk, and their degree of willingness to engage in risk-taking or risk-averse behaviours is important.

To summarise, the examination of the data included in this paper assumes that Bandura's (1971) modelling within SLT is crucial for learning. A key attraction of the theory is the attention given to the role of modelling as a social dimension, where mimicry and synchronisation guide learning. Bandura suggests that where complex sets appear (as in a mathematics classroom), new behaviours are best learnt through social cues, which is integral to modelling. The risks, if any, that the teachers take will be described, and how these relate to emotions. In order to exemplify the degree of willingness, three different yet experienced teachers will be considered in turn.

1.1 Source of the data

Eight secondary school mathematics teachers, from Norfolk (UK) participated in the wider research and were visited up to three times. Data collection consisted of three stages; career storytelling, one or more classroom observations and measurement of galvanic skin response (GSR), (used to roughly indicate internal emotions), and each lesson was followed by discussion meetings using video stimulated recall. The four-phase analysis process identified episodes in the transcripts and video records that centred on interactions with an affective interpretation, especially external affective expressions. These data were then used to produce connections and any repeating strong affective themes. The analysis used Goldin et al.'s (2011) Engagement Structures combined with Positioning Theory (Harré & van Langenhove, 1999), which resulted in a series of four discussions; self, play, modelling, and storytelling, since expressions of positive emotions frequently appeared in conjunction with all four categories. These included teacher enjoyment at recall of episodes of teaching within interviews, class laughter when teachers were being playful, and examples of teachers modelling enjoying engaging with mathematics. Additionally, characteristics of excitement were seen, such as when teachers used storytelling, either to emphasise the mathematics, or to change the rhythm of the lesson.

In what follows, I draw from these data to illustrate the varying degree of risk in the choices made by the teachers. The episodes are described before examining the place of risk-taking for the teachers in the study. The examples are from the classes of experienced secondary school teachers who are working from a solid base of tried and tested practices that are inseparable from other classroom practices that affect learning. The participants were aware that my research explores teacher emotions, and within the context of normal classroom practice. There was minimal risk identified from participation such as embarrassment or loss of trust. No actual names were used, and all the data was kept anonymous. There was no compulsion to participate, even if line managers gave consent, and the voluntary nature was made clear to participants both verbally and in writing.

The wider study (Lake, 2018a), through discussion of the roles of self, play, modelling, and, storytelling, concludes with the idea of F.R.E.S.H. The idea that the affective dimension of experienced teachers in action is centred on five key elements: Focus (where teacher intensity determines what is important in mathematics), Risk (the component that is discussed in this paper), Experimentation and modelling (which includes the roles of emotions within novelty and deviation), Shift and transition (positive emotions and change), and High intensity (stronger emotion use at critical moments).

The episodes discussed below were selected from lessons where shared laughter or banter, as exemplar of expression of positive emotion, were visible to an observer. There was no intention to assess the teachers or to judge their ability to engage their students other than as observed and the following offers a variety of examples to illustrate how and where risk may be located in teaching.

1.2 Episodes of risk-taking and emotions in mathematics classrooms

The episodes are from three teachers who at the time of the data collection taught classes from year 7 (age 11) to year 11. Helen, Freddie and Adam (pseudonyms) provide a spectrum of risk-taking across the participants; a spectrum from avoidance of risk to what might be deemed active seeking of risk. To provide some context, Helen has had a varied career path, having also taught primary and her degree is in accountancy. She has a secondary PGCE in mathematics. She has taught in various schools and has been teaching for 14 years, across the 11 to 16 age group in mathematics. She is now a classroom teacher in a well-regarded rural school. Adam is a head of department and had been teaching eight years. He pursued a different career after qualifying, before returning to teaching and moving into his current role. Freddie teaches in a larger urban school. The data was collected during his fourth year of teaching, when he had recently taken on additional pastoral role. His degree is in mathematics and physics. He has a secondary PGCE in mathematics and teaches ages 11 to 18.

Helen presented as a cheerful and dedicated teacher. In interview, she told of choosing mathematics because she enjoyed it at school. The discussion on both occasions was dominated by discussing individual students and the place of exams within mathematics teaching. For Helen, there was no evidence in any observed lessons of engagement in risk-taking. For example, she used a game as a teaching tool to offer variety. The introduced activity was groups of students solving mathematics problems, rewarded by points, as part of the preparation for a forthcoming exam. The teacher gave instructions, positioned within the interactions as rule setter, as for some board games. This activity was a teaching tool, intended to alter pace and was not considered as risk-taking by Helen or myself-as-observer, partially as the primary purpose was product orientated; it was a game designed to directly support exam success. Helen, established in the wider study as a strategic, outcome orientated teacher, seemed to see risk in play when exam results were at stake, when playing becomes time wasting, she said in interview after the lesson,

> "...we are coming up towards a test ... you kind of want to make every second really focussed and really count, and really relevant and really going to help them with that test rather than perhaps being a bit more exploratory and a bit more outside the curriculum, outside the box."

Helen's selected position was as judge and adjudicator for the game, rather than as participant. The adopted position enabled her to monitor behaviour and offered variety with little perceived risk in terms of behaviour management. This view is supported by what she says in interview, about being able, as a teacher, to play with the curriculum,

> "I like doing games. I am quite a fan of games. I do sort of an auction activity where kids bid for equipment and they have to do a task. I quite enjoy doing that..."

Moving on to the next teacher, we get a sense of how Adam sees his role and a sense of his personality from how he talked in interview,

> "But then, as I was growing up it was like, I don't actually want to be an accountant as I perceived it as being a bit boring. Well, what do they do, they just play with numbers all day... um [Indicating disbelief?]. So, I didn't really think about what I wanted to do. I wanted to be like a policeman or a fireman. Then...I didn't really think about it until I was older. Then at school I was always just good at maths and that was it. I used to help students with their homework in the mornings, on the bus, in payment, [laughs] give me like a can of coke or a chocolate bar and I'd help them with their maths homework."

One might assume from this quote that he wishes to avoid boredom, and hence suppose that he is likely to be open to risk-taking. In the selected episode, Adam engaged in a foolish scenario about a shepherd counting his sheep, as a means to attend to the natural numbers. Once he had selected a student to be a shepherd, the class inevitably began to bleat.

> "(Baa) Mark just... can you check all your sheep? (Baa) Can you do...? [Pointing to each one gesture] ...count the sheep. Alright." [Teacher writes the counting numbers on the board, there was some laughter and inaudible banter at this point, humorously suggesting that the counting was difficult for Mark]

Adam could have made himself shepherd, with a different relational impact, and potentially less risk, but this raises the question as to whether the affective impact would be less. He said afterwards,

> "I don't usually have to kind of settle them, but him and M, the other student, they are both kind of on the cusp of being... dodgy characters in the school... [...] I've never got anyone to actually get out of their seat and actually be a shepherd. But he just seemed to be 'Alright, I'll do that', [laughs] So that's really quite a... a better way of explaining it than before, and I'll probably use that again. I like that. "

Teachers also experience boredom. There is some evidence which suggests that risky activities can counter boredom (Mandler, 1984). This is perhaps illustrated by Freddie. Prior to the observed lesson he said,

> "I wanted to be a really, really good teacher. Um... and... ... I like pushing myself, um... and I think, like I try and become a better teacher. I try and do new things in the classroom...."

In the selected episode, Freddie included himself in a face measuring activity, when he could, as is common, have monitored the student activity. Instead, he allowed students to measure his face, and to record his data to compare with a perfect face using golden ratios. He was however engaged in modelling what he wanted the students to be doing. This is a risky action as it potentially allows non-engagement for the rest of the class as some measurements involved covering his eyes with a ruler. However, his reward for this risk may have been the reduction of distance between teacher and students, even though, as he comments afterwards, he felt like “a plum”. As he says afterwards,

> “And I sort of... and looking at... I knew I was about to talk about the Golden ratio, I thought that... I always find that um... I didn’t want people to feel bad if their ratios were different to the Golden ratio so I kind of wanted to use myself as an example, to say it doesn’t matter. Like if they could see my measurements up on the board, then it’s sort of... and the fact that I don’t care [...] So I thought that tied in very well together, if I have my measurements on the board then it’s like I’m part of them. So, I can discuss with them rather than just... I think it just brings us more onto sort of an equal playing field, so we can sort of discuss it. Like ‘I’ve got my results, you’ve got your results, how did you do? ‘Oh, my ears are a bit off proportion’ or something like that. [...] I was also trying to keep an eye that there wasn’t anyone hitting each other with rulers or anything else.”

2 Considering willingness to engage in risk

This section considers what can be learnt from the risk-taking in the classrooms of these teachers and in particular both willingness and what competencies (Palmer et al., 2014) are being modelled by the teachers in each example.

From interview, we know that Helen had poor behavioural management experiences in former schools,

> “I was still a teacher ten years into my profession, but I really had some struggles, I really struggled with some of my classes because they were so difficult, we had windows smashed and I was kicked by a pupil.”

It is reasonable to assume such experiences would increase her awareness that showing emotions in class has associated risk, thus perhaps forming a block to risk-taking in the classroom, and hence to expressing positive emotions. As for humour, “We learn by experience whether or not it [humour] is a tactic we can use effectively” (Ziv, 2010, p.12). In the lesson, Helen followed common procedures associated with the role of a teacher. There is security for teachers from teaching mathematics in a textbook form, as balance is not then risked by experiment; doing different. The balance in Helen’s case lies between assessment requirements (upon which the students, and hence the teacher, are judged ‘good’ or not), and individual needs (school mathematics is not only for assessment).

There is little evidence of the competency of creativity in this example, as she related through a story from her first year of teaching,

> "I also realised that I just needed to be one step ahead of them, rather than know the whole syllabus inside out. But what I did do in preparation was I went through the whole textbook in the summer holidays, so I did every single question in the textbook, just to reassure myself that I could actually do this... and teach this."

There is security in a choice such as this, as such knowledge can reduce risk, although without tolerance for ambiguity. In the observed lesson she attended to behaviour, with little responsibility or autonomy given to the students.

For Adam, there is a high degree of risk involved in creating the spontaneous scenario. There is a degree of vulnerability involved, as engaging in such playfulness involves revealing self. In the episode, Adam required a degree of confidence so that the older students would not think it silly or childish to become sheep in their mathematics lesson. This implies that to engage fully in risky behaviour, a teacher needs a perception of some reward for the endeavour. Adam also has to carefully judge how far to go before returning to the task, so it requires careful management too, which is an important characteristic of experience.

Looking foolish in front of students is not the only risk. European culture is one where childishness can be a criticism, so teachers also risk criticism of neoteny (behaving in a childish way), as Adam does in the episode above, but in a negative sense. Yet Brown (2008) suggests that humans are adaptable in terms of problem solving just because they are among the most neotenous species on Earth; that when an activity becomes habitual, and therefore easy, the risk reduces. An implication for these experienced teachers is that they need to continually engage in reflexive re-positioning. They need to be willing to keep making the game different to maintain the risk and reward balance; an ideal 50% balance (Clifford, 1991). This suggests a motivation for Adam to create the sheep scenario, which modelled creativity for the students. Whilst his experience enabled him to balance a further risk, that whilst engaging in extended scenarios that give students autonomy, a teacher must manage behaviour carefully, as these may seem to students to be a relinquishment of expected routines; students might easily lose sight of any mathematical purposes. It takes courage to do this.

Freddie actively sought risk, as he says, he liked pushing himself to do new things. The risk he took in placing himself in the role of participating student in the activity is significant. The shift in the power relationship, "like I'm part of them", created by this choice is notable, as, if repeated, this type of inclusion potentially creates a safe environment, a 'riskable classroom' (Kellermeier, 1996). It also allows development of autonomous students. Freddie also seemed to be motivated by care for the emotional needs of the students, that he did not want them to feel 'bad'. Such choices do however change the teacher role and the usual boundaries, a risky move perhaps, but one which indicates the competency of courage (Palmer et al., 2014). He is assuredly stepping into a situation in which he is not fully comfortable in order to model collaborative learning.

2.1 The place of roles and boundaries

On occasion, a teacher might be challenged in an unacceptable way, especially if the rules are ill-defined. In the lessons where the teachers were seemingly taking risks, such as Freddie and Adam, it was notable that this was in combination with strong ground rules, and expectations that were frequently reinforced. This suggests risk-taking teachers know from experience the importance of clearly defining the boundaries within which risks, in the form of changing the rules, might be taken. For example, they might model playfulness, as Adam does, in conjunction with engendering potentially controversial situations through use of questions, or through creating surprising connections or revelations. This may require abandoning some of the expected roles of a teacher, which again is potentially risky. Yet Goffman (1997) suggests that when some expected roles have been abandoned, there may be less potential for conflict between teacher and students, with a potentially beneficial impact on student learning and engagement. A teacher can choose to be creative or to digress from expected role norms. In doing so, they need to accept any associated risk and associated vulnerability, and any associated emotions.

2.2 Risk-taking and emotions

Prior experiences may have shown the participant teachers whether risk-taking is a successful stratagem, and implies that, if shown to be successful and that if they are willing to accept the risk of failure, then they can expect enjoyment. The wider study suggests some teachers appear to continually seek freshness, fluidly re-positioning to generate and support positive learning, in order to respond to student needs and engage them in learning. It may be that a self-aware and reflective teacher seeks different ways to gratify and entertain both the students and themselves. The expression of positive emotions evoked by anticipation of enjoyment is likely to make the risk-taking successful; students can see this modelling and expectation (one of Bandura's (1971) elements of learning via vicarious experiences) and may respond positively.

Classroom management includes assessing the balance between losing control and safety in the familiar. Judging how much risk-taking is appropriate is challenging for teachers. Teachers are open to student, parent, and institutional judgements, whilst playfulness includes potential criticism of neoteny, in a negative sense. Other judgments include assessing the likelihood of rejection by students, for whom perhaps playfulness is either not the norm, or who only see such behaviours as an opportunity to push limits. Teachers function within systems of rules for behaviours and may perceive taking risk averse choices as reducing the likelihood of external criticism because these rules are not broken. For example, the duty within the role is to meet curriculum requirements, usually in the form of examination success, as illustrated by Helen. Within these constraints are individual dispositions, whilst, within what is already often an effortful role, using positive emotion requires intensity, and hence effort. Further, it takes energy to put oneself 'out there'. For example, to attempt humour, which will not necessarily be accepted, means there is vulnerability too, as experiencing the rejection of attempts is painful to an individual. In a mathematics classroom, the existing norms may not be conducive to the use of positive emotions. If used unsuccessfully, teachers may not repeat, and may withdraw further attempts, as it seems to be perfectly possible to teach mathematics without any emotional displays, either positive or negative. Similarly, in the UK, teachers are frequently externally assessed, so the connection between joyfulness and teaching (such as Adam's neoteny), and the essential creativity to engage in the action of play, might disappear through too much criticism from self or external judgements.

A further role of positive emotions in regard to risk-taking has emerged from the criteria for storytelling (Lake, 2018b), as storytelling includes a degree of removal from real-life which reduces risk and potential anxiety (as not 'real') for both teacher and students. Similarly, Perry and Dockett (2007) suggest that in childhood, many early mathematical understandings that create meaning are formed through play. They emphasise the role of play in creating situations supportive of innovation, risk-taking, and problem solving. Separation from reality creates a safe place for risk-taking or developing curiosity, by supporting a reduction of potential shame or embarrassment, and for predictive exploration into uncertainties.

Used well, positive emotions, which are more than humour as discussed by Ziv (2010), do not endanger a teacher's authoritative position, and hence reduce risk,

> "Part of the pleasure that is created by every humorous message stem from the awareness that "this is not for real." This awareness offers a respectable way out of expressions or actions that threaten the group. If these were taken seriously, punishment or rejection would follow, but when exactly the same message is conveyed humorously, it is more easily withdrawn. It is enough to say, "But I didn't mean it seriously," and the threat is removed." (p.13).

Further, Morreall (1983) suggests laughter is indicative of security for group members, and conflict becomes unlikely. The implication is that teachers who use positive emotions can safely engage in teaching and learning without threat to self-esteem or status in that place and time, and indeed this suggests positive emotion use is an effective means of social management applicable to a mathematics classroom. Yet each teacher needs to assess how much risk-taking to incorporate for themselves, and to come to find the pleasures, playfulness, stories, and modelling of enjoyment of mathematics (Lake, 2018a) at unique and appropriate points in their teaching career.

The role of positive emotions within pushing boundaries as part of experiment acts to cushion and even functions (as suggested by Ziv (2010) for humour) to stretch the important skill of adaptability within a mathematics classroom, stretching the boundaries of what is possible before "irreversible sanctions kick in" (p.13). A teacher can model pushing boundaries as supportive of learning mathematics, or model risk-taking on the part of students. Students can imitate teacher actions, such as their use of positive emotions, and mimic behaviours seen as successful on the part of the teacher, ideally as 'thoughtful imitation' (Sfard, 2007, p.610). There is some bias in the data, as a position of adopting risk averse choices in teaching, and hence reducing risk in the form of unpredictability (Sutton & Wheatley, 2003), is likely to be common. This view of mathematics teaching did not appear often in the study, as such teachers would be unlikely to risk being 'discovered' (in what might be perceived as a form of duplicity) through observation, and hence are less likely to engage in emotion research. This is purely supposition perhaps, but likely, and a limitation of the data.

2.3 Risks in a system not built for it? Barriers, both professional and personal

This summary highlights some reasons why not all the participant teachers used positive emotions combined with risk-taking in their teaching of mathematics. Any teacher should be careful how positive emotions are used for several reasons, as there are perceived threats and risks involved. I have explored through the above examples how participant teachers that use positive emotions allay such risks, or at least manage them.

There are counter positions to many of the positions taken by the participant teachers. For example, if current practices are considered sufficient, for a risk averse teacher, the motivation for change is limited. Within managing behaviour for learning, handing over control to students is often seen as risky behaviour, especially as this potentially leaves a teacher open to criticism. The main reasons may be based within perceptions of discomfort or lack of confidence, in expecting chaos, so that the perceived risk outweighs perceptions of benefit.

Knowledge about the effect of experienced teachers can be used to support new entrants to the profession, a high-risk point because of the changing role. The participant teachers in this study are experienced and secure in their classrooms, and their perceived risk from external factors is perhaps less than for a new teacher. In future research, I would like to explore what moved these experienced teachers out of their ‘comfort zone’, where and when they pushed their limits, and importantly, what made them willing to do so. This has implications for teacher retention, as characteristic of all the participant teachers was a commitment to, and satisfaction from their role (Lake, 2015). It is worth noting that “By comparing preferences of new teachers with those entering other professions, we find that individuals choosing to teach are significantly more risk averse” (Bowen et al., 2014). This is important, as, if we are seeing risky practices within ordinary lessons of experienced teachers, then this implies that teachers become more risk-taking as their confidence develops and they move towards becoming better teachers. Yet the constraints remain. As Clifford (1991) said, “there is a real possibility that we are too culturally addicted to success to sell students on the notion of moderate intellectual risk-taking and too convinced that learning is inherently aversive to create exciting, enticing, and enjoyable risk-taking conditions” (p.274).

The challenge then becomes how to support and encourage teachers through the transition from trainees to experienced, and if considered desirable, more risk-taking teachers. The literature suggests that teachers should engage in risk-taking as beneficial for student learning, these episodes show that doing this in context is complicated, diverse and demanding of emotions, which means the accounts raise yet more questions about what it means to be a ‘good’ engaging teacher.

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Pre-Service mathematics teachers' beliefs regarding topics of mathematics education

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Beliefs are known to influence learning processes and thus become relevant in the instruction of pre-service mathematics teachers with regard to the pedagogical content knowledge taught in courses of mathematics didactics at university. In exploring beliefs about mathematics didactics of pre-service teachers training for secondary school, 50 bachelor students (ca. 5th semester) responded to two open-ended tasks in which they were asked to express their understanding of mathematics didactics. With the help of qualitative content analysis, topics related to mathematics didactics as identified by the participants are categorized. The category system shows that beliefs of participants differ in some respects from what selected research associates with mathematics didactics. Also, technical aspects of lessons like designing lessons are frequently mentioned within the answers, whereas topics with regard to learners or curriculum are rarely addressed.

Keywords
mathematics didactics,
beliefs,
pre-service teachers,
inquiry,
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1 Introduction

Within this paper we will refer to 'mathematics didactics'. This term is used as translation for 'Mathematikdidaktik'. Biehler et al. (2002) "call the scientific discipline related to this research [of international organizations such as ICMI] and the research-based development work didactics of mathematics" (p.1). Further explanations of this concept can be found in the following.

