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Research article

Navigating challenges in teaching the tower problem: Pre-service teachers' learning experiences of problem-solving through the lens of TDS

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Abstract: This study highlights the challenges faced by pre-service teachers in navigating mathematics problem-solving instruction at the university level. The activity of problem-solving is central to mathematical sense-making, crucial from the elementary grades onward. However, problem-solving often occupies a marginalised position in elementary school classrooms. This issue can be partly attributed to the fact that many pre-service elementary teachers possess (1) limited mathematical knowledge regarding problem-solving strategies and (2) counterproductive beliefs about how to effectively teach these skills. Building on an intervention with a group of pre-service elementary teachers addressing these two critical barriers to teaching problem-solving, this study explores the challenges that these teachers identified as they prepared and delivered lessons focused on problem-solving. Drawing on the Theory of Didactical Situations (TDS), key concepts were employed to analyze the data with a focus on *the target knowledge*. The findings add to the growing body of research highlighting challenges that teacher education programs can address to better prepare pre-service teachers for teaching problem-solving in mathematics.

Keywords: mathematics problem-solving, algebra, teacher education, TDS, intervention

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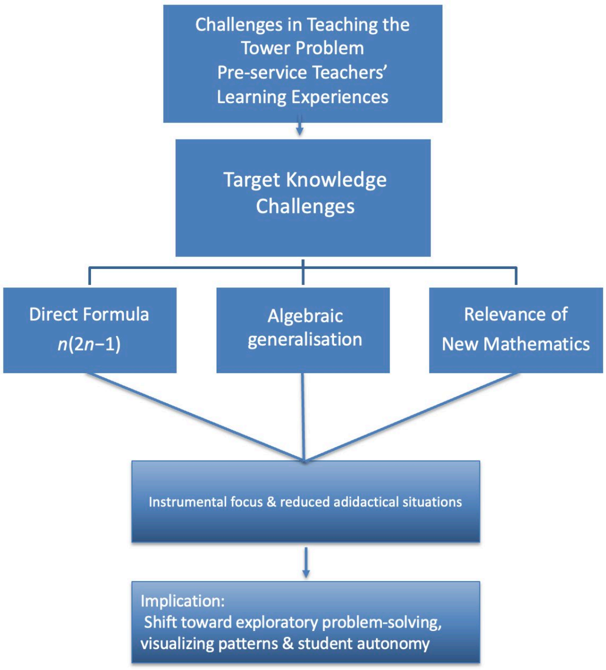

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1 Introduction

Mathematics is widely regarded as one of the most stimulating scientific disciplines, yet it is also one of the subjects that causes great challenges and anxiety among learners (Hembree, 1990). Even though solving and posing mathematics problems is a professional skill for primary school teachers, pre-service teachers often struggle with mathematics content at the university level (Weber et al., 2023). One reason found in previous research for this is that primary teachers do not necessarily identify themselves as mathematics teachers (Pereira, 2005). Rather, primary teachers form a class-teacher identity early, focusing on teaching various subjects (Livy, Herbert, & Vale, 2018). When it comes to mathematics content, pre-service teachers see themselves as university students learning advanced mathematics rather than as future teachers who will teach mathematics (Palmér, 2016). Furthermore, they do not perceive university mathematics as relevant to their future profession (Buchholtz et al., 2013). This group of university students can be especially challenging to engage in content during their teacher education (Goulding et al., 2002; Ambrose, 2004; Buchholtz et al., 2013). Often, mathematical content is perceived as relevant when it comes from within classroom practice (Carrillo-Yáñez et al., 2018).

Professional knowledge for mathematics teachers has been addressed by numerous researchers for decades (e.g., Putnam et al., 1992; Fennema & Franke, 2001; Ball & Forzani, 2009), emphasizing the importance of both mathematical content and teaching skills. The role of a teacher educator is, among other things, to connect school mathematics to advanced mathematics (Kaiser et al., 2017; Masingila, Olanoff, and Kimani, 2018), for instance, by making teaching practice the core of the course (Ball & Forzani, 2009).

Problem-solving is often viewed as an effective method for establishing connections in mathematics education (Bishara, 2016; Weber et al., 2023). This approach enables students to navigate common challenges encountered in school mathematics, encompassing both rich mathematical content and relevance to specific grade levels. In this article, I refer to such challenges as school-relevant problems, a term commonly used in research to describe problems that are appropriate for specific grades due to their contextual suitability and cognitive demands (e.g., Henningsen & Stein, 1997; Weber et al., 2023). The emphasis on school relevance in mathematics problem-solving is underscored by policy documents and materials from the Swedish National Agency for Education (2022). Additionally, there is a web-based repository of problems tailored for specific grades within compulsory education, as research has firmly established the importance of problem relevance for different age groups (Silver, 1997; Decker & Roberts, 2015).

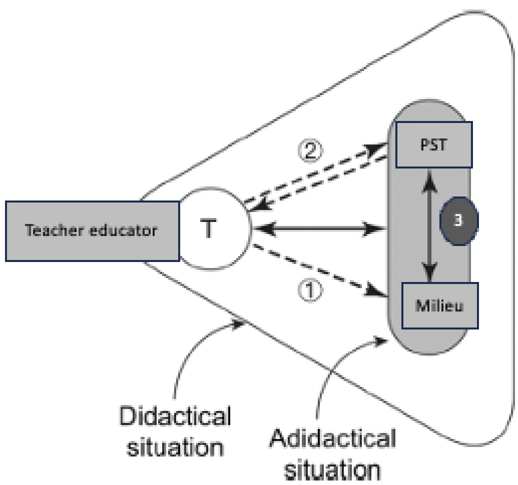
While there are varying perspectives on the nature of the subject of mathematics, both mathematicians and teacher educators share a common goal: enhancing the mathematical knowledge of pre-service teachers to equip them for their future profession. For example, Leikin et al. (2017) examine the complex role of mathematicians in teacher education, emphasizing the importance of establishing a connection between higher education mathematics and school-level mathematics. Similarly, Bragg (2015) advocates for the selection of problems that are not only mathematically rich but also relevant to the teaching profession. Despite previous studies underscoring the significance of problem-solving in teacher education (Morris et al., 2009; Ding, 2016; Leavy & Hourigan, 2018), there remains a surprising dearth of research focused on the challenges that pre-service teachers encounter as they learn to teach mathematical problem-solving. This gap in the literature highlights the need for further exploration into the specific difficulties faced by future educators, paving the way for more effective teacher preparation programs.

2 Theory

A theory particularly helpful when considering mathematical knowledge and its relevance in the classroom is the Theory of Didactical Situations in Mathematics (TDS), grounded by Brousseau (1997) and developed by many others over the past decades. The epistemological assumptions of the TDS are based on seeking answers to the question about the conditions for the acculturation of particular knowledge within the mathematical community (Artigue & Houdement, 2007; Margolinas & Drijvers, 2015). TDS is empirically grounded, epistemologically sensitive in the context of teaching at the university level (Artigue, 2014), and helpful in attempts to understand mathematics teaching. In this study, key notions of TDS illuminate the interactions among the educator, pre-service teachers, and mathematics, with the target knowledge defined as the content that is possible to learn (Brousseau, 1997).

The didactical transposition begins with the establishment of a didactical contract, where the educator shapes the learning environment and clarifies intentions while also encouraging exploration by the learners (Herbst, 2003; Gonzáles-Martin et al., 2014).

Figure 1. A schematic model adapted from Lendínez Muñoz et al., (2023), showing teaching situations including interaction between teacher education and milieu (1), teacher-student dimension between teacher educator and pre-service teachers, PST, (2) and pre-service teachers and milieu (3).