During the teacher training period pre-service teachers are learners of subject specific as well as pedagogical knowledge. Pedagogical knowledge is taught and learned in educational science courses. Whereas content knowledge is conveyed in courses of mathematics; pedagogical content knowledge is the subject of instruction within the mathematics didactics courses. Those courses therefore pursue the aim of initiating learning processes of pre-service mathematics teachers regarding topics of mathematics didactics. According to constructivist theories about learning, Terhart (2003) defines learning as a process that "is never controlled in its course and result but always involves an individual – but in social contexts – constructing and reconstructing inner-worlds" (p. 32). As an individual process, learning is affected by inner conditions of the learner. During learning, different processes of cognitive, affective, and motivational nature interact (Op't Eynde et al., 2006). As beliefs are a part of the affective domain of an individual (Goldin, 2002), they influence learning processes. "A person's beliefs [...] what (s)he finds interesting or important will, as such, have a strong influence on the situations (s)he will be sensitive to, and whether or not (s)he will engage in them" (Op't Eynde et al., 2002, p. 15).

While other studies focus on beliefs about mathematics or its teaching and learning, the rational of this study is to research pre-service mathematics teachers' beliefs about mathematics didactics. Within studies regarding beliefs about mathematics and its teaching and learning, effects of beliefs towards the learning process have been approved (cf. Müller et al., 2008, p. 268). Theoretical and empirical considerations therefore lead to the assumption that beliefs pre-service mathematics teachers have about mathematics didactics, have an effect on their experience in learning pedagogical content knowledge at university. The objective of researching beliefs of pre-service mathematics teachers about mathematics didactics derives from this assumption. Bar-Tal (1990) mentions four areas of research regarding beliefs: It is possible to explore acquisition and change, structure, effects or contents of beliefs. This study focuses on contents of beliefs.

2 Theoretical background

2.1 Dealing with beliefs

With regard to constructivist theories about learning, beliefs can be seen as reflections of the individually constructed reality of each learner. They can be defined as being "mental constructs that represent the codifications of people's experiences and understandings" (Schoenfeld, 1998, p. 19). Rokeach (1975) characterizes each belief as having three components: a cognitive, an affective and a behavioural component. In this paper, we will focus on cognitive components of beliefs about mathematics didactics. Regarding this component, Rokeach (1975) claims that beliefs represent a person's knowledge. Accordingly, beliefs can be labeled as "subjective (personal) knowledge" (Furinghetti & Pehkonen, 2002, p. 43) or "internal knowledge" (Lester, 2002, p. 351). Although concentrating on cognitive components, each time a participant makes a choice about mentioning a content of mathematics didactics his or her own evaluation on the acceptability of this content plays a role. Therefor those beliefs also contain an affective dimension (Pehkonen, 1994).

Within the definition of Schoenfeld beliefs are mentioned to be codifications of people's experiences. Pre-service teachers made individual experiences during their time as students, during their teacher education or during their private life, for example when giving extra lessons to students. Those experiences form and influence their beliefs. According to Blömeke's (2003) review of literature, pre-service teachers seem to know what lessons should look like and thus intend to learn just a repertoire of methods when entering teacher training. Within their review of literature about conceptual change in teachers' conceptions of learning, motivation and instruction, Pintrich and Patrick (2001) state pre-service teachers appearing to pay less attention to their role in facilitating learning and understanding students.

2.2 Mathematics didactics as object of beliefs

As beliefs can be seen as part of the "internal knowledge" (Lester, 2002, p. 351), there also exists "external knowledge" (Lester, 2002, p. 351). The latter can be defined as "knowledge resulting from the consensus of some community of practice" (Lester, 2002, p. 351). In order to compare the answers of the participants with something that could be seen as external knowledge, we will have a closer look at selected works of researchers (see Table 1). These works deal with mathematics didactics as a scientific discipline or pedagogical content knowledge as the knowledge that is dealt within this discipline.

Table 1. Dimensions of pedagogical content knowledge (Ball et al., 2008 & Kunter et al., 2013) and areas of research in mathematics didactics (Vollstedt et al., 2015).

Ball et al. (2008)	**Kunter et al. (2013)**	**Vollsted et al. (2015)**
Knowledge of content and curriculum	Knowledge of mathematical tasks	Contents of mathematics
Knowledge of content and teaching (KCT)	Explanatory knowledge	Learning and teaching environments such as lessons
Knowledge of content and students (KCS)	Knowledge of students' mathematical thinking	Learner
		Teacher

Within their scientific work Ball et al. (2008) as well as Kunter et al. (2013) created models, in which pedagogical content knowledge is differentiated. Exploring the knowledge that is needed to fulfill the tasks of teaching, Ball et al. (2008) created a model of 'mathematical knowledge for teaching'. Within their model, they distinguish different parts of pedagogical content knowledge (see Table 1). One dimension of pedagogical content knowledge that Ball et al. (2008) distinguish in their model is 'Knowledge of content and curriculum'. They claim this knowledge to be part of pedagogical content knowledge, but it is not considered in their further research. Furthermore, pedagogical content knowledge as well as mathematics didactics as a scientific discipline are in context with what happens within a mathematics classroom. Accordingly, Ball et al. (2008) frame one facet of this knowledge by 'knowledge of content and teaching' (KCT). Within other publications this knowledge is also called "mathematical knowledge of the design of instruction" (Hill et al., 2007, p. 132). It is for example about finding examples for the access to a new topic, evaluating different forms of representations and using methods (Ball et al., 2008). Besides dealing with mathematical contents and designs of lessons, pedagogical content knowledge and research with a special view to students is needed. Therefore, 'Knowledge of content and students' (KCS) contains knowledge about common student errors, students' understanding of a content, their developmental sequences and their common computational strategies (Hill et al., 2008).

The research program "Professional Competence of Teachers, Cognitively Activating Instruction, and the Development of Students' Mathematical Literacy (COACTIV)" (Kunter et al., 2013, p.1) aims at identifying the individual characteristics that teachers need to solve their professional tasks successfully (Kunter et al., 2013). Professionalism is seen here as an interaction of specific, experience-based, declarative and procedural knowledge which form a competence in the narrow sense (Kunter et al., 2013). Within their model they distinguish different aspects of professional competence. Besides motivational-affective components, professional knowledge as one component is separated into different domains of knowledge. One domain is framed by pedagogical content knowledge, which is further separated into different facets (see Table 1). The model of the COACTIV-program does not mention a special facet of knowledge concerning the curriculum, but they claim 'knowledge of mathematical tasks' to be one part of pedagogical content knowledge. It is further defined as "knowledge of the didactic and diagnostic potential of tasks, their cognitive demands and the prior knowledge they implicitly require, their effective orchestration in the classroom, and the long-term sequencing of learning content in the curriculum" (Kunter et al., 2011, p. 33). "Knowledge of explanations and multiple representations" (Kunter et al., 2013, p. 33) or 'explanatory knowledge' is in a similar way paid attention to within the ‘knowledge of content and teaching’ (KCT) by Ball et al. (2008). Misconceptions, typical errors and strategies, ways of assessing students’ knowledge and their process of learning are also mentioned as a part of pedagogical content knowledge within the facet of ‘Knowledge of students’ mathematical thinking’.

Finally, we refer to an article by Vollstedt et al. (2015) that describes research objects and objectives of mathematics didactics as a scientific discipline by representing four areas of research (see Table 1). Research regarding mathematical tasks or the curriculum is seen by Vollstedt et al. (2015) as part of an area of mathematics didactics that deals with the role of the contents of mathematics. Choosing, legitimizing, and preparing mathematical contents for schooling are research objectives regarding this area (Vollstedt et al., 2015). According to Vollstedt et al. (2015) mathematics didactics as a scientific discipline further needs to research mathematical classroom-settings and learning materials, for which explanations and representations are used. Furthermore, the structure and development of mathematical competences of students and their conceptions need to be understood (Vollstedt et al. 2015). Therefore, learners frame objects of research within mathematics didactics but teachers also provide this. Research within mathematics didactics also deals with exploring the personalities of teachers, their professional competence, and their professional development (Vollstedt et al., 2015). The last area comprising teachers only becomes a relevant area when dealing with mathematics didactics as a scientific discipline. Corresponding aspects are not mentioned within the models of pedagogical content knowledge by Ball et al. (2008) or the COACTIV-program (Kunter et al., 2013).

3 Research question

Learning contents corresponding to pedagogical content knowledge, however, depends on beliefs that pre-service teachers have towards the subject of mathematics didactics. Accordingly, research is needed to better understand the beliefs held by pre-service mathematics teachers with regard to mathematics didactics.

With this study, we want to explore contents of beliefs pre-service mathematics teachers have by answering the following research question: “Which content-related beliefs regarding topics of mathematics didactics are mentioned by participants when expressing their understanding of mathematics didactics?”

4 Methods and Sample

Having the aim in mind to explore the contents of beliefs, the pre-service mathematics teachers were asked about their understanding of mathematics didactics within the study. Answers can be seen as written representations which provide insights into the subjective knowledge of mathematics didactics. Those answers were analyzed by having a closer look at topics of mathematics didactics that are mentioned by the pre-service mathematics teachers. According to Schoenfeld's definition, the mentioned topics represent the codifications of people's experiences and understandings. They are part of the mental image and subjective knowledge the pre-service teachers have about mathematics didactics and thus are part of their beliefs. As beliefs in terms of subjective knowledge are held individually and thus subjective, they "can never be judged to be correct or incorrect" (Op't Eynde et al., 2002, p. 24). Instead, the answers are compared to selected works by researchers, who tried to examine what pedagogical content knowledge and mathematics didactics are.

4.1 Participants, sampling and data collection

Teacher training in Germany is tri-parted in stages of learning: Bachelor of Education, Master of Education and subsequent in-school preparatory service. This research was undertaken during the Bachelor period of studies which generally lasts approximately 3 years, each year consisting of a winter and summer semester. Within the Bachelor program pre-service teachers take courses in the two subjects' areas they will teach, as well as educational science courses, and must also complete three internships. Those pre-service teachers who chose mathematics as one of their subjects have to take courses and pass exams in mathematics as well as in didactics of mathematics. When examining beliefs held by pre-service teachers in Germany with regard to didactics of mathematics, one aspect should be considered. The word 'Mathematikdidaktik' (didactics of mathematics) in Germany is unknown for most of the pre-service teachers at the beginning of their teacher training. This means that our research intentionally excluded first semester students, assuming that pre-service teachers in subsequent semesters already have experienced the word 'Mathematikdidaktik' and have an idea about its meaning. Due to the fact that there is not much knowledge about the beliefs of preservice teachers regarding mathematics didactics, an exploratory study is realized that aims to explore these beliefs.

In order to answer the research question, 50 pre-service mathematics teachers studying at a German university participated in an inquiry (age: M=23.02, SD=3.15; semester: M=5.62, SD=2.84; sex: female=62,5%, male=37,5%). They were at least in the fourth semester and all had previously passed one exam in mathematics didactics. Within the corresponding lecture of this exam, the topics taught included topics regarding mathematics lessons (such as use of media), topics regarding learners (such as specific learning theories) and topics referring to mathematical contents (such as general objectives of mathematics education). All of the pre-service teachers who took part want to become teachers in secondary schools and have completed one three-week internship.

The inquiry was made at a preparation meeting during the semester break. We wanted to ensure that the participants were not working on a topic of mathematics didactics at that moment because this might have influenced their answers. The researcher, who is not and has never been an instructor for the participants, implemented the inquiry. The participants of the preparation meeting were free to take part, but there no one refused the participation. Furthermore, the inquiry was realized as a paper-pencil-questionnaire and the answers were anonymous. Within the framework of the inquiry, the participants had to answer questions regarding sociodemographic information (sex, age etc.) and two open-ended tasks. By answering the first task (Please describe what you think mathematics didactics is.), the participants were asked to formulate their personal definition of mathematics didactics. The second task (What kind of mathematical-didactical requirements must be fulfilled by a mathematics teacher?) focused on mathematics didactics as a field of competences in the context of mathematics lessons. The inquiry was placed at the very beginning of the preparation meeting and the participants had as much time to answer as they needed.

4.2 Data Analysis

The answers were analyzed using qualitative content analysis. Regarding the research question, it was designed to elicit beliefs regarding topics associated with mathematics didactics as described by the participants. For this reason, a thematic qualitative text analysis (Kuckartz, 2014a) was chosen.

We decided to form categories in a deductive-inductive way. Therefore, we used the four areas of topics derived from Vollstedt et al. (2015), mathematical content, lessons, teacher, learner, and used them as our main categories. As can be seen in Table 1, it is possible to structure the facets of pedagogical content knowledge and research objectives of mathematics didactics as a scientific discipline by using these four areas. The topics that were mentioned by the participants within their expression of beliefs about mathematics didactics were matched to one of these main categories. Subcategories were formed inductively.

Giving insight into the process of coding, Table 2 and Table 3 show two translated answers chosen as examples of codings in different categories.

Table 2. Translated answer to the task: "Please describe what you think mathematics didactics is."

Answer	Codings
With mathematics didactics, I understand the doctrine of how to transfer certain contents to students.	transfer of knowledge
Therefore, content knowledge is needed.	professional knowledge
Teachers must know how to realize problems of students and how to solve them effectively.	diagnostic work
Furthermore, a teacher should use different solution paths for a task	preparation of contents
and different methods for designing lessons.	designs of lessons

Table 3. Translated answer to the task: "What kind of mathematical-didactical requirements must be fulfilled by a mathematics teacher?"

Answer	Codings
Orientation towards the learners' individuality	learner
Being able to reduce mathematical contents in a way that preferably all students can understand them choosing appropriate tasks	preparation of contents preparation of contents
Choosing appropriate contents	curricular tasks
Considering aspects regarding the psychology of learning, like preconceptions of the students	learner
Knowing a variety of teaching techniques Choosing appropriate methods Using different methods	designs of lessons designs of lessons designs of lessons

Qualitative content analysis is a systematic proceeding governed by rules and oriented towards quality criteria of validity and reliability (Schreier, 2014). In terms of validity, it is a requirement to create a category system in such a way that it is able to capture essential aspects of meaning. Schreier (2014) claims that at least one category needs to be formed inductively. In line with that claim, all subcategories within this research were formed inductively. Regarding reliability, a text apprehension needs to be as intersubjective and consensual as possible (Schreier, 2014). Therefore, each participant's answer was coded by three different persons (researcher and two student assistants, who were trained for this task) followed by a discussion of each coding. If there was a disagreement, it was discussed from different points of view and then a decision was made, and the guideline of coding was adjusted. This approach is called "consensual coding" (Kuckartz, 2014, p. 211).

The study follows a mixed-methods-design. As also frequencies of codings will be mentioned, qualitative findings will be quantified. This transformation of data reasons a transfer design of the study (Kuckartz, 2014b).

5 Findings

According to the research question, we will first consider the category system (see Fig. 1). As it was mentioned in the methods section, all topics were subordinated to one of the four main categories: mathematical content, lesson, teacher or learner.

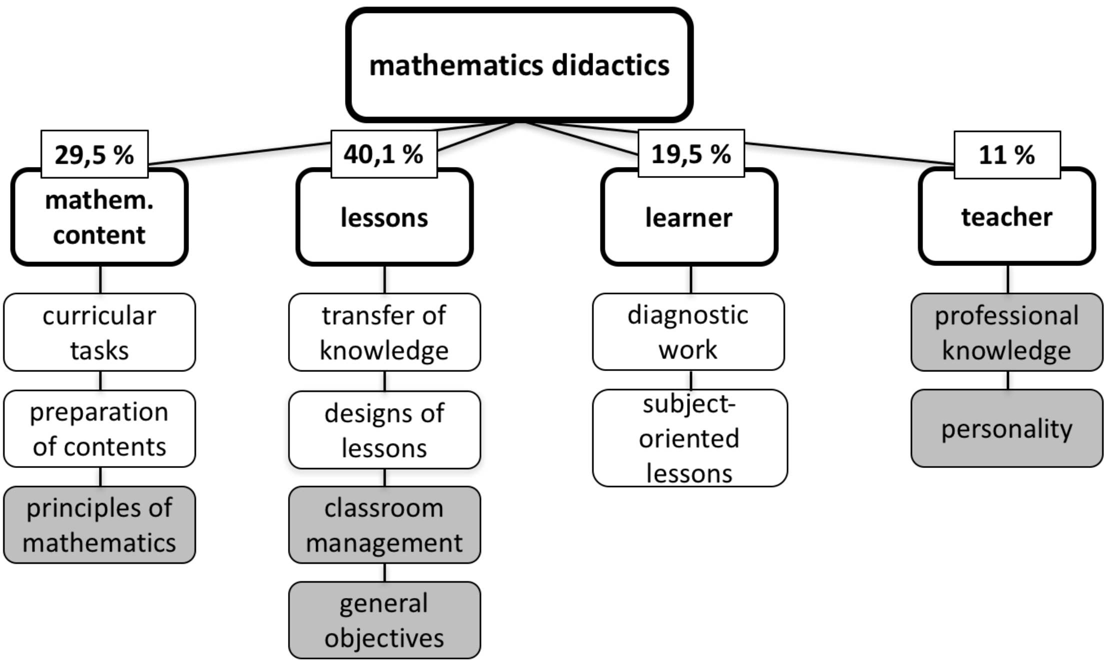

Figure 1. Category system and distribution of codings regarding the main categories.

As seen in Figure 1, some subcategories are marked in grey. This is because they do not fit with what was derived as being topics of mathematics didactics in the theoretical background. In terms of the first main category, 'mathematical content', in line with 'knowledge of content and curriculum' (Ball et al. 2008) 'curricular tasks' were mentioned. One participant, for example, stated: 'A mathematics teacher should be aware of principles of mathematics didactics that are needed to understand the curriculum'. As well 'preparation of contents' is mentioned, in which knowledge of mathematical tasks (Kunter et al., 2013), for example, is needed. An exemplary coding for this category is the following: 'Mathematics didactics is about illustrating mathematics for students in different ways'. But there were four participants explaining their understanding of mathematics didactics by defining it as 'principles of mathematics' (e.g. 'By mathematics didactics I understand a science that represents principles of mathematics'; 'In mathematics didactics important terms of mathematics are learned.'). This understanding differs from what was defined as being topics of mathematics didactics. Principles of mathematics are seen as being part of content knowledge (Kunter et al., 2013). 29.5 % of all codings are matched to the category 'mathematical content'. Furthermore, nearly every answer of a participant (46 of 50) has at least one coding within this main category. A sub-category frequently used is 'preparation of contents', coded within 39 answers. In contrast, the subcategory of 'curriculum' was used within 18 different answers.

The second main category deals primarily with technical aspects of 'lessons', such as the 'knowledge of content and teaching' (Ball et al., 2008) and 'explanatory knowledge' (Kunter et al., 2013). Again, three sub-categories were built, one of which is about the central idea of 'transferring knowledge' in mathematics lessons ('Mathematics didactics should explain how to transfer content knowledge in the best way.'). Secondly, one subcategory collects all passages that deal with 'designs of lessons', such as using methods or materials, time management or the structure of lessons. Accordingly, one participant wrote: 'Mathematics didactics offers tools which help teachers to design their lessons.' Regarding the subcategories 'classroom management' (e.g. 'I think within mathematics didactics we will be taught about disturbances during lessons and how to solve them.') and 'general objectives', (e.g. 'Mathematics didactics is not only about teaching mathematical contents, but also about promoting aspects like communication'), topics were mentioned that are not specific to mathematics classes. Because of not being specific to mathematics teaching, classroom management, for example, is seen as part of pedagogical and psychological knowledge in the framework of the COACTIV-program (Kunter et al., 2013). Concerning the distribution of codings (see Figure 1), more than one third of them are found in the category ‘lesson’ (40.1%). It can be stated that the answers of each participant have at minimum one coding within the main category ‘lesson’. By including the subcategories, it can be said that especially ‘designs of lessons’ and ‘transfer of knowledge’ were frequently coded (at minimum one coding within 43 or 45 answers). Non-mathematics related issues such as classroom management and general objectives of education have in comparison been rarely coded (within three and five answers).

The main category dealing with the ‘learner’, comprises all topics mentioned by the participants that have to do with student individuality. Based on this, the lessons and actions of a teacher are adapted to the special audience they face within a classroom. 9.5% of all codings were matched to this main category. There were 10 participants writing about mathematics didactics without mentioning anything that could be matched to the category of learner (mentioned by 40 of 50 participants). One sub-category is framed by the ‘diagnostic work’ of a teacher which includes understanding, assessing, and reacting to students’ learning processes, like it is mentioned in the following: ‘Mathematical-didactical requirements of a teacher are about realizing talents and weaknesses of students and dealing with them’). Another subcategory is framed by passages regarding a ‘subject-oriented lesson’ (e.g. ‘Mathematical-didactical requirements are about teaching content knowledge in a way, that individual needs of each student are paid attention to.’). Half of the participants did not mention anything that could be coded within the subcategory of ‘subject-oriented lessons’ (mentioned by 25 of 50 participants) while 20 did not connect mathematics didactics with topics deriving from ‘diagnostic work’ of a teacher (mentioned by 30 of 50 participants).