Within the theoretical tradition of the TDS problem-solving is seen as the source of learning mathematics. If the mathematics educator is set out to teach mathematics via problem-solving (Shroeder & Lester, 1989; Erkan & Kar, 2021), the choice of the problems as well as how they are presented and discussed with the learners becomes a part of the didactical situation. Within a didactical situation, the devolution of an adidactical situation, where the learners get an opportunity to attempt the problem, takes place (Brousseau, 1997; González-Martín et al., 2014). The adidactical situation can be seen as a space of learning if the educator manages to provide the important conditions for the pre-service teachers to accept the problem as their own and make active attempts to solve it. In adidactical situations in problem-solving, the learners are required to be active, "to test, reject and progressively adapt and refine their models and solutions thanks to the potential offered by the milieu of the situation in terms of action and feedback, without relying on teacher's guidance, without trying to guess the teacher's expectations" (Artigue & Houdement, 2007, p.3).

In an evolving adidactical situation, educators adopt a facilitative role rather than a direct instructive one, creating an environment where learners are empowered to actively engage in mathematics problem-solving driven by their own curiosity and reasoning (Artigue, 2014). This approach promotes autonomy, encouraging learners to explore concepts, formulate explanations, and develop solutions independently, which significantly enhances their mathematical understanding and critical thinking skills. Such learner-centered dynamics foster a deeper engagement with mathematical ideas, as students are motivated to construct meaning through their own efforts and interactions with peers, however, the role of the educator remains crucial as a facilitator and guide. If, during this process, the teacher feels compelled to re-enter the activity, perhaps by providing hints, clarifications, or corrections it may signal a misalignment between the intended learning goals and the learners' current level of understanding (Brousseau, 1997). This mismatch can indicate that the learners have not yet internalized certain mathematical concepts or that the task is not appropriately calibrated to their prior knowledge. Recognizing such moments allows educators to adapt their strategies, either by scaffolding further or by adjusting the difficulty of tasks, to better support learners' progression without undermining their agency in the learning process. This interplay underscores the importance of balancing learner autonomy with timely pedagogical intervention to foster effective mathematical learning experiences.

In the presented study, *the target knowledge* concerns the content perceived by pre-service teachers as a learning experience, taking the perspective of the learners in the didactical situations they take part of during a university course. The key notions of the TDS provide analytical tools to move from pre-service teachers isolated experiences and actions towards a research approach on problem-solving in teacher education.

3 Aim and question

The aim of this study is to identify the learning experiences of pre-service teachers as they engage with mathematics problem-solving in higher education. Guided by the theoretical framework of Theory of Didactical Situations (Brousseau, 1997), the research question is:

- What target knowledge does pre-service primary teachers identify as challenging in didactical situations related to mathematics problem-solving in higher education?

4 Research design

The study was carried out at a large Scandinavian university, during one semester of a compulsory mathematics course at the final year of a primary teacher education program. The cohort consisted of 49 pre-service teachers, of whom 16 volunteered to participate in the observations and 6 in the follow-up interview. The author of this article is the teacher educator and one of the examiners of the course. For ethical reasons the oral presentations were graded by another examiner. The follow-up interviews were audio recorded. Written consent was given by the participants to use all course material.

4.1 The instructional development

In contrast to previous teaching, the instructional development involving a model including collaborative planning took place, engaging teacher educators in discussions about the selection of the target knowledge suitable for lectures and workshops. The teaching team consists of one mathematician with a PhD in mathematics, an assistant who is currently a PhD student in mathematics, and a mathematics educator with over ten years of experience in teacher education and a PhD in mathematics education. The design included a didactical situation where 20 to 30-minutes oral presentations are given by the pre-service teachers in pairs, and a follow-up with 40 to 60-minutes semi-structured individual interviews. The instructional model designed for this study included didactical situations (Brousseau, 1997) developed in a collaboration between the mathematician and the teacher educators in the course. The target knowledge was chosen collaboratively, with a view to the preservice teachers' preparation to teach a lesson on mathematical patterns. The problems were chosen from the national problem bank as well as course literature. The mathematics teacher educator referred to in this study is the author of this article.