While the other main categories, including topics that frame objects and objectives of mathematics didactics as a scientific discipline, are connected to competences that are claimed in the context of mathematics lessons or/and are learning contents within teacher training, the implementations of the category ‘teacher’ differ. The participants expressed neither research areas of mathematics didactics nor competences or learning contents regarding pedagogical content knowledge but rather aspects of ‘personality’ and ‘professional knowledge’, especially content knowledge, as mathematics didactical qualities a (mathematics) teacher needs to have in their opinion (e.g. ‘A teacher should have a strong personality.’; ‘A teacher should have a high level of content knowledge.’). Those topics differ from the topics of mathematics didactics derived by selected research. Regarding mathematics didactics as a scientific discipline, researching the personality of mathematics teachers is mentioned, but having a certain kind of personality is not connected to mathematics didactics or pedagogical content knowledge within the literature. 11% of all codings are matched to this main category. Sub-categories such as 'professional knowledge' and 'personality' of a teacher are coded within 25 and 20 answers.

5.1 Different focuses

Concerning each individual answer, 22 participants focused on topics that were coded within the category of 'lessons'. 'Focused' means in this case that most of the codings in the answers of a participant are made within this category. A focus is only mentioned when one main category has at minimum two more codings than all the others (Mcodings=11,84). In contrast to the use of this category, the others are underrepresented (rarely or not used) in the answers of those participants. Similarly, there were 13 participants focusing primarily on 'mathematical contents' when expressing their understanding of mathematics didactics. Another two participants focused on 'learner', while 13 participants used two or more main categories similarly frequently.

6 Discussion

Regarding the research question, we were able to see that especially the contents of mathematics lessons were frequently mentioned within the answers of the participating pre-service mathematics teachers. This shows that those topics are strongly connected to the mental image and subjective knowledge (beliefs) pre-service teachers have towards mathematics didactics. Blömeke (2003) mentions pre-service teachers intend to learn just a repertoire of methods when entering teacher training. This might be a reason why the participants in this study frequently stated topics of mathematics didactics that can be coded in the main category of 'lessons' and also nearly half of the participants (22) focused on those topics when describing their understanding of mathematics didactics. In contrast, curricular topics and those dealing with learners seem to be underrepresented within the beliefs of the participants regarding topics of mathematics didactics. Pintrich and Patrick (2001) report similar findings regarding the underestimation of topics related to learning processes of students. The results also show that some topics the participants mentioned cannot be matched to mathematics didactics from the point of view of research that was derived by selected research works. Mathematics didactics being a discipline where principles of mathematics, general pedagogical or personality issues are learned neither fit into the models of Ball et al. (2008) or Kunter et al. (2013) nor are in line with the remarks by Vollstedt et al. (2015) as explained in the findings-section.

It should be noted that 7 of 584 codings in total represent topics that were learned during the first lecture of mathematics didactics (e.g. being aware of the different phases of a modelling process), thus possibly influencing their thoughts, in addition to the general topics of mathematics didactics already known.

Furthermore, we found that participants focused on different areas of topics when talking about mathematics didactics. This might be a sign of different, more general beliefs regarding topics of mathematics didactics. In the next step, selected participants are interviewed to enable a deeper view into the beliefs hold about mathematics didactics as dealing with the topics connected to this discipline is only to be seen a superficial first step to approach to these beliefs.

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The issue of 'proudliness': Primary students' motivation towards mathematics

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In this paper, we study year 2 and year 5 students' expressed motivations for doing mathematics. The responses were analysed using thematic analysis; first with a deductive approach using themes from previous research, and then an additional inductive analysis searching for new themes. The results show that the children express both intrinsic motivation (cognitive-oriented and emotional-oriented), as well as extrinsic motivation (including outward and compensation). Two new categories of cognitive intrinsic motivation were found—normative and personal. The results also indicated an interplay not only between the different categories but also within categories, signalling that expressed motivation is double-layered. Some implications are discussed.

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affect,
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1 Introduction

Motivation is a central component of learning, including mathematics learning (Gerholm, 2016; Schukajlow, Rakoczy, & Pekrun, 2017). Simply put, "to understand students' behaviour we need to know their motives" (Hannula, 2006, p. 165). A reduced description is that if learning is to take place, we need some incentive (Ryan & Deci, 2000a), and this incentive is pivotal if students are to direct their behaviour towards learning (Radford, 2015). Therefore, it is relevant to investigate individuals' expressed motivations if we want to understand why, for instance, some students are positively or negatively disposed towards mathematics, or understand the differences in performance in mathematical tests. One possible consequence of students' perceiving maths to be hard, boring and useless is students' choosing not to continue studying mathematics as soon as they are given a chance to opt out (e.g. Brown, Brown, & Bibby, 2008).

Even though there has been much research in psychology and general education generating several theories aiming to support explanations about motivation and learning, such as Bandura's (1977) theory about self-efficacy or Wigfield and Eccles' (2000) expectancy-value theory, the topic has not received much attention in mathematics education (Hannula, 2006; Schukajlow et al., 2017). Two relatively recent studies that focus on younger students' motivation for learning mathematics—year 2 (age 8) and year 5 (age 11)—both report that most students in year 2 express mainly positive motivation towards mathematics whereas students in year 5 express significantly more negative motivation, such as stress (Blomqvist, Elamari & Sumpter, 2012; Dahlgren, Johansson & Sumpter, 2010). This contrasted with an earlier Swedish report that found interests and positive emotions peaked in grade 5 (Skolverket, 2003). Such results, potentially indicating a rather rapid change in expressed motivations and emotions, signal the need to know more about young students' motivations and other affective constructs. Therefore, the aim of this paper is to explore a qualitatively nuanced understanding of expressed motivation and potential relationships between different affect-related constructs. The following research questions are posed: (1) "What different types of motivation do primary school students express?" and (2) "How are these different types of motivation interrelated in the students' responses?"

2 Theoretical background

As stated earlier, motivation has been of academic interest for decades, and we can only give a limited background. Motivation can be understood as 'the engine that keeps us going', the drive to accomplish things that in one way or another are of some importance to us. More specifically, one definition is: when a person "is energized or activated toward an end" (Ryan & Deci, 2000b, p. 54) and inversely, when you feel no drive or inspiration to act you are considered unmotivated. Another definition:

> "Motivation is the process whereby goal-directed activity is instigated and sustained." (Schunk, Pintrich, & Meece, 2010, p. 4).

Both definitions relate motivation to a goal or an end which implies it to be understood as conscious, although not necessarily positive, since a goal can be either positive—striving for attainment, or negative—seeking to avoid. The word 'activity', also used in both definitions, connects the concept of motivation to a starting point, meaning that motivation needs to be triggered somewhere and somehow (c.f. Ryan & Deci, 2000b; Schunk et al., 2010). One explanation for the origin of these triggers and the energy of driving forces is the concept of need (Hannula, 2006; Ryan & Deci, 2000a), and more specifically the need for relatedness, autonomy and competence as inherent driving forces for human motivation.

In Deci and Ryan's (1985) theory of self-determination the notions of extrinsic and intrinsic motivation are considered as parts of the same motivational spectrum that range from amotivated, where a person has neither positive nor negative motivation, to extrinsically motivated, to intrinsically motivated. What distinguishes between these levels are the levels and origin of self-regulation, a construct primarily linked to autonomy, but also to competence in the form of self-efficacy. When we see people doing something for the pure satisfaction of doing it, because it is fun or challenging and not because it renders you any type of external consequence, this person is considered intrinsically motivated, as opposed to an extrinsically motivated person being active in order to attain a certain outcome (Ryan & Deci, 2000a). However, in the way most schools are organised, factors such as grades and exams are linked to extrinsic motivation. This means that teachers also need to regard extrinsic motivation as an influence on learning. It has been suggested that intrinsic and extrinsic motivation should not be perceived as a bipolar construct, at least not in classroom situations (Harter, 1981), and research has shown that elementary school children express both intrinsic and extrinsic motivation when asked about their motives for studying (Lepper, Corpus & Iyengar, 2005). In relation to achievement, Hattie (2009) establishes a link between motivation and achievement but without potential direction differentiation. Further work on the relation between motivation and achievement is done by Garon-Carrier et al. (2016). Their results indicate that there is no correlation between motivation and achievement, but that the correlation exists in the opposite direction—from achievement to motivation—and they discuss both temporal links as well as linkage to academic self-concept in relation to this result (Skaalvik, 1994). Hence the importance of understanding the various levels of both intrinsic and extrinsic motivation (Ryan & Deci, 2000a) as well as the potential interplay between the two alongside other factors (Hannula 2006, 2012; Prat-Sala & Redford, 2010).

Both intrinsic and extrinsic motivation can be further divided into subthemes (Amabile, Tighe, Hill & Hennessy, 1994) with the two main themes falling into two subscales each: extrinsic motivation is split between Compensation (e.g. personal gain such as grades) and Outward (e.g. personal appearance such as status). Further, Intrinsic motivation is divided between Challenge (e.g. trying to solve problems) and Enjoyment (e.g. positive feelings). However, when applying these subthemes to the study of upper secondary school students' indicated beliefs, Sumpter (2013) saw the need for an expansion of the intrinsic motivation subthemes. The subthemes were called Cognitive and Emotional respectively, allowing for more cognitive-linked motivation such as wanting to learn and an expansion of the emotional spectrum to include the negative.

Also, studies focusing on students' motivation point towards constructs being intertwined or in some sort of interplay (e.g. Gerholm, 2016; Jansen, 2006). One example is the longitudinal case study of Rita, reported by Hannula (2002, 2006). Rita's comments concern emotions, beliefs and motivation in such an intertwined way that it seems impossible to separate them. Other research concludes that the constructs in themselves can be internally intertwined with their sub-constructs and hence difficult to separate. The case of Sam, an upper secondary school student reported by Sumpter (2013), illustrates how negative cognitive intrinsic motivation appears to be compensated by extrinsic motivation. Another study, focusing on high achieving students in upper secondary school, stresses that the feeling of joy that comes from being socially accepted has a pivotal motivational aspect (Gerholm, 2016). But the very opposite can also be true: students can actively avoid participating in social classroom activities for fear of producing a socially stigmatising image to peers or appear to the teacher to be a poor student (Jansen, 2006). This behaviour was linked to acting on negative compensational motivation, meaning that emotions and beliefs linked to motivation could be both negative and positive. Studies like these can illustrate how social factors are at play in school-related situations and activities, both perceived (Sumpter & Sternevik, 2013) and expressed in task-solving sessions (Sumpter, 2013). According to Radford (2015), understanding the connection between the individual and socio-cultural realm is of value:

> "/…/ the affective domain in general and motives and motivation in particular are not only subjective but also sociocultural phenomena. They are subjective and sociocultural in the sense that on the one hand, motives are the motives of a concrete and unique person but, on the other hand, they relate to a sociocultural and historical world that transcends the individual. In its transcendence, the sociocultural historical world indirectly – albeit in a decisive manner – shapes and organises the individual's motives and emotions. "(Radford, 2015, p. 26).

In this way, motivation can be considered an individual-based construct as well as something that is shaped and organised in interplay with the cultural and social environment where the individual exists and acts.

2.1 Methods

The present study seeks to build on the results from Blomqvist et al. (2012) and Dahlgren et al. (2010) where students in years 2 and 5 answered a seven-item questionnaire (for a complete list of the questions see end notes) and the data was analysed using mainly quantitative methods. This meant that the results were limited: both studies only reported generally using a positive/negative scale while nuances or other aspects of motivation were not explored and no possible explanation for the reported differences was offered. In this study we used Blomqvist et al.'s questions for semi-structured interviews in search of a qualitatively nuanced understanding of the motivational construct. Thus, we collected students' descriptions of their mathematics education experiences and of mathematics in general. The data consist of transcripts of interviews containing the respondents' explicit utterances. In addition, their implicit communication, including exclamations, sighs and pauses have been taken into account since this taps into a person's possibly subconscious driving forces (Bryman, 2016; Kvale & Brinkmann, 2014). As this study focusses on motivation, responses in relation to Blomqvist et al.'s question number four, "Why do you do maths?", will be discussed here. Our decision to use students from year 2 and year 5 enabled comparison between our results and those of Blomqvist et al. (2012) and Dahlgren et al. (2010).

2.2 Participants

Three schools were sampled for this study in a way that captured the distribution between city and suburban as well as the socio-economic settings of the urban area where this data was collected. At each school, teachers teaching the intended grades were asked to participate, their pupils were informed, and consent was obtained from parents. Shortly before each interview, the teachers were asked to pick out, from the group of volunteering pupils, individuals that they considered to be neither particularly good nor particularly poor at mathematics so we could get a large and diverse group that lay between the two extremes. The teachers were also asked to consider whether pupils would react well in the interview situation, to avoid some of the common difficulties associated with interviewing children (Hritz, Royer, Helm, Burd, Ojeda & Ceci, 2015). Prior to the present study a pilot study was conducted with six students from year 2, four students from year 4 and five students from year 6. The pilot study indicated that the questions worked on a general level but follow up questions were needed especially when the respondents started talking about other subjects such as Star Wars or computer games.

All the interviews were conducted by the first author of this article over a period of four weeks in the participating schools during lesson time and in a separate room close to the classroom. In total 19 students participated—ten girls (four in year 2 and six in year 5) and nine boys (six in year 2 and three in year 5) were interviewed. Each interview lasted for about 25 minutes, including time for a drawing task (item number seven), and was audio-recorded. This limited sample size does not allow for any general conclusions, but it will be sufficient to see nuances in how students describe their experience of mathematics and their emotional and motivational relation to it (e.f. Guest, Bunce, & Johnson, 2006).

The ethical considerations stipulated by the Swedish Research Council through Codex (Vetenskapsrådet, 2017) were followed, meaning that all respondents had written parental consent and were informed that they were participating voluntarily and at any moment could stop the interview without having to provide any explanation. The Council also stipulates how data is managed and reported, and therefore all names have been changed.

2.3 Interviews

Semi-structured interviews allow follow up questions to be posed, providing the respondent with opportunities to provide richer detail (Bryman, 2016). The follow up questions were mainly of the "please expand" type, for instance "could you explain a bit further?" or "can you give an example?" but questioning also facilitated following a respondent's line of thought. This technique also allows the intended meaning of questions to be reciprocally verified; either the respondent asks the interviewer back, "[H]mm, what do you mean...?" or the interviewer rephrases the question when the response reveals a misunderstanding of the question. In addition to revealing nuances the follow up/please expand questions provide the interviewer with a method for triangulation through a set of responses around one topic, thus a means of countering common methodological issues associated with interviewing children, e.g. neutrality issues, interviewees feeling intimidated, etc. (Hritz et al., 2015; Talmy, 2011).

2.4 Methods of analysis

The interviews were transcribed verbatim, including non-verbal communication such as exclamations or stresses of words, as well as sighs and extended pauses. Instances where the respondent highlighted and stressed a word were marked with italics. Non-verbal instances of a more temporal nature such as pausing were marked with (...) when the pause was short, approximately less than two seconds, and with rectangular brackets ([thinks]) if the pause was longer. For readability the transcripts followed standard spelling conventions, for example, an utterance like: "ifju-no-wadda-meen" was transcribed as: "if you know what I mean". In this paper, when sections are omitted to shorten excerpts these places are marked with (/.../).

In the complete transcripts, responses to a selected question were marked and the analysis was subsequently carried out in three steps, the first two deductively and the third inductively (e.g Braun & Clarke, 2006). A first categorisation was made by connecting responses to either of the two main themes, extrinsic or intrinsic motivation (e.g. Ryan & Deci, 2000a). Coding for extrinsic motivation focussed on answers connected to the outside world, like: "you[everybody] have to know", or more explicitly connected with reward or punishment: "if I get a job as a cashier when I grow up" or "[if I can't do it] it will be embarrassing", respectively. The second theme was intrinsic motivation—interest, enjoyment and satisfaction for instance, where one example of a response could have been: "it feels good when I do it".

The second, deductive analytical phase used a four-part mapping scheme: first, data connected to the Extrinsic motivation category were mapped against the sub-themes Outward and Compensatory, here understood as social-gain values (e.g. "I'm concerned about what other people think of me.") and personal-gain values (e.g. "I'm motivated by what I can earn.") respectively, as described in Amabile et al. (1994). Second, data connected to the Intrinsic motivation category were mapped using the same subscales as described by Sumpter (2013) as linked to intrinsic motivation: Cognitive and Emotional. The subtheme Cognitive frames issues of knowledge and personal development, and Emotional contains statements like "I think it's fun doing it". In the third and last step of the analysis we followed the inductive thematic analysis as described by Braun and Clarke (2006), searching the total wealth of the data for recurring themes that captured "something important about the data /.../ and represent[ed] some level of *patterned* response ..." (Braun & Clarke, 2006, pp 82, italics original). One example was various references to "the future". However, further analyses of the data reveal variations, like explicit mentions of “a job” or making “calculations”, that contribute to form a larger and more nuanced pattern. This phase of the analysis is an iteration where potentially enhanced variation is balanced by a condensation of themes, looking for the point where the themes are still coherent and descriptive, but not overlapping. In the cases where a respondent’s utterance contained more than one theme, the utterance was split.

The analysis was made primarily by the first author, and any unclear responses were analysed by the two authors separately before being discussed to increase the reliability of the analysis.

4 Results

First, as an overview and a guide for the reader, we present in the form of a table (Table 1), the different themes that resulted from the analysis, including the different subscales which informed the deductive analysis, and the subthemes which came out of the inductive analysis. Below, the contents of this table are discussed.

Table 1. Explicit motivation expressed by students in years 2 and 5. Outward to be interpreted as social-gain values and Compensation as personal-gain values. (Total number of instances within brackets).

Main themes	Subscales	Subthemes
Extrinsic Motivation	Outward (27)	Important for the future (12)
		To be able to calculate things (8)
		To get/manage a job (6)
		To produce an ‘answer’ (explicit) (1)
	Compensation (5)	Not to make calculation errors (3)
		You succeed if you make an effort (2)
Intrinsic motivation	Cognitive (15)	To learn (13)
		Want to try new things (1)
		Maths makes you better (1)
	Emotional (9)	Fun/I like it (7)
		Exciting (1)
		No-stress (double negative) (1)

Looking at Table 1, we see that the most common response (n=27) was regarding extrinsic motivation with the Outward subscale. These subthemes, including similarities and differences, will be discussed in three themes that reflect the result of the study. The aim of this paper is not to study differences between the two age groups.

However, because we believe that the respondents' age is an important part of the context, we have chosen to include this in the excerpts below.

4.1 Future needs—important but often vague

Most responses were about a specific work task or a profession in the future and were analysed under the Extrinsic motivation Outward scale. Louise uses the example of working at the till in a supermarket/shop:

> "I should learn so if I work at the till when I'm grown up for instance and there is a fruit that cost 20 crowns, and the person gives perhaps 40 crowns, then I should be able to calculate how much he/she should get in return." [Louise, Y2]

Here, it is about a specific situation (being able to calculate change at the till) more than it is about the profession 'shop assistant'. Anton, also year 2, talks about both the situation and the profession; here civil engineer:

> "To me it's like that you should learn for the future, if you will for instance have a job as ... an engineer or something because then you need to know a lot of maths; you need lengths and all. Shapes and figures."
> **Interviewer**: "...mm, and what do you need to use it for?"
> **Anton**: "To construct blueprints. And it's important to know mathematics." [Anton, Y2]

Anton not only states the profession but also gives examples of what types of mathematics (e.g. 'lengths') and why you need it (constructing blueprints). Often the responses are related to professions connected to being able to calculate things, for instance Melker (Y2) thinking about becoming a doctor: he "must know all the calculations" and it "has to be easy when you are grown up." The ability to calculate is also mentioned without links to any profession:

> "To be able to learn and so on. Because for example if it's your birthday and you made a cake and have invited seven people you must be able to split the cake in eight pieces and then you need the maths where you learnt to know how to split the cake." [Frida, Y5]

Minna has a very vague idea of any links at all, but nonetheless has a strong sense of necessity:

> "Because you have to learn. When you're grown up and you must pay, and if you work in a food store then you don't know how much money you should give

back or how little. So, you have to know, anyway, to also know ... you have to have maths.” [Minna, Y2]

The situations described by Frida and Minna are more about everyday situations, but what these four examples (including Louise and Anton) have in common is the conviction that mathematics is very important for managing situations connected to adult life and responsibility. The statements above serve as examples of the Compensation category. A further analysis shows that some statements contain indications of a social rather than a personal reference, here illustrated by the following response:

Interviewer: “Why do you do maths?”
Samira: “What? Why I do maths?”
Interviewer: “Yes, why do you do maths?”
Samira: “At school?”
Interviewer: “Yes, we can start there, why do you do maths at school?”
Samira: “To learn. And you’re going to have knowledge, it’s good if you have it later in the future if you’re going to buy stuff it’s good to be able.”
Interviewer: “Ok. And why is it so good to be able?”
Samira: “Because otherwise you can’t know. For example, if I have a one-hundred bill and take more stuff than I can buy it’ll be embarrassing at the till.” [Samira, Y5]

In Samira’s response, two matters are indicated. The first is the importance of knowing some mathematics in order to avoid embarrassing situations. The second is that due to this embarrassment, there is a risk that others might view you as a less knowledgeable person. In her response, there are traces of intrinsic motivation, negative emotions connected to avoidance, and outward extrinsic motivation.

4.2 “To learn” — of normative or personal origin

One common reason that pupils give for why they do mathematics, with slight variations, is ‘to learn’. It could be an indication of extrinsic motivation such as you need to learn to be able to calculate, such as Frida’s response above with an example of when and why, or it could be given without any further explanation. But it could also be about the individual’s own wish, thus indicating a more intrinsic motivation. Comparing Pakisa’s with Christopher’s statements illustrates a delicate but important difference in their responses:

> **Pakisa**: "To learn how you calculate ... and ... I don't really know ... you know, you learn maths and you need it in the future sometimes.
> **Interviewer**: "Why do you need it in the future?"
> **Pakisa**: [Thinks] "I don't know ... "[Pakisa, Y5].
>
> **Christopher**: "I want to learn things. Try new things ... and stuff." [Christopher, Y2]

Both statements reveal a connection to a cognitive dimension, but at the same time a subtle difference regarding intrinsic motivation; Pakisa's response is neither typically extrinsic, nor does it show the same intrinsic qualities as Christopher's does. In order to capture these slight differences in their expressed motivation, we chose to divide the subtheme 'To Learn' that is sorted under Intrinsic motivation and the subscale Cognitive (i.e. Intrinsic Motivation-Cognitive-To-Learn, shortened: ICL), into two: ICL-Personal and ICL-Normative. ICL-Personal is then defined as Intrinsic motivation with linkage to personal desires or issues of self-fulfillment whereas ICL-Normative is considered as Intrinsic motivation linked to convictions of right and wrong. In the above examples, Pakisa's statement is considered ICL-Normative based on "you need it in the future sometimes", implying that her driving force is based on complying with a norm, whereas Christopher's is considered ICL-Personal due to his "wanting" to learn. Here, the ICL-Normative group is dominant among the respondents. Some students offered both arguments when asked to differentiate between personal and normative and some stuck to one (either ICL-Personal or ICL-Normative), regardless of the way the questions were posed or whether the follow-up question took a slightly different track.

4.3 Interplay

Another result is the interplay of factors that can be interpreted as Intrinsic motivation with a mix of Cognitive and Emotional origin, here illustrated by Matteus and Casper:

> **Interviewer**: "Can you explain why you do math?"
> **Matteus**: "I think it ... is because it's fun to calculate. You can take your time and calculate, it's no rush. You have the time."
> **Interviewer**: "Is that the *reason why* you do maths?"
> **Matteus**: [nods]
> **Interviewer**: "... because it's fun ...?"
> **Matteus**: "Yes, it's also exciting."
> **Interviewer**: "How is it exciting?"
> **Matteus**: "... umm ... if there's a task that is rather difficult, you don't think you're going to make it, but when you try you *do* make it." [Matteus, Y5]

In the above example Matteus' feeling of contentment is not explicitly expressed, but we sense it implicitly in his emphasis on the word '*do*'. The interplay is confirmed by Casper, from grade 2, who gives a similar but more explicit description:

> **Interviewer**: "Why do you do maths?"
> **Casper**: "Why I do maths?"
> **Interviewer**: "Mm?"
> **Casper**: "Because I like it very much ... and it just feels good when I do maths."
> **Interviewer**: "Aha, does it?"
> **Casper**: "Mm ..."
> **Interviewer**: "Can you try to describe that feeling of 'good'?"
> **Casper**: "It feels warming. Proudliness [Swedish: 'stoltlighet']. And it feels like you're going to make it." [Casper, Y2]

Here the motivation for doing maths is the "warming" feeling of pride when you manage to complete a (difficult) task. This is indicating an interplay between motivation and emotion.

Both Casper and Matteus express a similar motivational combination: one part being a positive and Emotion-driven motivation linked to Cognitive aspects: "it just feels good" and "it's fun" respectively, and the other part expressed as Extrinsic motivation—that there are tasks that require calculation and you have time to do this. But the last quote, from Matteus: "because it's fun to calculate, you can take your time and calculate, it's no rush. You have the time" at the same time reflects a third and temporal type of motivation: that if you do not have the time and you have to rush when calculating the tasks, then different emotions, most likely negative ones, also come into play. This positively expressed type of interplay consists of missing a negative emotion, borrowing from the arithmetic rule of thumb: "two minuses make a plus", we termed this category 'double negative'.

Figure 1 summarises the relationships between the different constructs in Table 1.

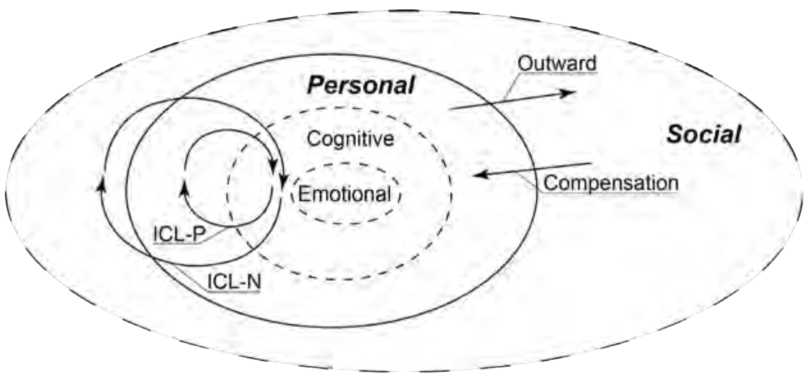

Figure 1. the relationships between the different constructs.

Figure 1 illustrates how the three Emotional and the three Cognitive components are located within the personal where the Emotional dimensions could be seen as part of a core concept. The two new constructs ICL-Personal and ICL-Normative both connect the cognitive components with the other components, here seen as an interplay. The two constructs related to extrinsic motivation, Outward and Compensation, can also be understood as an interplay between the personal and the social levels and with the two constructs working in opposite direction as each other. The Outward theme is the persons' motive to make social gains, and the Compensation theme is the person's motive to make some personal gain from the social or outside world.

5 Discussion

The aim of this paper was to study younger students' expressed motivation for doing mathematics. Looking at the main motivations that these students expressed, the responses are about future needs associated with the responsibilities of adulthood, some explicitly in connection with a profession or job, some in relation to shopping and some in very general terms, expressed as "I need it". While all respondents communicated the innate importance of mathematics, they sometimes struggled to give examples. This is somewhat different to previous studies where the most common response was mainly intrinsic motivation (Blomquist et al., 2012; Dahlgren Johansson & Sumpter, 2010). One explanation for this difference could be attributed to different research methods. In this study, the interview format and the possibility that the posing of follow-up questions allowed the respondents to offer several answers which deepened their explanations, thereby giving multiple motives for action. In some cases, this led to a seeming ambiguity in the responses, as illustrated with Samira's explanation about shopping. This ambiguity indicated that the categories Outward and Compensation could either be expanded in their definitions to include social situations or be expanded with a third category that focusses on social situations (c.f. Amabile et al., 1994). Here, based on our limited results, we cannot make any definitive conclusions. It would require additional investigations in order to determine whether it is different from the previous two categories or a refinement of the definitions. The result—that students in this study assign personal as well as social components to mathematical skills—is interesting given it has been noted in previous studies, both with prospective teachers (Sumpter & Sternevik, 2013) and upper-secondary school students (Sumpter, 2013), and it could be seen as a confirmation of Radford's (2015) conclusion about the importance of social and cultural environments in motivation to study. Also, the findings of two new subthemes, ICL-Personal and ICL-Normative, means that a further investigation of sub-constructs could lead to a better understanding of what motivation is and how it can be expressed.

When looking at the Intrinsic category, we see that the subthemes Cognitive and Emotional were present just as in Sumpter (2013). In the present paper, the results also reveal nuances within each category. One example is the difference between joy and excitement. Both could be considered emotionally positive, but there is a difference between enjoying a mathematical problem and being excited and challenged by problem solving. Some students describe finding the working situation enjoyable because it is nicely framed, which is different to being excited by the prospect of winning a multiplication game or challenged by solving a tricky problem. Another, and perhaps complex, example from the Intrinsic category is the subtheme No-stress. This can be considered a double negative; the first negative being the absence of stress, and then, defining stress itself as a negative emotion, hence double negative. It should be emphasised that this No-stress theme is something other than the other two: the first two are clearly positive, albeit for different reasons, but this third theme is an interpretation of positive because it lacks negative loading. Given that the No-stress theme is linked to a comparison of previous mathematics schooling, it could be interesting to study older students' expressed motivation since they would have more experience to draw upon.

The combined conclusions around intrinsic and extrinsic motivational factors, as discussed above, is that even though they are very helpful in providing a tool for separating the inner from the outer sources of motivation, these concepts do not seem as separable and linear as those presented by Ryan and Deci (2000a; 2000b). Rather, the results from this study support the idea that the intrinsic/extrinsic constructs are intertwined and "messy", both in relation to each other and also to other affective constructs like emotion (c.f. Hannula, 2006, 2012; Radford, 2015).

When analysing the relationship between different types of motivation in the students' responses we found instances of interplay, such as being proud (i.e. Emotional) when solving a very difficult problem (i.e. Cognitive) or feeling embarrassed (i.e. Emotional) when being in a social situation (here Outward Extrinsic motivation). Our results support research that has looked at the interplay between intrinsic and extrinsic motivation (e.g. Lepper et al., 2005; Prat-Sala & Redford, 2010). But the link between other affective constructs such as emotions are also present, and just as Hannula (2012) concluded, it can be hard to separate them. Here it is illustrated by Casper's own invented word 'proudliness' ('Stoltlighet'). His strong feeling of pride when completing a challenging task appears to be so important to him that he invents a word appropriate for the occasion. This could be seen as a confirmation of Emotional being a subtheme under Intrinsic motivation, but also that emotions are an integral part of motivation, making interplay between affective constructs a probable by-product (c.f. Hannula 2012; Prat-Sala & Redford, 2010; Sumpter, 2013). One conclusion is that students' expressed motivation seldom appears to be one-dimensional, and one possible implication could then be that mathematics teaching cannot approach students' motivation in a one-dimensional way. Based on the qualitative differences themes within the Extrinsic category, we then suggest that researchers and teachers might need to reevaluate the role of motivation in mathematics education—that extrinsic motivation is also important, and constantly striving for students' positive intrinsic motivation could result in missing out on other motives and needs that are important.

Note

The questions in the questionnaire from Blomqvist et al. (2012) were:

1. What do you think about maths?
2. How do you feel before a maths lesson?
3. How do you feel before a Swedish lesson?
4. Why do you do maths?
5. How do you feel when you do maths?
6. What do you do when you do maths?
7. Please draw a picture of yourself when doing maths.

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Prospective mathematics teachers' self-referential metaphors as indicators of the emerging professional identity

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Ideals play a key role in a student teachers' identity work. They form targets to strive for and a mirror for reflection. In this paper, we examine Finnish mathematics student teachers' metaphors for the teacher's role (N=188). We classified the metaphors according to a model that identified teachers as subject matter experts, didactical experts, and pedagogical experts, with the addition of another two categories, self-referential and contextual. For the exploration of emerging professional identities, we studied the self-referential metaphors, which formed the most common category in the data. We observed that every third metaphor described either student teachers' personalities or their incompleteness as teachers, or new beginnings or eras. Although these aspects were expected, they also inform us as teacher educators of the values and ideals that student teachers have in terms of teaching and being a teacher. The metaphors that mathematics student teachers produced illustrated their identity processes and their emerging identity as a mathematics teacher.

Keywords
metaphors,
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1 Introduction and purpose of the study

In teacher education, prospective teachers' teacher identity is considered a complex and dynamic construction in which personal and professional features complement each other (Beijaard, 2017). Student teachers ponder questions such as: "Who am I as a teacher or what kind of teacher do I want to become?" (Beijaard & Meijer, 2017). Ideals play a significant role in an individual's identity work because they form targets to strive for and help set goals. Ideals reflect what is valued (Poom-Valickis & Löfström, 2018), but if they conflict with reality, they come under doubt (Anspal, Leijen, & Löfström, 2018).

Beijaard and Meijer (2017, p.178) state that beliefs are the building blocks of a teacher's professional identity because they guide teacher's engagement, commitment and actions both in and out of the classroom (see also Vermunt, Vrikki, Warwick, & Mercer, 2017). Cross (2017) describes teacher identity as both a process and a product, a relationship between people and professional contexts, socially situated and differentiated from a teacher's role, but not clearly differentiated from a teacher's self.

Ketelaar, Beijaard, Boshuizen, and Den Brok (2012) studied eleven teachers' positioning regarding educational innovations, and their findings showed that three concepts were useful for describing the similarities and differences in teachers' positioning, namely ownership, sense-making, and agency. Ownership acts as a facilitator when expressing who one is as a teacher. Sense-making is about the interaction between one's identity and what one is striving for, resulting in maintenance or change in one's identity. Agency in turn gives the teaching career a direction. Most teachers, particularly in secondary and higher education, derive their identity first and foremost from the subject they studied themselves (Beijaard, 2017). Thus, it is important that student teachers can experience autonomy during teacher education and have space for negotiation about both their former and their emerging intentions (Beijaard & Meijer, 2017). Moreover, they must have the opportunity to participate authentically as teachers, which means that they must have autonomy and responsibility for others as well as themselves (Moate & Ruohotie-Lyhty, 2014).

Different practical methods have been used in teacher education to support student teachers' identity work. Bullough (2015) listed several: life writing autobiographies and life histories, journals, diaries, blogs, logs, and scenarios. Moate and Ruohotie-Lyhty (2014) mentioned action research, teachers' auto-biographical stories, and the use of metaphors. A recent study by Hamilton (2016) offered prospective teachers opportunities to consider and articulate metaphorical ideas through multimodal means by taking pictures. Lynch and Fisher-Ari (2017) collected periodic metaphors. Their students wrote a metaphor after every class and saved the series of metaphors on Padlet-boards (padlet.com). Tait-McCutcheon and Drake (2016, p.1) asked their participants to draw metaphorical pictures of jackets: 'Imagine for a moment that the relationship between yourself and your professional learning and development is a jacket. What jacket would it be and how would you wear it?'.

In this paper, we examine prospective mathematics teachers' metaphors for the teacher's role. We used the same Manual as was used for the NorBa Project (Löfström, Poom-Valickis, & Hannula, 2011), which is based on Beijaard, Verloop, and Vermunt's (2000) tripartition of teachers as subject matter experts, didactical experts, and pedagogical experts, expanded with two additional categories, self-referential and contextual. Our definition of the concept of metaphors is this:

A metaphor is 'an application of a word or a phrase to an object or concept it does not literally denote, suggesting a comparison to that object or concept' (Random

House Webster's College Dictionary, 1990). A metaphor is most often a noun, and common categories are an inanimate or animate object, a person, or an animal.

In this study, we concentrated on student teachers' self-referential metaphors which comprised the most common category in our data, and our research questions were:

1. "What self-referential metaphors do the prospective mathematics teachers use most often: inanimate objects, animate objects, persons or animals?"
2. "What aspects of prospective mathematics teachers' emerging professional identity can be traced from self-referential metaphors?"

2 Metaphors in teacher education

Kasoutas and Malamitsa (2009) stated that metaphors are valuable in teacher education because they elicit teachers' personal theories of learning and teaching. They also identified three reasons for using metaphors: (1) they enable us to express things and ideas that are difficult to describe through literal language, (2) they capture the complexity and multiplicity of our experiences and ideas, and (3) they are more concise and vivid than literal language. For many prospective teachers, as well as for in-service teachers, metaphors offer insights into the ways in which teachers may understand themselves and their actions in the class (Thomson, 2015).

By using metaphors, prospective teachers can articulate their understanding of complex ideas and concepts regarding teaching and learning. Metaphors also serve as tools for fresh perspectives on experience, new ways for making meaning, and reflection (Bullough, 2015; Hamilton, 2016). Lynch and Fisher-Ari (2017) describe metaphor creation as innovative and rooted in linguistic and conceptual forms. According to them, metaphors serve both literary and aesthetic purposes.

In teacher education, metaphors can mediate the relationship between espoused beliefs and enacted practice and provide potential for changing both (Tait-McCutcheon & Drake, 2016). Therefore, prospective teachers can apply metaphors when exploring, examining, and understanding their own emerging professional identities (Hamilton, 2016).

Sometimes people feel more comfortable when they can disguise their thoughts using metaphors. Tait-McCutcheon and Drake (2016) found in their study that lead teachers willingly used third person positioning and humour. This enabled them to be more honest and authentic with their reflections. For teacher educators, metaphors can serve as a method to trail students' engagement, to notice students who need additional support, or to picture a more holistic knowledge of individual students. (Lynch & Fisher-Ari, 2017).

3 Metaphors for the teacher's role

At the beginning of their professional growth, prospective teachers tend to focus on themselves. They still lack experience for wider understanding of teacher roles, membership in school organizations, collegial co-operation, and learning. Poom-Valickis and Löfström (2018) noticed that student teachers had doubts about possessing the qualities and skills necessary for a teacher. They wondered whether they were clever enough, or able to present their case or manage a class. Some students also wondered whether they had made the right career choice. In her study, Hamilton (2016) found three themes, namely teacher and teaching as guide and guiding, teacher dispositions, and the multiplicity of teaching. During teacher education student teachers also changed the metaphors they had chosen in the beginning of their studies. Löfström and Poom-Valickis (2013) found that a third of the metaphors stayed the same, another third changed completely, and in the rest of the cases some element was added to the initial idea which expanded the view of the teacher's role.

Löfström, Poom-Valickis, and Hannula (2011) have developed a manual for analysing teacher metaphors. They identified five categories: (1) subject matter experts, (2) didactical experts, (3) pedagogical experts, (4) self-referential metaphors, and (5) contextual metaphors. Subject matter experts possess vast, detailed knowledge which they transmit to their pupils. Didactical experts know how to split the content into comprehensible parts and how to facilitate pupils' learning. Pedagogical experts focus on caring and nurturing pupils' holistic development. Self-referential metaphors indicate the characteristics of the teacher's personality, personal characteristics or features (c.f. Portaankorva-Koivisto, 2013). And finally, contextual metaphors describe where or in what kind of setting or environment the teacher works.

An earlier study examined Finnish pre-service and in-service mathematics teachers' metaphors for the teacher's role (Oksanen, Portaankorva-Koivisto, & Hannula, 2014). The most common metaphor category (46%, 33/81) used among pre-service mathematics teachers was self-referential, in comparison to in-service teachers, of whom only 15% (10/94) presented a self-referential metaphor. As their analysis progressed, Oksanen et al. (2014) noticed that self-referential metaphors could be further classified into four different subcategories. Metaphors describing personality or characteristics (24%), metaphors describing hesitation (33%), metaphors describing a new beginning or new era (18%), and metaphors describing something 'big' waiting ahead (24%).

Interestingly, a group of experienced teachers in Norway rarely used self-referential metaphors (Grevholm, 2018). Only about one out of six teachers gave a self-referential answer, which is in line with the findings of Oksanen, Portaankorva-Koivisto, and Hannula (2014) mentioned above. Some such examples were an actor and a midwife, who are qualified, experienced, professionals. Other examples were a rock in an agitated sea, or a potato. The idea of the potato seems to be rooted in a specific Norwegian saying which claims that a potato is very useful and can be of help in many different situations (used in many dishes).