The instructional model consisted of four phases, beginning with a collaborative planning session in which the mathematics education lecturers worked alongside a mathematician to design activities centered on pattern problems. The mathematician reviewed the problems, emphasizing the importance of achieving algebraic generalization in relation to the selected tasks (Taplin, 1998; Rivera, 2010; 2013). He also supported the students in collaboratively deriving the general formula after approaching the problems in an adidactic manner (Brousseau, 1997).

Pre-service teachers were tasked with determining the number of dots in a triangular formation, where each layer adds an increasing number of dots (1 in the first layer, 2 in the second, and so on). The objective was to find the total number of dots for n layers. During a planning session, the lecturers decided to use colored dots as manipulatives to help students visualize the shapes and guide them toward algebraic generalizations through targeted questions. PTs were organized into small groups and provided with manipulatives to construct layers (1, 2, 3, etc.). The questions designed to facilitate exploration included: How many dots are in a triangle with 1 layer? 2 layers? 3 layers? How does the number of dots increase with additional layers? How can this growth be represented as a numerical sequence?

Generalization and algebraic representation were planned to evolve after exploring the pattern hands-on. The PTs could share their observations with the whole group, as they noticed that the total number of dots corresponds to the sequence of triangular numbers:

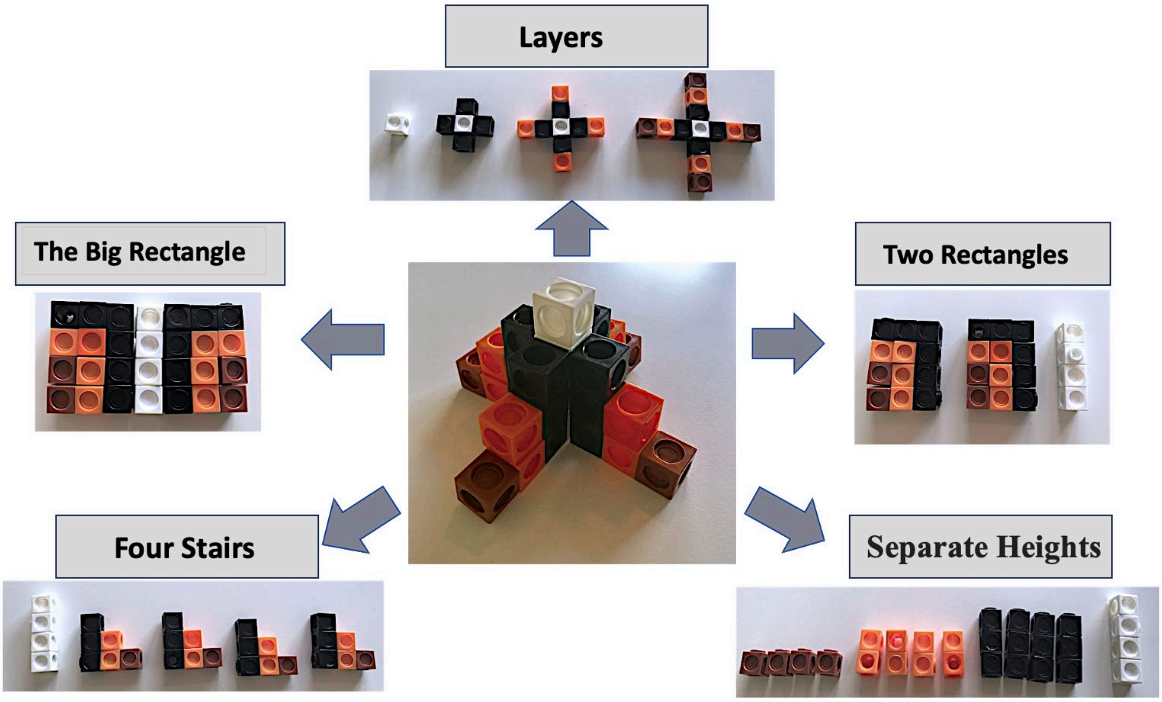
- 1 layer: 1 dot ($T_1 = 1$)
- 2 layers: 3 dots ($T_2 = 1 + 2 = 3$)
- 3 layers: 6 dots ($T_3 = 1 + 2 + 3 = 6$)
- 4 layers: 10 dots ($T_4 = 1 + 2 + 3 + 4 = 10$)

The mathematician assists in articulating discoveries and guiding the group toward a formula for the *n*-th triangular number, through discussion based on observed patterns. Lecturers facilitate a discussion on the significance of algebraic generalization, connecting it to real-world applications and other mathematical concepts.

Solving equivalent problems during the lecture was also considered important and therefore included in the instructional design. The mathematics educator presented different solutions from anonymous pupils in primary school and discussed how the pupils solve such problems. This was done during a workshop with authentic pupils' solutions. During the second phase, the mathematics education lecturer presented the instructional model and discussed the target knowledge with the cohort. The mathematician presented problem-solving from a mathematical perspective. During the third phase, a lecture combined with a workshop were held on the topic of algebra, providing opportunities for pre-service teacher to analyze pupils' solutions and misconceptions with the mathematical content in mind. Different pattern problems were chosen for the mathematics education lecture. The cohort got acquainted with problems that the Swedish National Council (2022) suggests for teaching mathematics in primary school (grades 1-3).

The tower problem involves determining the number of cubes needed to build a tower of a given height, including one that is ten cubes high (n=10). This problem is significant for primary education as it promotes mathematical content and pattern generalization (Rivera, 2010), which enables different ways of discovering and structuring objects (abductive-inductive actions on objects) and translating these discoveries into algebraic generalization (symbolic actions). It encourages pupils to explore and structure objects, leading to algebraic generalizations. Solutions to the problem (Figure 2) include example-related generalization (Lynch et al., 2022), allowing students to derive patterns and formulate a direct equation for height n.

Figure 2. Different methods of approaching the tower problem with example-related generalization.



Problem-solving in the TDS is attached to progression within a specific mathematical area (Artigue & Houdement, 2007). TDS helps to understand problem-solving as a collective process. The mathematical area in this study concerns arithmetic series (the number of cubes in the "Layers" is a 4-difference arithmetic series); the sum of the first *n* natural numbers (the sum of the cubes in the wings of the "Four Stairs"); quadratic series (the second difference in the series of all bricks is constant). The lecturer focused on the mathematical content of the problem, motivated by the likeliness of these patterns being discovered by the pupils. Building upon the lecture, PTs were presented with a set of authentic pupils' solution from 3rd grade. A whole-group discussion with different solutions explicated to the board and discussed in the cohort ended the workshop.

In the third phase, the PTs went into depth regarding the mathematical content of the problem during a lecture on patterns and arithmetic. At the final stage, PTs held a lesson with pupils, using the problem from the problem bank, and presented their lesson in pairs.

4.2 Data collection and analysis

Data were collected during and after the oral presentations of the fieldwork (Phase 4). Each pair of participants delivered a presentation lasting 20 to 30 minutes, wherein they outlined their lesson and provided an analysis of pupils' solutions to the tower problem. Following these presentations, a 20-minute group discussion was conducted with all eight pairs to further explore the problem and their respective lessons. Observations of the eight oral presentations were independently documented by two observers to enhance the reliability of the findings. The observation protocol included a detailed account of the strategies employed to solve the problem.

The six participants who selected the tower problem subsequently engaged in individual post-observation semi-structured interviews, each lasting between 40 and 60 minutes, conducted within a two-week period following the observations. These interviews were recorded and transcribed within one week of the observations, resulting in a comprehensive transcript of 63 pages. The core questions posed during the interviews included:

- What aspects of the instructional model were significant? How were these aspects significant?
- In your presentation, you mentioned [...]. Could you elaborate on that and provide further details about [...]?
- What insights can you share regarding the content of the mathematics lectures?
- Please describe the content related to the tower problem. What lessons do you believe can be gleaned from the tower problem?