4 The study and its methodology

4.1 Context

Secondary teacher education in Finland consists of a five-year programme (3 BSc and 2 MSc, 300 ECTS). The students major in one school subject and minor in one or two others. Prospective mathematics teachers have pedagogical studies (60 ECTS) as a minor subject which can be completed in one academic year. At the University of Helsinki, these studies include 20 ECTS of supervised teacher training in university teacher training schools, 17 ECTS courses in general education, 15 ECTS didactical courses in students' major and minor subjects, and 8 ECTS of students' pedagogical thesis. Pedagogical studies combined with subject studies provide qualifications to teach at the secondary level.

4.2 Participants and data collection

The data for this study was collected at the University of Helsinki at the end of student teachers' pedagogical studies, in 2012–2016. Mathematics student teachers were asked to write a metaphor and expand the statement 'as a mathematics teacher I am …', and to continue with an explanation for their statement. We only gathered the metaphors for which students had granted permission to use as data. All in all, 188 metaphors were collected.

4.3 Analysis and data selection

The analysis can be seen as a deductive content analysis with the given categories mentioned above. Metaphors were categorized on a case-to-case basis, using two independent raters, the coding of which were compared at the end. In contradictory cases, a third rater was used. The metaphors and their explanations were analysed as a unit. Sometimes the metaphor was unambiguous and in line with the explanation, and sometimes it could be used to express different meanings and the explanation became firmer. Metaphors that had no explanation at all were excluded. After categorization, only the metaphors categorized as self-referential were selected for further analysis. These 67 metaphors (36.4% of the total 184 metaphors with explanations) made up the data for this study. In some cases, students replied with something that could be literally true (e.g. “myself”), in which case the answer was excluded. One reason for the focus on self-referential metaphors is that these metaphors focus mainly on what teaching represents for the students as individuals. They describe features or characteristics of the teachers’ personality. The metaphors describe who the teacher is, part of the teachers’ identity. And our interest in this paper is the emerging teacher identity. A second reason is that according to the earlier study by Oksanen et al. (2014) mentioned above, the self-referential metaphors are specific to pre-service teachers. Therefore, they are important to study in detail. A third reason is that the rest of the metaphors are analysed as a basis for other papers.

5 Analysis and findings

First, we analysed mathematics student teachers’ self-referential metaphors according to the metaphor words they had used. The student teachers in this study most often used an inanimate object (26 items) such as a rainbow, a ship in the fog, a diary, or red wine, or a person (20 items) such as a Buddhist monk. Less common was the category of animate object (10 items) such as a seed or a bun dough or an animal (8 items), such as a young foal, or “my dog Karu”. Thus, 26 metaphors represent static objects and 38 metaphors (59.4 %) refer to a changing situation.

Second, we analysed the metaphors and their explanations and looked for their meaning. We divided them into three categories. About 40.3% were metaphors describing the student teachers' personalities, and 40.3% described incompleteness as a teacher. The third, and smaller category, 10.4%, were metaphors describing a new beginning or era.

Cross-tabulation revealed that the metaphor's word analysis could not describe the whole meaning of the metaphor. When the student teachers used inanimate objects as metaphor words, they could express either their personality or their incompleteness as a teacher.

Inanimate objects describing personality:

> "As a mathematics teacher, I am *a rainbow* full of a beautiful spectrum of colours that bring diversity and joy but are still in a certain order and bring similarity."
>
> "As a mathematics teacher I am *a circle* because I am soft and friendly."
>
> "As a mathematics teacher I am *a clock* – precise."

Inanimate objects describing incompleteness as a teacher:

> "As a mathematics teacher I am *a log cabin* under construction, the foundations and building blocks are a solid piece of work, but it takes a lot of time to get it ready. And indeed, cottage projects are such lifelong projects, they can never be fully ready."
>
> "As a mathematics teacher I am *a prototype*, not yet ready but a model that needs to be refined and developed over and over again."
>
> "As a mathematics teacher I am *a product or service*, formally ready, yet something to be further developed."

When describing their personality as a teacher, student teachers often referred to characteristics that they expected teachers to require, such as calmness, friendliness, or creativeness. They also reflected on their capabilities and pondered if they had already forgotten what was difficult in learning elementary mathematics or how they could remember their pupils' names.

Animate objects describing personality:

> "As a mathematics teacher I am *a calm sea*, I am a very calm person, but I recognize the potential power within me (authority)."

> "As a mathematics teacher I am *an onion*, because I have acquired a lot of experience in how to apply mathematics and at the same time, I have forgotten how it all started."

An animal describing personality:

> "As a mathematics teacher I am *a fox*, I rely more on intelligence and wisdom than learning by heart."

> "As a mathematics teacher I am *a cat*, always curious and enthusiastic with delicious ideas."

> "As a mathematics teacher I am *my dog Karu*, who gets excited about new things and is fine with people."

A person describing personality:

> "As a mathematics teacher I am *a Buddhist monk*, while I teach mathematics, I am rather quiet and reflective not the 'let's get going' type of person."

> "As a mathematics teacher I am *Charlie Parker*, I do not play notes."

> "As a mathematics teacher I am *an Alzheimer patient*, I do not remember anyone's name and constantly demand reasons for everything."

When describing incompleteness, student teachers often referred not to their new identity as a teacher but to the fact that teachers must develop their work throughout their careers.

Animate objects describing incompleteness as a teacher:

> "As a mathematics teacher I am *a newly planted tree*, I have every opportunity to grow into a tall and solid tree if I get help and support when I have difficult times. In other words, I still need help and support as a mathematics teacher."

> "As a mathematics teacher I am *a crocus* that blossoms in early spring. I'm trying persistently to develop and 'flourish' as a teacher, but still need time."

> "As a mathematics teacher I am *a banana*, at first, I might seem a bit raw, but I will mature over time."

An animal describing incompleteness as a teacher:

> "As a mathematics teacher I am *a foal* who is learning to walk, and every day gets more self-confident and more robust, I have little experience in teaching, but every lesson I learn new things to improve my following lessons."

> "As a mathematics teacher I am *a new-born lamb*, I can get up, that means I have the basic skills, but I am still a bit unsteady and there is a lot to be learned."

> "As a mathematics teacher I am *a butterfly chrysalis*, the caterpillar phase lasted from the day I was born until the beginning of my pedagogical studies. Now I'm in the housing phase where I still have a lot to develop, and maybe one day I will be a teacher, a butterfly, even though the teacher develops throughout his life."

Here the choice of the animal, in a developing phase or young and new, transmits the image of incompleteness.

A person describing incompleteness as a teacher:

> "As a mathematics teacher I am *a small child*, helpless but eager to learn."

> "As a mathematics teacher I am *a child taking his first steps*. I feel I'm not ready to teach or to go back to school again. If I ever become a teacher, I must first find an awful lot of self-confidence and confidence in my own skills. And if (or when?) this happens, who knows what will happen. But I still need support."

> "As a mathematics teacher I am *a little child* inspired to learn new things."

Again, the choice of metaphor word, indicating something young and new-born, exposes an image of the start of building a teacher identity.

Every tenth metaphor described a new era or new beginning. Sometimes student teachers embraced it and saw their role as valuable. Sometimes they felt uncertain and hesitated about whether they wanted to become teachers at all.

> "As a mathematics teacher I am *a hiker who has serviced his bicycle*, ready for a longer trip."

> "As a mathematics teacher I am *a glue stick*, everything sticks, but I can pull my head in and hide."

> "As a mathematics teacher I am only *a puppy*, growing up as a teacher, but willing to learn new things."

> "As a mathematics teacher I am *an autumn leaf on the wind*, let's see where I'm going."

> "As a mathematics teacher I am *a ship in the fog* which I hope finds its way to the harbour."

> "As a mathematics teacher I am *a flame that lights the candle*, because there is no better lighter than a man who has fought against the same wind."

> "As a mathematics teacher I am *a long-distance relay runner* who has received a baton from his teachers."

Here the chosen metaphor words are dynamic and indicate new changing situations. The student teachers sense a new beginning of a process that will develop and mature.

6 Discussion

Teacher identity is both a process and a product (Cross, 2017). This was confirmed by the answers of the mathematics student teachers in our study in Finland. For example, the teacher is seen as a final product in the metaphors of a rainbow, a circle, and a clock. On the other hand, the teacher is seen as a process in the metaphors of a log cabin under construction, a prototype, a crocus, or a newly planted tree. The product cases descriptions of teacher ideals or desired characteristics for the future were, for example, bringing diversity and joy, being soft and friendly, and being precise. The ideals reflect what is valued (Poom-Valickis & Löfström, 2018) and strongly point to the future as a teacher.

In the process cases we found links to teacher development over time: building blocks being solid pieces, but the building of a cabin being a life-long project; a prototype that needed to be repeatedly developed and refined; a crocus persistently trying to develop and 'blossom' as a teacher, but still needing time; a newly planted tree that would grow into a tall and solid tree if it receives help when needed.

The answer to our first research question is that six out of ten students chose a dynamic word when describing the teacher's role: a live object, a person, or an animal. That could be interpreted as if they are having a sense of the changing situation as prospective teacher. The second question searches the answer to what aspects of emerging teacher identity can be found. One of the aspects is that the student teacher is taking part of a process, is developing and growing, maturing and getting more complete. The student can sense the new beginning or start of a learning process. Some realize that the learning process as teacher is life-long. Another aspect is that students give hints about how they see ideal teaching and learning, for example rather using wisdom and intelligence than learning by heart, rather teaching in a quiet and peaceful way than in 'let us get going style'. The metaphors also reveal many personality traits that are seen as valuable, such as helpful, kind, humoristic, willing, improving, and so on.

According to Kasoutas and Malamitsa (2009), metaphors are valuable in teacher education because they elicit teachers' personal theories regarding learning and teaching. For example, in the metaphor of a fox, the student's theory is that it is more important to rely on intelligence and wisdom than learning by heart. In the case of the Buddhist monk the student seems to believe in teaching in a quiet and reflective way rather than being the 'let's get going' type of teacher. The students' own learning is illustrated by the metaphors. An example is the foal, which is learning to walk and becoming more self-confident and stable; or the puppy, which is still growing as a teacher but willing to constantly learn.

We found that it was much more common for student teachers to use self-referential metaphors than it was for experienced teachers. This result coincides with the study by Oksanen et al. (2014) and observations by Grevholm (2018). In our study almost every third student teacher chose a self-referential metaphor. Among those who used self-referential metaphors, about four out of ten used metaphors describing student teachers' personalities or characteristics. Four out of ten used metaphors describing their incompleteness as teachers. One out of ten preferred metaphors describing a new beginning or era.

The results were not surprising. It is understandable that prospective teachers focus on teacher personality or identity and on their own incompleteness as teachers. During their education, they are reminded of the preferred characteristics of teachers and what they themselves must develop in order to achieve the goal of becoming a 'good' teacher. More experienced teachers have a deeper understanding of teachers' working conditions and contexts, and they can relate better to phenomena other than the self. They have a wider view of the profession. The use of metaphors to describe the teacher's role helped make these differences between experienced teachers and student teachers visible. The metaphors also illustrated and indicated some aspects of what prospective teachers saw as professional development for teachers. One of these aspects was the willingness to learn, while understanding that a teacher is a life-long learner, patient, soft, friendly, calm, joyful, self-confident, knowing, and able to withstand hard use when needed.

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Investigation of Finnish and German 9th grade students' personal meaning with relation to mathematics

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This study focuses on a comparison of personal meanings that students from Finland (FIN) and Germany (GER) assign to (learning) mathematics. Participants are 256 Finnish and 276 German ninth graders. The survey consists of 18 scales that are based on the theory of personal meaning. The original German version was translated into Finnish. Using item response theory (IRT) partial credit models, the psychometric properties of the scales were found to be good. As statistical procedure, Differential Item Functioning (DIF) analysis and mean comparisons were conducted to compare the two groups' (FIN and GER) responses. Indicators of educational system and curriculum could be found in students' responses to explain similarities and differences between the two samples. In both countries, social inclusion is meaningful for most of the students (*Support by teacher*, *Experience of relatedness*, and *Emotional-affective relation to teacher*). In addition, it is personally meaningful for Finnish students to do well in mathematics. This shows a link to identity-related questions such as confirming important aspects of the self. Hence, personal meanings related to mathematics are more common in Finland than in Germany (*Active practice of mathematics*, *Cognitive challenge,* and *Self-perfection*).

Keywords
comparative study Finland / Germany,
curriculum,
differential item functioning
educational system,
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1 Introduction

The claim for meaning in education has been raised for many years and meaningful learning is assumed to be a central impetus (Biller, 1991) as well as one of the major goals (Vinner, 2007) of education. Hence, one of the challenges of (mathematics) education is to find convincing answers to the quest for meaning as well as to develop learning environments that enable and foster meaningful learning for the students. Yet, even when only the field of mathematics education is considered, the notion of meaning is complex and multifaceted (Kilpatrick, Hoyles, & Skovsmose, 2005b). This article elaborates on a facet that considers the perspectives of the students and asks what is personally relevant for them when they are involved with mathematics in a school context. Vollstedt (2011b) terms this facet personal meaning (c.f. also Vollstedt, 2010, 2011a). Studying personal meaning is necessary to describe how students relate mathematics to their biography in order to better understand their learning processes from a research perspective (Meyer, 2008). Our research intention presents a small step towards an improved understanding

(Lester, 2005) of students' personal meaning when learning mathematics in order to take adequate account of them in lessons.

In a former qualitative study (Vollstedt, 2011b), 17 different personal meanings were reconstructed from interviews with secondary students in Germany and Hong Kong. Subsequently, a reliable instrument was constructed with the aim to assess those different personal meanings (Vollstedt & Duchhardt, 2019). This paper is a report of a study in which this survey was used with German and Finnish ninth graders to investigate which personal meanings they relate to (learning) mathematics and what similarities and differences can be found between the Finnish and the German sample. A comparison between two countries can help to get a better understanding of the theory of personal meanings and the instruments to measure it. Are the theory and instruments applicable only in the context in which they were developed, or do they persist also in a different cultural context? To test whether the construct personal meaning and the developed survey were specific to Germany, we conducted a comparative study in Germany, the country in which the theory and the survey were developed, and Finland. Finland was chosen as counterpart for this study as it is another European country in which the school system is quite different from the German one. Thus, although there might be similarities as both countries exemplify Western cultures, there are also structural differences that might contribute to different preferences and perceptions of mathematics. On these grounds, we first examined if it is possible to assess personal meaning: "Does the survey, which was originally developed in German, assess the different kinds of personal meaning with reliable scales in both countries?" Secondly, we conducted a Differential Item Functioning (DIF) analysis to examine if the latent variable models work equally across the two samples in Finland and Germany. In the last step, we compared the German and Finnish students' personal meanings using *t*-tests. Moreover, with the necessary diffidence, we provide tentative explanations for our results based on both countries' educational systems and curricula.

2 Theoretical framework

2.1 Personal meaning

There is a rich diversity of meanings of meaning (cf. Kilpatrick et al., 2005b, Reber, 2018). Thus, different interpretations of the term are often used synonymously although they are not synonymous at all. Kilpatrick, Hoyles, and Skovsmose (2005c) present different facets of meaning of a mathematical concept X which can be grouped as the meaning of X from a content perspective, the meaning of X within different spheres of practice, and the meaning of X from the perspective of different individuals involved in its construction. They conclude:

> “These views are actually different meanings of meaning insofar as different methodological tools are needed to explore them, different theoretical frameworks, etc. They insist on several different dimensions of meaning: psychological, social, anthropological, mathematical, epistemological or didactical. But all these dimensions must not be seen as isolated, one from the other. In fact, they constitute a system of meanings whose interactions shape what may be seen as *the* meaning of a mathematical concept.” (2005c, pp. 14–15).

In addition, Birkmeyer, Combe, Gebhard, Knauth, and Vollstedt (2015) relate meaning to cognition and affect: For the individual learner’s acts of consciousness, meaning represents a dimension that – apart from the areas of experience and action – focuses on a sphere of self-assuring clearance and clarification in the process of learning. The attainments of the consciousness with respect to giving meaning, as well as its affective embedment, create effects of meaningfulness in learning processes. These are to a greater or lesser extent distinct or can be experienced as such. Hence, it is also a matter of an inner psychic experience of meaning. This is neither sensation only nor thinking without emotion, and neither pure and isolated cognition nor knowledge that is independent from consciousness.

Following this description, the global concept of meaning has a dialectical relation to psychological as well as cognitive aspects. Both aspects are conducive to the development of one’s own identity: when something is meaningful to an individual, the content somehow makes sense for him or her (in terms of sense-making and understanding) and she or he gains orientation from it (in terms of understanding oneself and the development of one’s own identity) (Birkmeyer et al., 2015). Hence, meaning is something different than pure sense-making of the content as it additionally relates the content to the individual's identity and biography.

The distinction between cognitive and affective aspects of meaning also becomes clear when one regards that "even if students have constructed a certain meaning of a concept, that concept may still not yet be 'meaningful' for him or her in the sense of relevance to their life in general" (Kilpatrick et al., 2005c, p.14). The first kind of meaning from the quotation is of cognitive nature as the construction of a concept's meaning. It usually involves sense-making processes. The second kind of meaning, however, involves an affective interpretation as the relevance of the concept is connected to one's personal life. To conclude, two very distinct aspects of meaning can be differentiated here, namely "those relating to relevance and personal significance (e.g., 'What is the point of this for me?') and those referring to the objective sense intended (i.e., signification and referents). These two aspects are distinct and must be treated as such" (Howson, 2005, p.18). In line with Howson's distinction (see also Reber, 2018 for a distinction between subjective and objective meaning), Vollstedt (2010, 2011b) coined the term *personal meaning* to designate the first aspect of relevance and personal significance. Personal meaning describes the personal relevance of a mathematical procedure, content, or the people involved in the learning process for an individual, in our case mostly a student of mathematics. Key questions in this realm of research include: What is personally relevant for me when I am dealing with mathematical contents? Why should I get involved with this? What relations do the contents have to my own biography? Thus, personally experienced meaning occurs in the shape of a personal goal, a value, an intention, a purpose, a reference, or a use that an object or an action may have for the self (Vollstedt, 2011b).

In addition, Vollstedt and Duchhardt (2019) further differentiate the second facet of meaning as objective sense described by Howson above - they distinguish between collective and inner-mathematical meaning. They characterize collective meaning as follows:

> "(...) the relevance of a mathematical procedure or content for a certain group of people in contrast to an individual. This group of people can be characterised by a set of shared beliefs about the use of mathematics e.g. in terms of application in a certain profession, in life, in other scientific areas etc." (Vollstedt & Duchhardt, 2019).

The main question to be asked is whether the mathematical procedure or content has relevance in professional contexts at work or in other sciences. Adding to this, inner-mathematical meaning is characterized as “the relevance of a mathematical procedure or content without a relation that refers to something else than mathematical theory” (Vollstedt & Duchhardt, 2019). Here, central questions discuss the role of a certain mathematical theorem for other mathematical areas, the judgment of the importance of theorems for other areas (e.g. fundamental theorems), or criteria of relevance (cf. Vollstedt & Duchhardt, 2019).

As the focus of our study is on the individual and his/her relation to mathematics, we concentrate on personal meaning understood as personal relevance as described above. Personally experienced meaning is contingent on the individual and a certain context (see below). It has an endogenous character, i.e. it cannot be delivered by the teacher but, on the contrary, must be constructed out of the learner’s individual biography (Meyer, 2008). With respect to mathematics, the need for meaning cannot be fulfilled altogether: personal meaning must be continually interpreted and subjectively constructed for each mathematical learning content anew (Fischer & Malle, 1985). Therefore, at the same time and in the same context, different students can assign different meanings to the same mathematical content

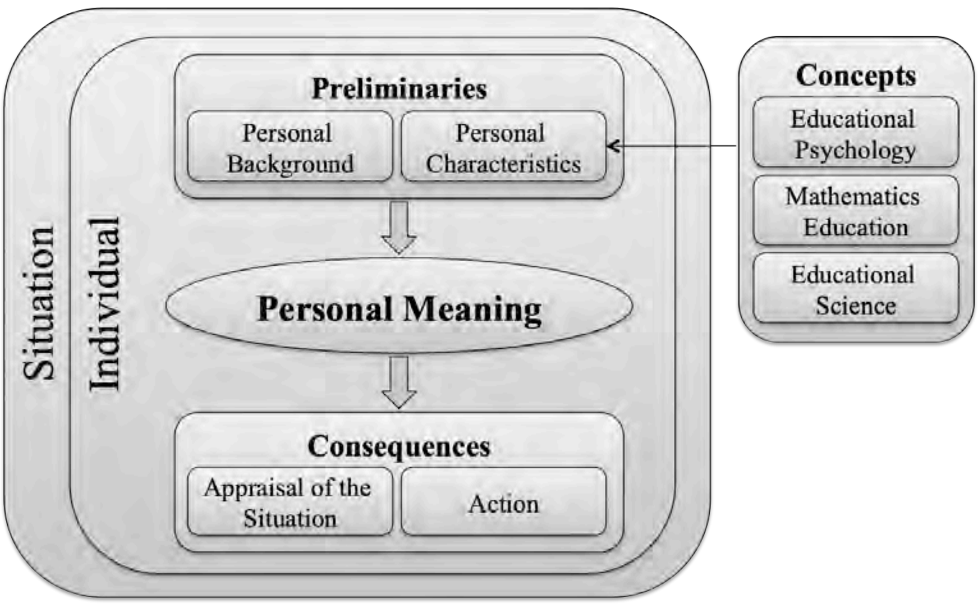

Figure 1. Relational framework of personal meaning (Vollstedt, 2011b).