Participants were also encouraged to articulate the relationship between the content and their learning experiences, as well as to relate the mathematical problem to their teaching practices in the classroom.

Beginning with the observation protocols, follow-up questions were formulated in accordance with semi-structured interview methodology. The analysis was informed by theoretical frameworks (Creswell & Guetterman, 2019), employing three primary concepts from the Theory of Didactical Situations (TDS) as proposed by Brousseau (1997): *target knowledge, adidactical situations,* and *milieu*. The analysis drew upon semantic content derived from both paired oral presentations and individual interviews. Observation protocols and transcripts were systematically analyzed for commonalities and patterns (Cobb et al., 2003), with a particular focus on identifying instances of target knowledge in the participants' verbal expressions and presentations. Common keywords extracted from the transcripts were synthesized into thematic headings.

5 Results

The thematic analysis resulted in three qualitatively different themes including challenges regarding problem-solving, focusing on the target knowledge such as the direct formula, general solution to a problem, and relevance of new mathematics for classroom practice. In this section, the themes are presented in detail.

5.1 Target knowledge 1: The role of the direct formula

Pre-service teachers faced challenges related to the direct formula for solving the tower problem, often using "The Big Rectangle" method introduced by the mathematician to derive the formula $n(2n-1)$. They relied on this direct formula as a solution for various school-related problems during the workshop. Anna, emphasizing the importance of mathematical explanations, shares her perspective in the following excerpt:

> Anna: I mean, he did not show it [the rectangle] just like that, poof,
> but he said there is a model of something that works for all towers.
> With this height or that height or something. It works in a way.

This excerpt conveys the learning experience of the target knowledge being the direct formula from a didactical situation during the mathematics lecture. Anna was one of the PTs who was able to work out the height and to show the general formula, *"Say the height is 5, 5 multiplied by 9, 45. Right? 10 minus 1. The formula is, mm… here, n(2n-1), mm… I'm saying what I'm showing you here"*. Although "The Big Rectangle" could guide educators to a direct formula, Anna and other PTs relied on a formula from their notes to solve the tower problem. This reflects an instrumental approach, mirroring the solution discussed during the workshop. The focus here is on numerical examples rather than on opportunities for students to explore "The Big Rectangle". Anna believes this target knowledge is useful for calculating the rectangle's area, emphasizing a quick move to symbolic generalization. Similarly, Elias' learning experience involved solving problems using the rectangle as demonstrated by the mathematician.

> Elias: Mmm. I solved [the problem] for five cubes height, like…
> Multiply 4+4+1 by 5, you know what I mean, yeah.
> With the rectangle, I don't want to show now, but it is 45, yeah.

In a similar vein, a symbolic action is displayed in Elias' illustration and Anna of his work with the tower problem. Elias' work with the tower problem exemplifies a symbolic action. He initially demonstrated confidence in finding a numerical solution, like when h=5, but later expressed a desire to reach the general solution, which he recognized as being suitable for 3rd grade. Elias acknowledged the value of the target knowledge he gained during the instructional intervention while also wishing to challenge his students to advance further in their learning. Similarly, all PTs in the study noted that none of their

7-9 year-old students in grades 1-3 approached the tower problem using this method; instead, they focused on direct formulas as the key knowledge in their mathematics learning.

During several presentations, PTs showed how the direct formula, being the target knowledge on a level above grade 3. Frode describes it as "*a lifeline*", supporting his calculations and "*a receipt*" to have learned new mathematics. One example of what was experienced as important during the course is provided by Frode:

> Frode: It was to be able to solve the tower [problem] of course... of course... I solved the problem first myself, my way. I actually did it in class by trying it out. For each figure, yeah. It was more effective his [the mathematician's] way and some other ways probably too.