Kilpatrick, Hoyles, & Skovsmose, 2005a; Vollstedt, 2011b). Vollstedt (2011b) proposed a model of personal meaning when learning mathematics and dealing with mathematical contents in a school context. In her theoretical framework she took the student's perspective. From this perspective, the following two main preliminaries influence the construction of personal meaning (see Figure 1): Firstly, the personal background of the student describes aspects which cannot be influenced by themselves like their socio-economic or migration background. Secondly, personal traits, i.e. aspects that concern the student's self, are relevant. They comprise concepts from various fields like educational psychology (self-concept, self-efficacy), mathematics education (beliefs), and educational science (developmental tasks). In addition to the individual preliminaries of a student, the situational context, i.e. context of the learning situation in terms of topic as well as classroom situation, is also a crucial factor for the construction of personal meaning.

The theory of personal meaning developed by Vollstedt (2011b) consists of 17 different kinds of personal meaning. They were constructed based on interview data with students from lower secondary level from Germany and Hong Kong. The aim of the study was on the one hand to develop a theory of personal meaning grounded in empirical data and, on the other hand, to investigate the role of (learning) culture for the construction of personal meaning (see Vollstedt, 2011a, 2011b for more information).

In total, 34 interviews were conducted with students from grade nine or ten (aged 15 or 16), 17 in each place. The interviews started with a sequence of stimulated recall (Gass & Mackey, 2000) in which the students watched a short video sequence of five to ten minutes from their last mathematics lesson. The sequence was chosen to show a situation in which the students dealt with something new for them as this might be a situation in which existing personal meanings might be reaffirmed or new ones might be constructed (e.g. in an "Aha" moment, cf. Liljedahl, 2005). The students were asked to reflect on the thoughts they had when they were attending the lesson as well as to name the thoughts they additionally had while watching the sequence. The subsequent interviews then addressed various topics inspired by the relational framework of personal meaning (see above, Fig. 1). They usually lasted for about 35 to 45 minutes with one exception in Hong Kong (90 minutes). Sample questions were for instance: How did you like this mathematics lesson? What was especially interesting? What feelings do you relate to mathematics lessons? Why do you learn mathematics? What can mathematics be used for? (cf.

Vollstedt, 2011c for the detailed interview guide). The data gained were coded following Grounded Theory (Strauss & Corbin, 1990; see Vollstedt, 2015 for a detailed description of the coding process and Vollstedt & Rezat, 2019 for the amendment of the coding paradigm). Theoretical saturation (Strauss & Corbin, 1990) was reached as the last two interviews did not provide any new categories and the relationships between the existing categories seemed well established and validated. In addition, the theory could be judged as dense for this age group from a theoretical point of view (Vollstedt, 2011b). This, however, does not mean that the theory may not require subsequent revision. Although the theory of personal meaning may be corroborated by future research, it may well be that it can also be elaborated further (Vollstedt, 2015).

The different kinds of personal meaning that were reconstructed from the data vary among the duty to deal with mathematics because it is a school subject, the cognitive challenge that is contained in mathematical tasks, and the experience of relatedness among the fellow students. The various kinds of personal meaning can be distinguished with regard to the intensity of the relatedness to mathematics and to the individual respectively, giving rise to seven superordinate types of personal meaning (cf. Figure 2).

Subsequent studies were carried out to develop a reliable survey on the basis of this theory (Büssing, 2016; Schröder, 2016; cf. Vollstedt & Duchhardt, 2019; Wieferich, 2016). At two stages, theoretical revisions were necessary. The first had to do with the fact that the personal meaning *Efficiency* from Vollstedt (2011b) combined two aspects: efficient classroom management and students' efficient ways of working, which could not be assessed with one scale. Instead, items addressing the latter aspect were merged with *Active practice of mathematics* and *Experience of autonomy*. The remaining scale was consequently renamed *Classroom management*. The second amendment followed from Büssing's (2016) results, which suggest splitting up *Relevance of application* into two facets, namely *Reference to reality* and *Application in life*. The resulting survey is used in the current study. The 18 kinds of personal meaning are presented in Figure 2.

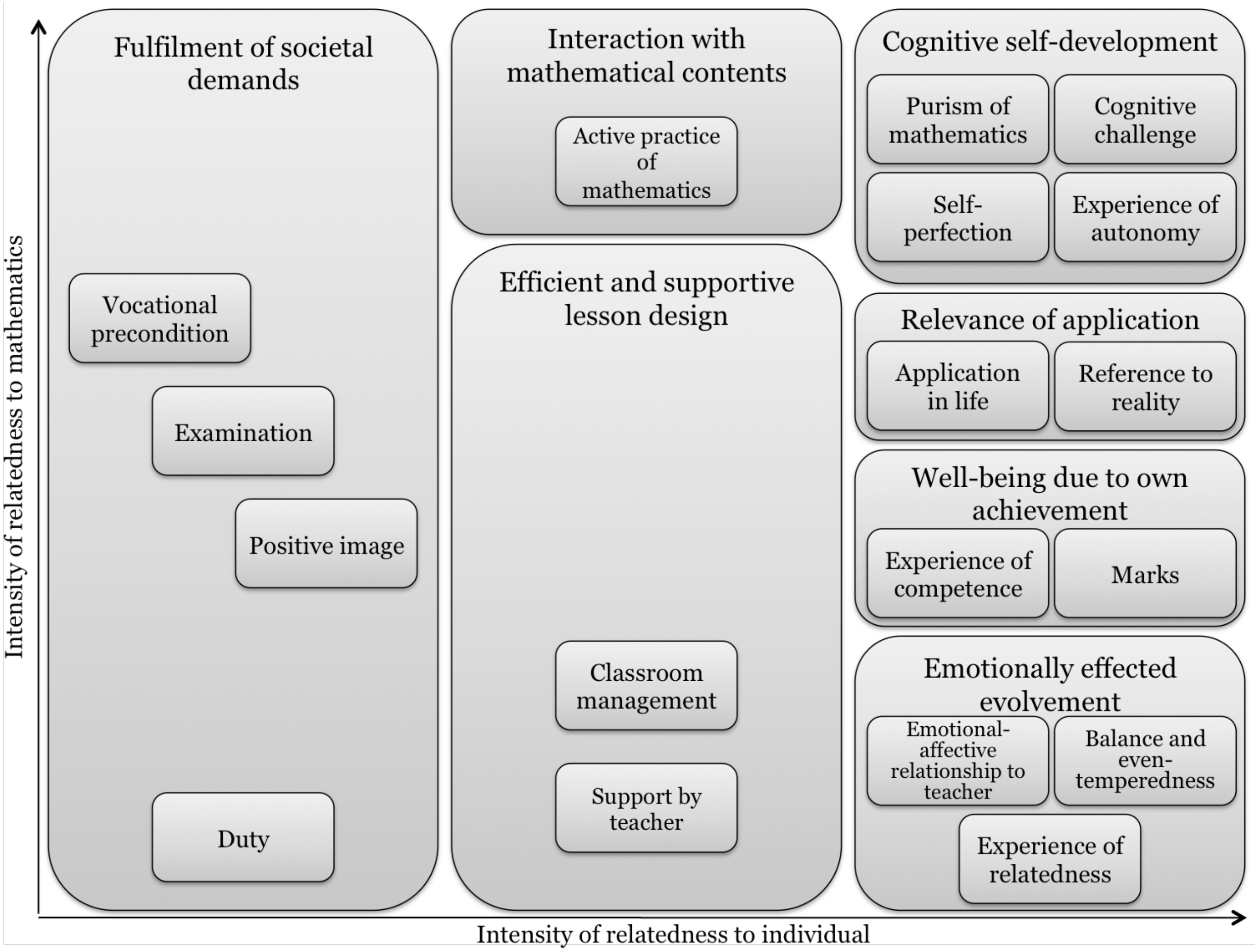

Figure 2. The typology of personal meaning with relation to the intensity of relatedness to mathematics and the individual consisting of 18 personal meanings. Note that amendments have been made with respect to the typology suggested by Vollstedt (2011b).

2.2 Educational systems in Finland and Germany

The previous section highlighted the construct of personal meaning and its relevance for learning mathematics in an educational context. In addition to students' background, foreground, etc., a countries' education reforms and policies may play a decisive role for learners' individual personal meanings in classroom. The educational context also affects and represents students' scholastic performance. Finland has been renowned for its high performing students in international comparative studies. However, in PISA 2015 Finnish students' mathematics performance, which shows a low correlation with their socio-economic status, had rapidly decreased compared to the former PISA studies. Now, Finland's PISA results are only a little better than the German results (OECD, 2018). The reasons for both countries' achievement level will not be discussed in the present study; the focus will rather be on their educational systems and mathematics curricula.

The mathematics curriculum of a country can be elaborated on different levels. Based on the results of the Second International Mathematics Study (SIMS; Travers, 1992) Valverde et al. (2002) describe the identified tripartite model of curriculum in detail: The intended curriculum comprises objectives in official documents for all students in a country. The implemented curriculum represents how these goals arise in mathematics lessons especially when teachers and students interact. The textbook is recognized as a mediator between intentions and implementation. The contents and processes that students deal with are focused in the attained curriculum. In general, the enacted mathematics curriculum comprises various stages within an educational system of a country, e.g. national objectives, educational goals, standards, syllabi, materials, teaching of a class (Thompson & Huntly, 2014). All stages are worthy of research as they would give an interesting insight into the development from intended to attained curriculum.

In our case, we intend to make a tentative analysis of the educational systems (attained curriculum) presented in the following sections. The analysis presented is not intended to completely meet criteria-guided empirical methods of comparing education systems. Nevertheless, it gives a first impression of interesting aspects of both countries' educational systems that might influence the construction of students' personal meanings.

A country's education is described by its educational system and core curriculum. These are essential to every education reform. Looking at the educational systems and curricula of both countries, disparities are noticeable.

According to its education philosophy "Every pupil is unique and has the right to high-quality education", Finland emphasizes students' individual growth, development and learning through equality, and high quality of learning. Finland has a comprehensive education for all (grades 1-9). Students visit one comprehensive school from primary level to lower secondary level independently of their academic performance. The low and high achievers sit in the same classroom and get individual support in accordance with their academic performance (Finnish National Agency for Education, 2017). Besides objectives of instruction and content areas related to the objectives, the national core curriculum for basic education defines seven transversal competence areas: Thinking and learning to learn (T1), Cultural competence/interaction and self-expression (T2), Taking care of oneself and managing daily life (T3), Multiliteracy (T4), ICT competence (T5), Working life competence and entrepreneurship (T6), Participation, involvement, and building a sustainable future (T7) (Finnish National Board of Education, 2016). Transversal competence areas are part of all subjects and play an important role in promoting students' individual development and their general learning. Another important fact is that during lessons students are engaged in reflecting about their own learning. Thus, during lessons students get support to understand their learning goals and receive help to recognize their own strengths and areas that need improvement. The Finnish teachers establish appropriate tasks that are necessary for learner's personal development. The teacher's role is to offer opportunities for the students to develop their skills for self-assessment and peer assessment in order to give and receive constructive feedback. Summarizing this, the focus of the Finnish curriculum is on cooperation rather than competition, and for teachers not to compare the students within a class. The continuous focus on these areas is considered to help students' life-long learning competencies.

In contrast, Germany focuses on the nascent need for skilled workers guided by their qualification initiative "Getting ahead through education" (*Aufstieg durch Bildung*). The federal government and the federal states had agreed on a common catalogue of objectives and measures whose aims are to raise the importance of education, to reduce the number of adolescents leaving school without vocational qualification, and to increase the number of students continuing education:

- Education is to have [*sic*] top priority in Germany
- Every child should have the best possible starting conditions
- Everyone should be able to gain school-leaving and vocational qualifications
- Everyone should have the opportunity to get ahead through education
- More young people should take a degree course
- More people should be filled with enthusiasm for scientific and technical vocations
- More people should take advantage of the opportunity for continuing education (KMK, 2017a, p.299)

In its federal states, Germany has many different school systems, which show variation e.g. with respect to the types of schools and the time span students visit them. One element that is similar in all systems is that there is an early selection into tracks according to students' academic performances (usually after four years, sometimes after six years of school attendance). The aim of this early separation is to provide optimal schooling with relation to the academic performance and needs of the students (KMK, 2017b). An analysis of the mathematical core curriculum shows that students' individual personal development is mentioned only superficially in the beginning of the educational standards (KMK 2005, p.6). The further description of the standards does not pick up this very fundamental topic at all. Hence, in the subject mathematics, the German education standards put a lot more emphasis on mathematical competences than on promoting students' individual development.

To conclude, the Finnish educational system focuses on every student's personal and academic development, as unique human beings and no separation is wanted. In contrast, the German educational system separates students early in order to respect their heterogeneous academic performances. In addition, the focus is on the development of mathematical competences rather than on personal development. These criteria are embedded in the core curriculum as a guidance for the mathematics lessons. Hence, the particular core curriculum of the country may also subconsciously influence teachers' actions in class and, thus, indirectly students' learning processes.

3 Research questions

In section 2.2, we provided a compact overview of the current Finnish and German education policy. The comparison illustrates remarkable differences in both countries' educational systems and curricula. Before it is possible to compare the results of two countries in a quantitative empirical study, we need to clarify some technical formalities. Therefore, we first examine technical questions focusing on the conduction of a comparative study on personal meaning in Finland and Germany. The following two major research questions are concerned with the compatibility of the results from Germany and Finland. The questions relate to the theory of personal meaning adapted for this study as described in section 2.1 above (Vollstedt, 2011b; Vollstedt & Duchhardt, 2019).

Validity of the instrument in Finland. The accessibility of personal meaning with a valid survey was confirmed in a former German pilot study with $N = 195$ ninth and tenth graders (Büssing, 2016; Schröder, 2016; Vollstedt & Duchhardt, 2019; Wieferich, 2016). The pilot study used data from the German federal states Bremen (two *Oberschule*, comprehensive school with school internal separation according to academic performance) and Lower Saxony (two *Gymnasium*, school for high achievers). This paper investigates ninth graders from Finland (comprehensive school) and Germany (different types of schools from various federal states). Hence, the first research question addressed in this paper is to test the validity of the Finnish survey of personal meaning (**research question 1**). In doing so, we investigate whether the Finnish translation of the instrument captures the same scales of personal meaning as the original German instrument.

Comparison between Finland and Germany. Building upon the results of the first research question, the second major target is a first approach towards a comparison of Finnish and German students' personal meanings (**research question 2**). The original survey was constructed and tested in Germany. Thus, students from Finland may understand and appraise the translated items in a different way than the original German formulation intended, or the German students rated the items. An interesting question in this comparison is what reasons can be detected for a different understanding or rating with respect to their educational system and curriculum. Therefore, it seems important to examine whether the latent variable models function equally across the samples in Finland and Germany. In the next step, meanings that are typically assigned to (learning) mathematics by Finnish and German students will be detected and significant differences will be discussed. This study considers crucial aspects of both countries' educational systems and curricula to make a tentative analysis of the formation of students' personal meanings.

4 Method

4.1 Sample

We collected survey data in Finland and Germany. In Finland, we collected data from 256 ninth graders (♀: 46%) in four comprehensive schools and 13 classes from the region Uusimaa. The 276 German participants (♀: 45%) from 17 classes were from different federal states (Bavaria, Baden-Württemberg, Bremen, and North Rhine-Westphalia) and attended different schools according to their academic performance (three Gymnasium (high achievers), one Realschule (middle achievers), three Oberschule (comprehensive school with school internal separation according to academic performance), and two Hauptschule (low achievers)). In our study, we are interested in all students' personal relevancies to deal with mathematics in an educational context. Therefore, we did not differentiate between high and low achievers to ensure having a heterogeneous group in Germany. In both countries, colleagues and private contacts of the researchers helped to find the schools.

4.2 Survey instrument

A research team at the University of Bremen (Büssing, 2016; Schröder, 2016; Vollstedt & Duchhardt, 2019; Wieferich, 2016) developed the German instrument to assess students' different personal meanings. The psychometric quality of the survey was good (Vollstedt & Duchhardt, 2019). For this study, the German version was translated into Finnish. We further validated the instrument in a cognitive lab (Zucker, Sassman, & Case, 2004). Based on the cognitive lab feedback we revised some items both in the Finnish and in the German version. Finally, the scale for the main study contained 131 items that were formulated as self-centered statements (like for instance "I deal with mathematics in order to…"). A 4-point Likert scale (0 = strongly disagree to 3 = strongly agree) was used to rate the survey.

4.3 Statistical procedure

Software. We conducted all statistical analyses in R (R Core Team, 2015). In particular, the R package TAM was used (Kiefer, Robitzsch, & Wu, 2015).

Scale analyses. Following an iterative approach, we tested the scales' psychometrical properties with item response theory (IRT) for both Finnish and German data. The IRT informed us about the validity of the survey in assessing students' personal meaning (see also Vollstedt & Duchhardt, 2019). For each scale of personal meaning, a partial credit model (PCM) with marginal maximum likelihood (MML) estimation was fitted to the data. We evaluated fit values of the PCM models ($0.8 < \text{Infit} < 1.2$; $-1.96 < t(\text{infit}) < 1.96$; $p(\text{infit}) > 0.05$) on item and scale level (Wu & Adams, 2007). Items with poor infit values were removed and the PCM model was refitted in an iterative procedure.

Differential Item Functioning (DIF). Differential Item Functioning analyses supported measurement equivalence of scales for people from two different groups. We compared the Finnish and German students' responses on items after adjusting for overall response tendency on the measured trait. This procedure was indispensable in this study in order to examine if the latent variable models functioned the same across the groups to ensure statistically fair comparison of any kind (Holland & Wainer, 1993; Monahan, 2007; Swaminathan & Rogers, 1990). An item had a DIF and was to be removed if two people from two different subgroups with the same overall agreement rate to this personal meaning had different agreement probabilities. We used the Educational Testing Service (ETS) classification system where an item has moderate to large magnitude of statistically significant DIF if the absolute DIF is >.638 and DIF significant >.426. Each item contributed to the estimation of the group means (Category C). The exclusion of a DIF item from a scale changed the estimate of the group difference. This allowed other items, which were previously inconspicuous, to show DIF. After removing the DIF items, the steps within the scale analyses were repeated.

Mean comparisons. We conducted Welch two sample t-tests (N = 532) to compare the Finnish and German students' personal meanings. Due to multiple comparisons, we controlled for false positives using Benjamini-Hochberg procedures (Benjamini & Hochberg, 1995) with a 1% (**) and 5% (*) false discovery rates. The computations were done using a published spreadsheet (McDonald, 2014).

5 Results and Discussion

Scale analyses. Psychometric properties of the Finnish and German surveys' scales were good. The estimated variances ranged from 0.49 to 5.73 (Finland) and from 0.58 to 5.30 (Germany) with most values around or above 1. Scale reliabilities ranged from an acceptable .65 to a very good .88 (Finland) and from .68 to .85 (Germany). Results from a former study with ninth and tenth graders from federal states Bremen and Lower Saxony show that it is possible to assess personal meaning in Germany with the valid German version of the instrument (Vollstedt & Duchhardt, 2019). Results relating to the assessment of Finnish personal meanings with a valid instrument (research question 1) show that the Finnish scales for the different personal meanings show all good psychometric properties. For most scales, two to four items were removed so that three to seven items remained. Exceptions were the two scales *Marks* (Finland reliability: .8, variance: 5.72; Germany reliability: .82, variance: 5.2) and *Duty* (Finland reliability: .83, variance: 5.73; Germany reliability: .83, variance: 4.73). They were assessed with only three items covering the core contents of the respective personal meanings with good values.