Frode's comment highlighted the significance of exploring various methods to determine the total number of cubes in an adidactical context. When learners are encouraged to work autonomously, they can discover solutions without educator interference. However, Frode exhibits an instrumental approach to learning, focusing primarily on arriving at a correct solution. He views solving the tower problem as essential for future teaching, indicating that the solution itself is the target knowledge. This reflects an instrumental use of knowledge, as Frode prioritizes effectiveness over the adidactical aspect of the learning situation.

5.2 Target knowledge 2: Algebraic generalization

During the oral presentations in pairs, another explicated challenge was connected to the target knowledge of generalization. The PTs presented various pupils' solutions and discussed them in relation to their teaching, emphasizing that the goal of problem-solving is to learn something to use when solving new problems, "*generalization, to go from one specific situation to any situation, any number of cubes*". All PTs took the target knowledge obtained from the workshop into consideration when designing their own lessons. The direct formula was no longer in focus since PTs considered it to be highly unlikely for a primary school student to come up with it. Frode emphasized the composition of "The Big Rectangle", which could serve as a point of departure in his own teaching to help pupils generalize. He took the drawing, showed a drawing of the tower divided up into four stairs and a height, and a pair of scissors and asked his pupils if he may cut it:

> Frode: It was quiet and when I cut out all the stairs and put them together to form a rectangle... I mean, you can see, they were not equally big, but you can see the rectangle. [...] I said, the most important thing is that now you can see how it will be if we put five in a row and build another, bigger tower, right?

Frode departure is pupil's own solutions and developed it one step further, towards a generalization. Frode's learning experience is explicated further during the interview, exemplifying how it is possible to challenge a pupil towards example-related generalization:

> Frode: The [big] rectangle is one way, but the two rectangles work fine.
> If you have a column here... Well, that is 100, the two rectangles, it is the best way to explain... Hm... It? Yes, it is, hm... To pupils.
> I mean, not to you, to pupils.

The comment indicates that that Frode reflected on potential approaches to solving the tower problem with a general solution. After resolving the issue, he used a manuscript of his lesson plan with his pupils. However, instead of encouraging exploration of the scenario's adidactical potential, Frode focused on the students' ability to reproduce a correct solution for generalization. The target knowledge mirrored what was demonstrated during the mathematics lecture, chosen because Frode felt comfortable with it. While he aimed to understand the problem deeply, he planned to discuss it superficially, offering hints to guide toward the solution he called "The Big Rectangle", while preparing them for the limitations of "The Four Stairs". In a later group discussion, Frode highlighted the connection between the tower problem and other pattern problems from the mathematics lecture.

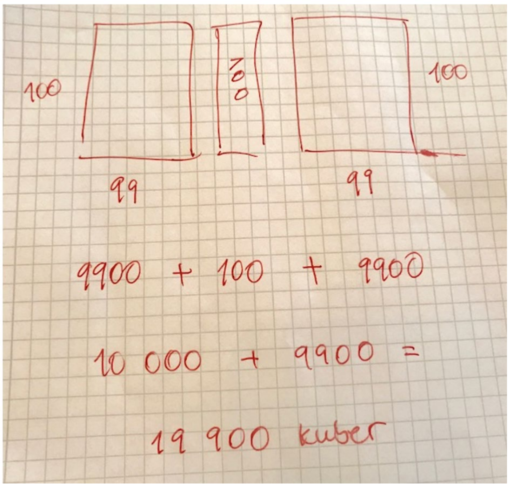
In contrast, some PTs, like Bea, did not see the value in general solutions. Although her pupils could not solve the tower problem, Bea was more focused on the lack of generalization than on the correct answer. In her presentation, she showcased pupils' attempts for n=100, asking them to confirm the method's validity and to explain why they obtained 98. However, generalization did not occur as she intended, as she sought to understand how pupils derived the total of 98.

> Bea: But I know it is much harder to write the expression for *n* than for *(n-1)* times *n*, it is the same again, *(n-1)* times *n* ... ehm, well, you know, no pupil in my second grade can do that! But if they, ehm, found a way that I know works, then I know it works.