Looking at the scales with removed items, in both countries the scale *Balance and even-temperedness* (sample item: "It is important for me to sometimes play games in math lessons.") had to be removed, as its psychometric properties were not acceptable. These results are rather surprising; this scale did not work at all in this study although it provided good values (variance = 1.07 and reliability = .68) in a former German study that was conducted in grades nine and ten (Vollstedt & Duchhardt, 2019). It was not yet possible to find a convincing explanation for why the current samples provided a different result.

The total reduction of 28 items provided a final instrument (in Finland as well as in Germany) with 103 items (17 scales) for further analyses.

Differential Item Functioning (DIF). Our second research question concerned the comparison between Finland and Germany. As a first step to this end, we conducted Differential Item Functioning (DIF) analysis. This analysis assessed whether the latent variable models function equally across the samples in Finland and Germany. The measurement equivalence of the survey is evaluated according to Educational Testing Service (ETS) classification (Monahan, 2007). Consequently, nine DIF items (Category C) were detected:

DIF items / Finland

- Active practice of mathematics (Act13): When I am actively challenged in lessons, I have the feeling to understand the math contents easily.
- Experience of autonomy (Aut5): It is important to me to organize the time for working on mathematical tasks on my own.
- Marks (Mar2): It is embarrassing when I have worse marks in mathematics than the others.
- Self-perfection (Sel3): It is important to me to perceive my learning progress.

DIF items / Germany

- Active practice of mathematics (Act2): I am happy when I can do mathematical tasks.
- Application in life (App3): I deal with mathematics so that I do not lack important knowledge later on.
- Experience of competence (Com2): I deal with mathematics because my learning success makes me feel good.

- Experience of competence (Com13): I am proud of myself, when I realize what I have learned in the last years in math lessons.
- Reference to reality (Rea6): I think that learning mathematics is important because it is of great importance for other sciences.

In total, nine items show DIF within the whole item pool, four from the Finnish version and five from the German version of the survey. To begin with Finland, Finnish students rated the items Aut5 ("It is important to me to organize the time for working on mathematical tasks on my own.") and Mar2 ("It is embarrassing when I have worse marks in mathematics than the others.") lower than German students. Item Aut5 from the scale Experience of Autonomy asks for students' procedures when they deal with mathematics. However, the Finnish translation of this item ("Minulle on tärkeää saada järjestettyä aikaa matemaattisten tehtävien yksinään ratkaisemiseen.") is more likely to be interpreted as referring to the time outside of school. So, the students may not be able to connect this statement with their classroom situation. Item Mar2 from the scale Marks deals with the topic of competition. As mentioned above (see section 2.2), both the Finnish educational system and curriculum do not support competition through comparison within the students in class. Thus, for Finnish students it is very strange to feel embarrassed. This may be a reason why they couldn't identify with this item.

In contrast, German students rated the two items App3 ("I deal with mathematics so that I do not lack important knowledge later on.") from the scale Application in Life and Rea6 ("I think that learning mathematics is important because it is of great importance for other sciences.") from the scale Reference to Reality lower than the students from the Finnish sample. We assume that these students may not see the connection between the contents in their mathematics class and their need for knowledge later on or for other sciences. For the other five items (Finland: Act13, Sel3, Germany: Act2, Com2, Com13; see the list of DIF items for details), no sound reasons related to the education reforms and curricula could be found. At this point, it could be assumed that the students faced problems of misunderstanding in relation to wording.

All nine DIF items were excluded from further statistical procedures so that the remaining 94 items of the survey were considered for the comparative analyses between Finland and Germany. The psychometric properties of the new scales were again analyzed with good results.

Mean comparisons. We conducted *t*-tests to answer the second major research question with respect to commonality of personal meanings and significant mean differences between the Finnish and the German sample. The results presented in Table 1. give an interesting insight into Finnish and German ninth graders' preferred kinds of personal meaning. The following table presents an overview of the mean comparisons in an alphabetical order.

Table 1. Descriptive results per personal meaning of the Welch two sample t-test between Finland (FIN) and Germany (GER).

Personal meaning	**Mean of item means**		**Standard deviation**		***t*-Test**			
	FIN	GER	FIN	GER	*t*	*df*	*p*	Cohen's *d*
Active practice of mathematics	1.91	1.75	0.61	0.53	-3.16	508.19	0.001**	-0.80
Application in life	1.92	1.86	0.70	0.60	-1.04	497.13	0.297	-0.09
Classroom management	1.99	1.96	0.59	0.57	-0.61	523.73	0.537	-0.05
Cognitive challenge	1.54	1.38	0.68	0.60	-2.98	511.67	0.003*	-0.25
Duty	1.42	1.58	0.87	0.80	2.30	514.91	0.021	0.19
Emotional-affective relation to teacher	1.88	1.87	0.58	0.60	-0.17	529.04	0.858	-0.01
Examination	1.72	1.83	0.55	0.55	2.31	526.62	0.020	0.2
Experience of autonomy	1.70	1.68	0.60	0.57	-0.50	522.93	0.611	-0.03
Experience of competence	2.01	2.04	0.62	0.53	0.65	503.31	0.511	0.05
Experience of relatedness	1.89	1.96	0.53	0.48	1.71	514.77	0.087	0.13
Marks	2.14	1.87	0.71	0.78	-4.13	529.77	0.000**	-0.36
Positive image	1.45	1.48	0.60	0.59	0.56	525.29	0.574	0.05
Purism of mathematics	1.30	1.24	0.79	0.69	-1.02	509.37	0.304	-0.08
Reference to reality	1.62	1.69	0.59	0.53	1.42	506.59	0.155	0.12

Self-perfection	1.78	1.61	0.67	0.50	-3.25	474.64	0.001**	-0.28
Support by the teacher	2.08	2.20	0.66	0.53	2.37	488.42	0.017	0.2
Vocational precondition	2.05	1.72	0.69	0.75	-5.22	524.88	0.000**	-0.45

Note. All Likert scales were coded from 0 (*strongly disagree*) to 3 (*strongly agree*). Negatively worded items were recoded. Items removed during the scaling procedure were not included. *Statistically significant according to Benjamini-Hochberg procedure on 5% false discovery rate, **statistically significant according to Benjamini-Hochberg procedure on 1% false discovery rate, N = 532.

In general, both, students from Finland and Germany like less personal meanings that are related to mathematics like Purism of mathematics and Cognitive challenge and prefer personal meanings with a social inclusive character like Support by teacher, Experience of relatedness, and Emotional-affective relation to teacher. At first glance, the Finnish students' preference is easily explainable when taking into consideration the inclusive character of the Finnish educational system and curriculum: These meanings are in accordance with what is emphasized in the latest Finnish education reform (Finnish National Board of Education, 2016). The Finnish educational system and curriculum give top priority to collective learning environment and students' individual development. The continuous focus of these factors in math lessons clarifies why the Finnish students prefer these kinds of personal meaning related to social-inclusive factors. On the other hand, the German educational system and curriculum do not emphasize the social aspects as being as important as the competences related to the subject mathematics. Therefore, it seems that this kind of personal meaning is typical for both Finnish and German student, and that the Finnish curriculum is better aligned with student's preferences than the German curriculum.

Significant mean differences between Finland and Germany could be detected for the five personal meanings *Active practice of mathematics* (t = -3.16; df = 508.19; $p < 0.01$; Cohen's d = -0,80), *Cognitive challenge* (t =-2.98; df = 511.67; $p < 0.05$; Cohen's d = -0.25), *Marks* (t = -4.13; df = 529.77; $p < 0.01$; Cohen's d = -0,36), *Self-perfection* (t = -3.25; df = 474.64; $p < 0.01$; Cohen's d = -0,28), and *Vocational Precondition* (t = -5.22; df = 524.88; $p < 0.01$; Cohen's d = -0,45). The differences between the two educational systems and curricula may explain why Finnish students give top priority for *Marks* and *Vocational precondition* while in Germany these are far less emphasized. In Finland, grade nine is when students make the decision for the academic or the vocational track. As the acceptance to these institutes is based on grade nine marks, students need to achieve good grades to continue to popular tracks and schools. In contrast, the decision for vocational training or further education for most German students has only to be made after grade ten. Thus, the students' marks and their plans with respect to their future professional career are not yet equally decisive as they are for their Finnish counterparts.

The personal meanings that refer to mathematics and show significant differences (*Active practice of mathematics*, *Cognitive challenge*, and *Self-perfection*) were more common among Finnish students than among German students. Interestingly, these personal meanings had a quite high level of intensity of relatedness to individual (see Figure 2). Hence, Finnish students liked to deal with mathematics in order to improve their own skills and their self, respectively. This is also an important aspect that has top priority in the Finnish curriculum. The task of the subject mathematics includes, beside technical components, instruction that supports a positive attitude towards mathematics and a positive self-image as learners of the subject (Finnish National Board of Education, 2016).

6 Summary and further perspectives

In this study, we investigated the personal meanings of Finnish and German ninth grade students. The concept of personal meaning is multifaceted and highly relevant for learning mathematics in an educational context. We identified some relevant differences between the educational systems and curricula in these two countries. The Finnish educational system and curriculum focuses highly on the social nature (i.e. students' individual development) whereas the German education reform emphasizes more the subject related competences. In this paper, these aspects of the particular education reform are referred to for the tentative explanations of the quantitative results. We established the instrument of personal meaning in Finnish and German ninth grades to detect learners' personal meaning when learning mathematics. The scale analyses provided, in Finland as well as in Germany, scales with good psychometric properties. The personal meaning *Balance and even-temperedness* was removed as the reliability and the variance were found not to be good. After this procedure, the valid instruments were used for the Differential Item Functioning (DIF) analyses to evaluate the measurement equivalence. Nine DIF items could be detected and for the clarification of four of those DIF items, the particular educational system and curriculum of Finland and Germany was consulted. The exclusion of these nine DIF items allowed us to use the remaining items for the comparison between Finland and Germany. The mean comparisons came up with five significant mean differences, namely *Active practice of mathematics*, *Cognitive challenge*, *Marks*, *Self-perfection*, and *Vocational precondition*. The comparisons showed that Finnish and German learners gave more importance to social inclusive aspects and German learners less importance to the subject mathematics related aspect. In addition, meanings related to mathematics were typical among Finnish learners. This can be related to confirming important aspects of the self as one of the notions of the Finnish reform (Finnish National Board of Education, 2016). Our results showed differences between Finnish and German learners' personal meanings depending on the respective educational reforms. Future research will examine the different preferences of the personal meanings between Finnish and German students with regard to other affective constructs like learning motivation and self-concept, independently of curricula.

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Exploring the influence of pre-service mathematics teachers' professed beliefs on their practices in the Sri Lankan context

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Research on impact of teacher beliefs on their practices has been recognized as one of the important aspects in the discipline of mathematics education. This study reports the results of a case study that gives an insight about the influence of professed beliefs of pre-service secondary mathematics teachers on their instructional practices in the Sri Lankan context. The pre-service teachers' professed beliefs were examined by using a questionnaire of six-point Likert scale items. Data on instructional practices were collected through classroom teaching observations and follow-up post-lesson interviews. Qualitative analysis of the audio-taped classroom teaching observation transcripts was performed, using a list of sensitizing concepts that reflected flexible and rigid beliefs aspects. The results reveal that professed beliefs encouraged them to adopt flexible practices, but to differing extents due to the influence of social expectations and contextual demands embedded within this educational context

Keywords
professed beliefs, pre-service secondary mathematics teachers, instructional practices

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1 Background

Several scholars have emphasized the significant role played by teachers' beliefs in shaping teachers' instructional practices (Leung, Graf, & Lopez-Real, 2006; Thompson, 1992; Pepin & Roesken-Winter, 2015). Leder, Pehkonen, and Törner (2002) discussed the complexity of beliefs, noting that they have been studied in different disciplines and from a variety of perspectives. Furinghetti and Pehkonen (2002), in their examination of how researchers from the 1980s and the 1990s characterized the concept of belief and which dimensions they used to do so, found that most focused on the static aspects of beliefs (e.g., how beliefs are constructed, what constitute beliefs etc.) and little on their more dynamic aspects (e.g., how beliefs function).

Mathematics teachers from different cultures have their own beliefs, which they translate into unique teaching approaches (An, Kulm, Wu, Ma, & Wang, 2006). In the literature, some studies (Swars, Hart, Smith, Smith, & Tollar 2007; Hart, 2002) paid attention to the impact of teacher preparation program on changing beliefs of the student teachers. Hart (2002), for example, showed that successfully completing a certification program in teaching changed pre-service teachers' beliefs. Van Zoest, Jones, & Thornton (1994), in their study examining the beliefs and practices of pre-service primary mathematics teachers engaged in a mentorship program, found that the primary pre-service teachers' professed beliefs had been influenced by the socio-constructivist approach to mathematics instruction promoted by the mentorship program.

Other studies on the relationship between beliefs and practice reported both consistencies and inconsistencies. Examining recent studies that addressed the dialectic relationship between teacher beliefs and practice, Goldin et al. (2016) pointed out that the discrepancy between beliefs and practices is still an open issue. In another former study, Cooney (1985) examined the professed beliefs of one pre-service mathematics teacher (Fred) and found his professed beliefs to align with a problem-solving approach to teaching mathematics; however, when Fred attempted to pose recreational problems, on certain occasions his practices did not match his professed beliefs. Cooney noted that, although Fred attempted to conduct his teaching in accordance with his professed beliefs, he was not always able to do so, as actual classroom teaching practices were affected by other factors, such as students' not being used to learning mathematics by using problem-solving heuristics in their learning context.

Raymond (1997) reported another example of teachers' beliefs and practices being inconsistent due to extraneous factors. He examined six teachers to determine the influence of beliefs on practice, and found that even a single element in the immediate classroom situation could influence teachers' mathematics teaching practice more than their mathematics beliefs; the topic at hand, time constraints and prior school experiences, such as teachers' preparation programs and prior experiences as students, were all factors that could change teachers' practices.

Moreover, the impact of mathematics teachers' beliefs about the nature of mathematics and of learning and teaching mathematics on classroom teaching practice has been suggested by previous research findings and theoretical papers (Ernest, 1989; Pajares, 1992; Thompson, 1992). Some, like Thompson (1992), reported both consistencies and inconsistencies between mathematics teachers professed beliefs and actual practices. However, systematic research related to mathematics teachers' beliefs and their practices is rare in the Sri Lankan context, and there is a void in the literature on the relation between teachers' beliefs and their practices within the sociocultural context of Sri Lanka. As such, the present study focuses on Sri Lankan pre-service teachers' professed beliefs about the nature of mathematics and of teaching and learning mathematics, and the impact of these beliefs on their actual teaching practices.

2 Theoretical Underpinning

In the literature different arguments have been made regarding the concept of teacher beliefs, and there is no clear consensus on their definition. The nature of different understanding of beliefs has been reported in the literature. Ernest (1989) suggested that knowledge is the cognitive outcome of thought and belief is the affective outcome, but beliefs also possess a slight but significant cognitive component. A similar interpretation was given in Pajares (1992) by going through a thorough review of the literature to distinguish between knowledge and beliefs which suggests that knowledge of a domain differs from feelings about a domain.

McLeod (1992) differentiated between beliefs, attitudes, and emotions, stating that beliefs are largely cognitive in nature relative to attitudes and emotions. As such, it can be noticed that although the term beliefs largely reflect the affective domain, it cannot be totally detached from the cognitive domain. Values are another crucial and emerging construct in mathematics education which has not yet been addressed largely (Seah, Andersson, Bishop, & Clarkson, 2016). The term 'values' refers to what one considers as important such as a certain belief being considered as something of importance and significance.

The current study focuses on the theoretical construct of professed beliefs of pre-service teachers. In this study, professed beliefs refer to what teachers express as their conceptions of the nature of a discipline (mathematics) and about a phenomenon (learning and teaching mathematics). This report focuses on professed beliefs about the nature of mathematics, and learning and teaching mathematics (Ernest, 1989). Teachers' beliefs regarding the nature of mathematics are referred to as teachers' conceptions of the nature of mathematics. Ernest (2004) paid much attention in elaborating on the absolutist-fallibilist distinction, assuming that the choice between the two would be the most influential epistemological factor on mathematics teaching. Therefore, it is important to explore mathematics teachers' beliefs about epistemology of mathematics and mathematics education; and their impact on teaching.

The term 'instructional practice' in this study indicates actual classroom teaching that facilitates students to acquire content by creating a conversation providing reasons, justifications, explanations, and conclusions. Referring to the above-mentioned theories, this study focuses on the instructional practices of pre-service teachers and the influence of teachers' professed beliefs and any other factors on those practices.

3 Method

The research question that guided this study is: What is the relationship between Sri Lanka's pre-service mathematics teachers' professed beliefs and their instructional practices? This is a part of a larger study that employed a survey and a case study. In the larger study, we examined teacher knowledge and their professed beliefs by employing a survey over a cohort (n=126) of the third-year, Sinhala-medium, pre-service secondary mathematics teachers enrolled in a three-year pre-service teacher education course at National Colleges of Education in Sri Lanka. Afterwards we conducted a case study, by selecting cases from the survey study, to explore the impact of teacher knowledge and beliefs on their instructional practices. We presented the results on teacher beliefs in an oral communication session at the 13th International Congress on Mathematics Education (Wadanambi, & Leung, 2016). However, we present a brief introduction about the beliefs questionnaire and the procedure of selecting cases below. The beliefs questionnaire consisted of four subscales adopted from Teacher Education and Development Study in Mathematics (TEDS-M) project's framework (Tatto, Schwille, Senk, Ingvarson, Peck, & Rowley, 2008) related to beliefs about the nature and learning of mathematics. We used two subscales – 'Mathematics as a set of Rules and Procedures' (Rules and Procedures) and 'Mathematics as a Process of Enquiry' (Process of Enquiry) to portray the beliefs about the nature of mathematics, and the sub-scales 'Learning Mathematics through Following Teacher Direction' (Teacher Direction) and 'Learning Mathematics through Active Involvement' (Active Involvement) to characterize the beliefs about learning and teaching mathematics. Statements in the Rules and Procedures scale and that of the Teacher Direction scale reflect rigid views while statements in the Process of Enquiry and Active Involvement scales reflect flexible views about mathematics as well as mathematics education as a discipline. We asked the participants to respond to each statement given in the beliefs scales by choosing among six-point Likert scale response alternatives from "strongly disagree" to "strongly agree". The study determined the overall degree to which each belief category was endorsed based on participants responses to each category. We considered the responses “Agree” and “Strongly agree” to be endorsements of the respective statements. We considered that the participant has endorsed the particular belief if his or her mean rating for a given scale was equal to or greater than 5. We selected the two cases described in this paper from the category those who professed to hold flexible views.

Flexible views meant those held by pre-service teachers who strongly endorsed mathematics as a process of enquiry and learning mathematics through active involvement. Rigid view holders meant those who endorsed mathematics as a set of rules and procedures and learning mathematics through teacher direction. However, there were no participants to represent rigid view category since none of the participants supported learning mathematics by teacher direction (see Table 2). Moreover, we defined the average of Process of Enquiry and Active Involvement scales as a measure to indicate the degree of flexibility of the participants’ views (beliefs), and to use that as a criterion for selecting even more flexible view holders among the participants.

We collected data on actual instructional practice during the internship year of the two pre-service teachers. Although secondary mathematics pre-service teachers are trained to teach mathematics to Grade 6-11 classes, during the internship training period they are more likely to be assigned to teach in Grade 6-9 classes. Therefore, we focused the topic of Algebraic Expressions which is generally taught in the second term of the year at the Grade 8 level, in this study. Depending on their teaching approaches, each teacher needed three or four 40-minute lessons to complete the topic. Throughout the lesson, we audio-taped each of the lesson and took field notes, particularly concerning situations that could not be audio-recorded (e.g., explanations written, or images drawn on blackboards, non-verbal student-teacher interactions, student responses to visuals used by the teacher, etc.). To clarify observed teaching and learning incidents, we conducted post-lesson interviews after each lesson. We audio-recorded the interviews and transcribed them for use in the data analysis process.

4 Data Analysis

When analyzing the beliefs questionnaire responses, we counted a participant to have endorsed a particular belief, if their mean rating for a given scale was equal to or greater than five. To spot the general pattern of the prospective teachers' professed beliefs, we also calculated the overall mean ratings for each beliefs scale, by averaging the mean ratings of all participants under each scale.

To analyze the classroom observation transcripts, we identified a list of sensitizing concepts including specific tasks, activities, or behaviors that reflected aspects of beliefs. We scrutinized the lists of sensitizing concepts used in previous studies (An, Kulm, & Wu, 2004; Leung, 1995) when analyzing classroom teaching observation data, particularly those that reflected aspects of teacher beliefs about mathematics and mathematics teaching and learning. Drawing on the key features mentioned in various previous studies, we created a theory-driven list of sensitizing concepts corresponding to different views of mathematics and mathematics teaching and learning. Then we modified the list to match the available classroom teaching observation data from the current study (see Table 1). Based on the list of sensitizing concepts the researcher coded the classroom observation transcripts; and the researcher invited a university academic from a Sri Lankan university department of mathematics to code a part of the data to verify and increase the reliability of the coding. The inter-rater coded three lessons out of seven transcripts. Whenever inconsistencies in the coding arose, the researcher and the inter-rater resolved them through keeping discussions between them. After coding the transcripts, we reported the frequencies for each type of incidents observed in the teaching/learning process.

Table 1. Sensitizing concepts that reflect Flexible View and Rigid View aspects.

		Sensitizing concepts	Description
Flexible View	31	Alternative methods	Teacher encourages students to perform alternative methods to figuring out the same problem or invent their own methods.
	32	Principle-oriented explanations	Teacher highlights the mathematical principles in explanations. Teacher not only states the facts but rather, focuses the issues such as why it works, how do we know, would it ever be true.
	33	Investigation or exploration tasks	Teacher engages students in investigation or exploration-based activities in a small group setting or individualized setting.

Rigid View	41	Fixed steps in solving problems	Teacher repeats or reminds students about the fixed order of steps to execute in solving problems, or teacher emphasizes fixed order of steps rather than letting students to attempt their own steps.
	42	Algorithmic or procedure-oriented explanations	Teacher highlights/focuses the steps in an algorithm or procedure without mentioning the principle.
	43	Drilling and practice-oriented tasks	Teacher wants students to practice a number of similar exercises rather than understanding the procedure.
	44	Memorizing facts and procedures	Teacher wants students to memorize the terminologies, facts or procedures.

5 Results

5.1 Results of beliefs questionnaire

A summary of the participants' endorsements on each of the beliefs scales is displayed in Table 2

Table 2. Participants' endorsement and mean rating of the degree of agreement on each scale.

Scale	No. of participants endorsed	% of participants endorsed	Mean rating
Rules and Procedures	29	23.01%	4.48
Process of Enquiry	91	72.22%	5.15
Teacher Direction	0	0%	2.81
Active Involvement	91	72.22%	5.15

The above results show that flexible views of both the nature of mathematics and of teaching and learning mathematics were rated more highly by participants than were rigid views. The two cases described in this paper were drawn from among those whose reported beliefs index was greater than five, indicating they held more flexible views (see Table 3). They are known as Sudam and Jayani in this paper.

Table 3. Beliefs indicators of the case study teachers.

Name	Process of Enquiry	Active Involvement	(PE+AI)/2
Sudam	5.17	5.17	5.17
Jayani	6.00	5.83	5.92

5.2 Results of instructional practices

5.2.1 Context of the instructional Practices

In general, chalk and blackboard were the major teaching/learning resources available to and used by the teachers in this research context. The two schools in which the instructional practice observation took place were mixed schools, with both male and female students learning in the same school. Inside the classrooms, male students sat on one side and females on the other. Each student had a desk and chair, and there were usually about 40 students in a classroom. There were no technological resources, such as computers or overhead projectors in the classrooms.

The observed lessons taught by the two participants both focused on constructing algebraic expressions, substituting values into algebraic expressions, constructing algebraic expressions containing brackets, and obtaining a method to remove brackets from an algebraic expression. Both schools were similar in most major respects. Both were mixed schools in which each class consisted of about 40 students, comprised of nearly equal numbers of boys and girls. In academic terms, the schools were neither higher- nor lower- level schools; rather, both were mid-level schools. Both schools were located in urban areas and, generally speaking, their students' achievement levels and their families' socio-economic statuses were also similar.

5.2.2 Results of observations of classroom teaching

Table 4 reports the frequencies of the teaching instances coded under flexible view (FV) and rigid view (RV) in relation to the practices of the case study teachers Sudam and Jayani.

Table 4. Frequencies of incidents related to Flexible View and Rigid View aspects.

	Sensitizing concepts		Sudam	Jayani
Flexible View	31	Alternative methods	1	3
	32	Principle-oriented explanations	4	3
	33	Investigation or exploration tasks	-	-
	Total number of flexible view incidents		**5**	**6**
Rigid View	41	Fixed steps in solving problems	1	
	42	Algorithmic or procedure-oriented explanations	2	1
	43	Drilling and Practice oriented tasks	-	-
	44	Memorizing facts and procedures	3	-
	Total number of rigid view incidents		**6**	**1**

A classroom incident that reflects the sensitizing concept Alternative methods is described below to make it sensible for the reader as an example.

5.2.2.1 Incident from Jayani's lesson (alternative methods):

Context: The objective of the lesson was to learn how to construct algebraic expressions containing brackets. The teacher drew a floor plan of a building with three rooms (Figure 1), of lengths p, q, and r, on the blackboard and asked students to construct an algebraic expression to represent the building's perimeter.

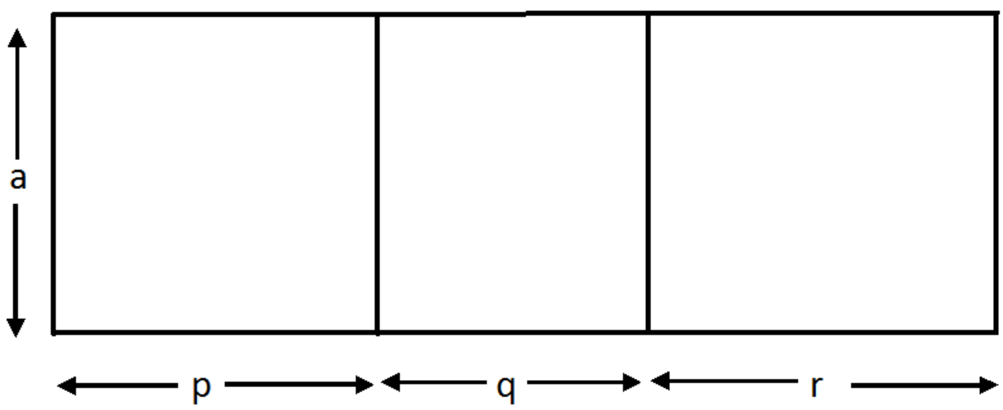

Figure 1. Floor plan of a building used as the context for algebra in Jayani's lesson (reproduced).

The teacher said, "Your friends have used two methods. Let's see what they are.", and then started a whole-class discussion about two approaches the students had used to formulate the expression and the appropriateness of each method.

> **Teacher**: "If this side can be expressed as $p + q + r$, this side also can be expressed as p+q+r [pointing to the longer sides of the rectangle], alright? Can it be written as [the teacher wrote $(p + q + r) + (p + q + r)$ on the blackboard] or can it be .... [writing $2(p + q + r)$ on the blackboard]? Are these two things or one thing?"
>
> **Students**: "One thing."
>
> **Teacher**: "Yes, equal or not?"
>
> **Students**: "Equal."
>
> **Teacher**: "Those who wrote this [pointing to $(p + q + r) + (p + q + r)$] and also those who wrote this [pointing to $2(p + q + r)$] are correct. If so, tell me what the width is. a unit? What is the width of whole building? $2a$... then this

is also added to this. Now, can it be simplified further? ...can, I now write the same thing without brackets? Now... see like terms are added. Two *p*s..."

Students: "Two *p*...two *q*...two *r*"

Teacher: "That means two of the long sides. Now, 2*a* is added to this (Figure 2). Can this be simplified further?"

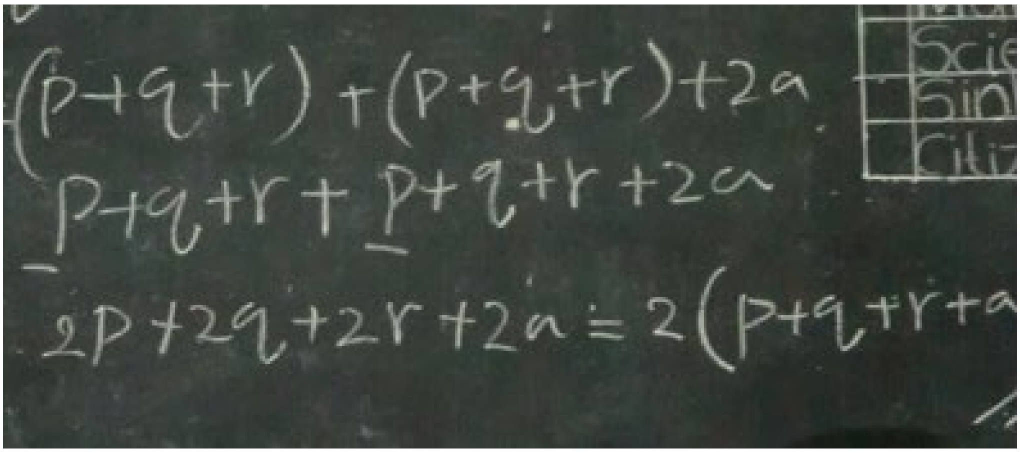

Figure 2. An illustration given in Jayani's lesson while constructing an algebraic expression.

Students: "Can't, ... can, can put two and brackets."

Teacher: "Some of you have done...these two are two different factors. Removing brackets by multiplying... have written so. Are all these methods correct or not?"

Students: "Correct."

By discussing with the whole class, the two approaches used by other students to solve the problem, the teacher encouraged the students to solve mathematics problems flexibly, without sticking to a fixed method.

A classroom incident that reflects the sensitizing concept Fixed steps in solving problems is described below to make it sensible for the reader as an example.

5.2.2.2 Incident from Sudam's lesson (Fixed steps in solving problems):

Context: This incident involves a situation in which the students did not use the required method their teacher expected them to use when simplifying an algebraic expression. The task had three parts. First, students were required to construct an algebraic expression to represent a given scenario, which they did, creating the equation: $4a + 16b + 40$. Second, they were to rewrite it as an algebraic expression with brackets, which led to the equation: $2(2a+8b+20)$. Third, they were asked to calculate the value of the algebraic expression, given that $a = 5$ and $b = 2$.

The teacher expected the students to substitute the values for a and b into the bracketed expression, $2(2a+8b+20)$. However, the students instead substituted $a=5$ and $b=2$ into the original equation, $4a + 16b + 40$. One student came to the blackboard and wrote down the following steps (Figure 3):

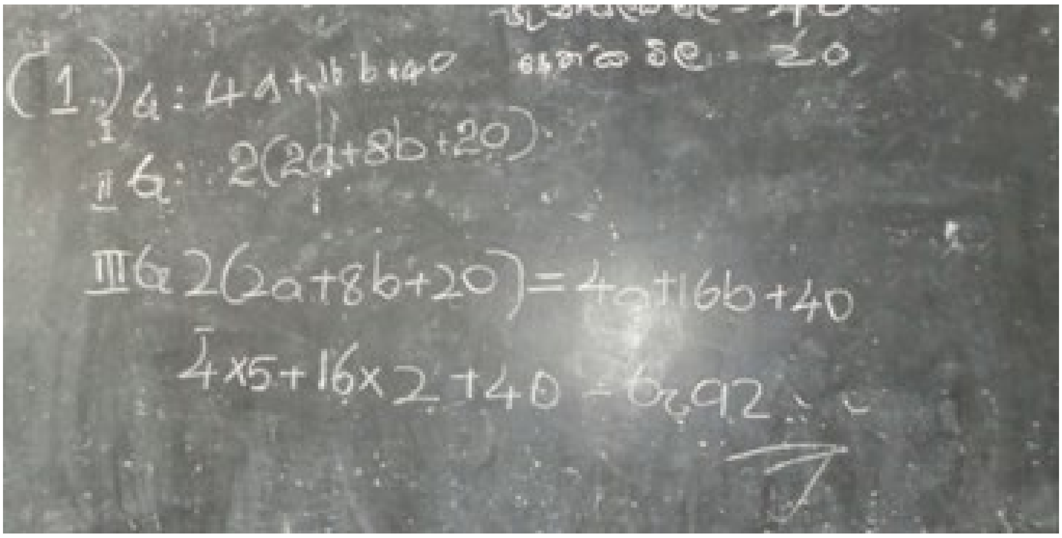

Figure 3. A student's calculation of the value of an algebraic expression by substituting the given values.

> **Teacher**: "You were supposed to substitute to this. [Pointing to $2(2a+8b+20)$.] I think none of you have done so." [The teacher showed the whole class how to do it, again.]

$$2(2a+8b+20)$$

$$2(2 \times 5 + 8 \times 2 + 20)$$

> **Teacher**: "Look here, how much...when simplifying the things inside the bracket."

$$2(10 + 16 + 20)$$

$$2(46)$$

> **Students**: "Forty-four, forty-six."

> **Teacher**: "Forty-six multiply by two gives how much? Ninety-two. **This is the process you were told to follow. Nobody did so. I didn't say you were to remove the brackets and simplify**. [a bit rougher voice than his usual tone.] This is the way to simplify when it contains a bracket. If you have not written it down, write it now."

5.2.2.3 Characteristics of instructional practices of Sudam and Jayani

The frequency of occurrences in each sensitizing concept (see Table 4) show the salient features of the teaching and learning approaches adopted by the two prospective teachers, with respect to the rigid view and flexible view aspects. An analysis of classroom teaching practices yielded evidence of both rigid and flexible practices, but none that was either highly rigid or highly flexible. For example, the two participants commonly posed questions to enable students to express their ideas or understandings, but activities such as exploration tasks, which encourage students to construct knowledge on their own, were rare.

Sudam adopted both slightly rigid and slightly flexible practices. In his practices, he attempted to build students' mathematical understanding of the content by providing principle-oriented explanations as a first step and having them systematically practice mathematics exercises. He seemed more likely to focus on mathematical understanding than on doing mathematics by drilling and practicing. Although there were Rigid View incidents, they basically happened to help students learn to solve mathematics problems systematically and to proceed more easily by keeping up their memory of technical terms, etc. He also showed concern about keeping slight variations (see Table 5) when selecting in-class exercises to practice, rather than repeatedly practicing similar problems. Altogether, these activities showed that his practices were somewhat more flexible than they might appear.

Jayani adopted mainly flexible practices and was very unlikely to implement Rigid View practices (see Table 4). She mentioned that knowing different methods to solve the same problem gives students opportunities to think about more efficient methods. She often focused on helping students gain mathematical understanding. Neither teacher employed purely inquiry-based learning activities; instead, they led discussions to encourage students to learn the content in a meaningful manner.

5.2.3 Evidence from the Post-Lesson Interviews of Sudam and Jayani

The instructional practice features observed can be further explained by the evidence gathered in post-lesson interviews. Based on the case study teachers' explanations, several other factors were identified as mediating variables that impacted on practices and are noted in Table 5.

5.2.3.1 Instances from the post-lesson interviews

Table 5. Explanations given by the cases to describe the particular features of their practices.

Feature	Explanations quoted from post-lesson interviews	Mediating variable
S1	**Sudam**: "Generally speaking, because a lesson like algebraic expressions seems more abstract in nature, such abstract matters should be explained …. because Grade Seven and Eight pupils … like relating to concrete materials. If it is done in a more practically-oriented manner, even better."	Psychological aspects of learning
S2	**Interviewer**: "Can you elaborate, what do you mean by practically oriented?" **Sudam**: "It would have been better if it was done with objects, but, see, the timing problem is there. See... in a lesson like on algebraic expressions, to introduce terms such as unknowns… for example, putting some pencils or such kind of objects in a box, so that pupils do not know exactly how many objects there are inside,… therefore, pupils would say x, they would say y,... such unknown terms would be expressed. Like that… It would have been better if it had done so, but the thing is, there is not sufficient time."	Time constraints
S3	After constructing algebraic expressions teacher gave students following exercises to simplify; 2x+5x+3x 5ab+ab+2ab 2p+2q+3q-p His explanation for selecting the exercises: **Sudam**: "It's about moving from simplicity to complexity. Now… first, understanding the like terms, next one is like terms too… but a bit more complex. See… the type of unknown is presented as multiplication of two unknowns. The next presents mixing like terms and unlike terms."	Simplicity to complexity
J2	**Jayani**: "When I was a student in school, a teacher of mine often had… no time to observe all our work when there were many… who comes first to show their books are correct and then … problematic situations were discussed. Perhaps we might have made the same mistake, then at the same time we can correct it. That is that."	Previous experiences as a student
	Interviewer: "Other reasons if any?" **Jayani**: "Yes, another thing is that the answer may be correct, but the	Exam

J3	method of getting the answer may be incorrect. … [Later] steps may correct final [result], simplification may be incorrect. I want students to do correct thing all the time and obtain the answer, while considering even small matters. The final goal is getting higher marks at the examination" [she says with a smile indicating a sort of happiness].	expectations

Similarities and differences in the case study teachers' practices on two perspectives (beliefs and mediating variables) are elaborated below.

5.2.4 Comparison of practices based on professed beliefs

The incidents related to Flexible View and Rigid View (see Table 4) show that both participants were likely to emphasize certain characteristics of flexible practices, such as promoting principle-oriented explanations. However, other characteristics such as alternative methods of solving problems and exploration activities were rare.

5.2.4.1 Beliefs and practices of Sudam:

Most of the Rigid View incidents in his practices were related to providing procedure-oriented explanations and emphasizing the memorization of factual knowledge. Moreover, in Sudam's practices, the frequencies of the Flexible View and Rigid View incidents were similar. However, as previously noted, his practices were mostly flexible as described in 5.2.2.3.

The results suggest that Sudam was likely to maintain both rigid and flexible practices, but his overall practices were more flexible in that they placed more emphasis on meaningful learning. His professed beliefs seemed more flexible view oriented, and his practices reflected his professed beliefs to a certain extent.

5.2.4.2 Beliefs and practices of Jayani:

In Jayani's practices, she attempted to make her pupils aware of the different solution approaches used by their peers, indicating her enthusiasm for flexible view teaching approach. She also wanted students to think of other ways of solving a given problem, rather than fixing on just one method. Moreover, Jayani's lessons featured substantially more Flexible View incidents than Rigid View incidents, suggesting she was more likely to maintain flexible practices than rigid practices.

The results suggest that Jayani's professed beliefs were more Flexible View oriented, as shown in her classroom teaching.

In sum, no exploration tasks were found in either prospective teacher's practices, but both often attempted to provide necessary mathematical arguments or facts to make pupils more aware of what they were doing, how certain procedural steps worked, and so forth. Both teachers also attempted to enhance students' mathematical understanding by adopting different representational strategies and by helping students overcome their difficulties through strengthening their conceptions. The above evidence shows that both case study participants exhibited certain flexible practice behaviors in their classroom teaching, although one also exhibited certain rigid practice behaviors. In other words, Flexible View -supported behaviors were common to both, while Rigid View -supported behaviors were not. Moreover, as revealed by the belief's questionnaire, both case study participants' professed beliefs about the nature of mathematics and of teaching and learning mathematics were more flexible view oriented. Therefore, it appears that their actual classroom instructional practices were shaped, to a certain extent, by their professed beliefs.

5.2.5 Impact of mediating variables on practices

Explanations given by the participants during post-lesson interviews provide further evidence that extend understanding of their instructional practices (Table 5). Both case study participants attempted to create a teaching/learning environment that was, to a certain extent, supported by flexible practices, due to the teaching and learning theories (psychological aspects of learning mathematics) they had learnt in their teacher education course which emphasized the need to create meaningful learning situations in their instructional practices. These facts are consistent with their professed beliefs. However, contextual factors such as time constraints, expectations of an examination-oriented culture and previous learning experiences as a student have also influenced their practices. In other words, their professed beliefs about the nature of mathematics and of teaching and learning mathematics encouraged them to adopt flexible practices, albeit to differing extents due to the influence of other factors.

6 Discussion and Conclusion

The overall analysis above indicates that the two case study teachers' professed beliefs had influenced their practices. Both participants held Flexible Views and the practices of both comprised more Flexible View than Rigid View aspects. The results also revealed that contextual factors such as examination-oriented expectations, time constraints and previous learning experiences of the teacher also caused differences in their actual practices.

To summarize, the study found that the participants' professed beliefs (flexible views on teaching and learning mathematics) affected their practices to varying extents but were restricted by the underlying social and contextual characteristics of the research site. Such consistencies between beliefs and practices were reported in prior studies in the area (Stipek et al., 2001; Temiz & Topcu, 2013) as well as the inconsistencies (Cooney, 1985, Raymond 1997). The study contributes to the extant literature (Thompson, 1992, Goldin et al., 2016) by adding evidence from the Sri Lankan context on the role of beliefs in mathematics teaching.

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