The excerpt above displays a critical reflection over the adidactical situation, as when the PTs use their own calculations to understand pupils' solutions, however, Bea mentioned that she did not expect the pupils' solution to identical to her own. Her own certainty of the solution, she said, made it possible to evaluate pupils' solutions and lead them towards a generalization. The general solution helps to foresee the answers generated by the pupils.

Several PTs acknowledged the concept of generalization in primary school mathematics to differ from mathematical generalization, not necessary connected to a direct formula and when it comes to generalization, the expectation on the pupils could not be related directly to the formula. Pupils in early school-grades find other way of expressing generalization (Figure 3).

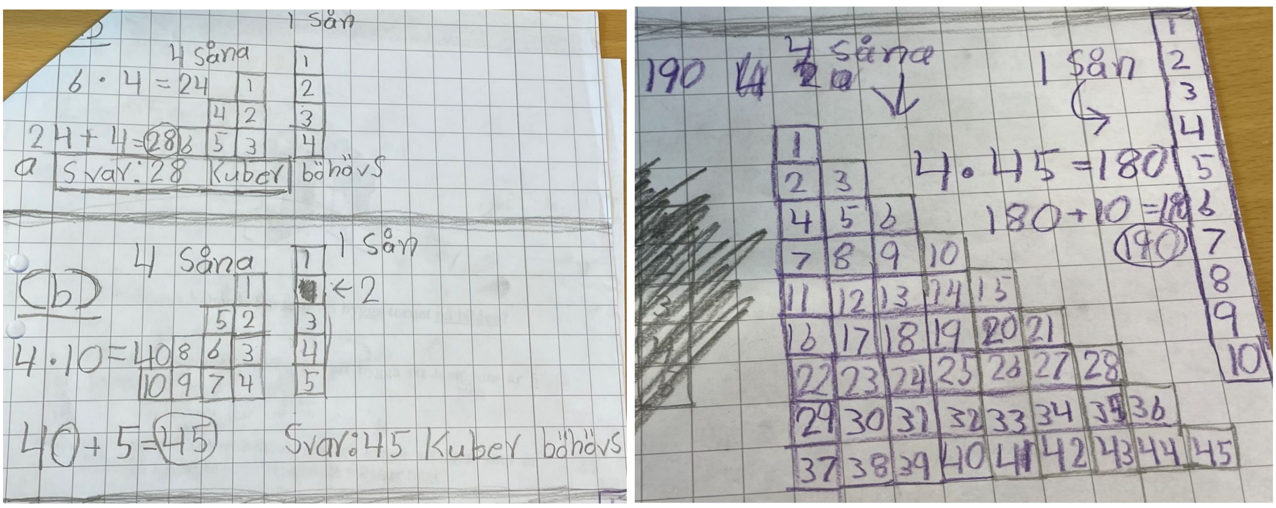
Figure 3. One of the most advanced pupil's solutions as it was presented by Bea during the oral presentation.



In one of the unexpected solutions from a third grade pupil (Figure 3) the method of solving the problem for *n*=100 by using example-related generalization is applied. By using "The Two Rectangles" the pupil in this example has found a way to generalize from earlier examples, *n*=4 and *n*=5. The pupil shows understanding of the total number of cubes in the middle (100) and can alternate between the number of cubes and the height of the rectangles. The graphical representation and the calculations made by this pupil reflect the way the pupil generalizes that the short side of the rectangle is *n-1* without expressing it algebraically.

Similarly, in a didactical situation in Anna's classroom, she formulated the problem to determine the number of cubes for n=4, n=5 but also "*for a tower twice as high as the one with five cubes where the height is 5*". In her presentation (Figure 4), a pupil's solution included calculations of a tower with *n*=4, *n+1* and *n*=10. She illustrated a type of generalization possible in second grade.

Figure 4. A pupil's solution to the version of the tower problem in Anna's classroom.



The majority of Anna’s students demonstrated example-related generalization by using visual representations of rectangles rather than algebraic expressions involving variables. In Figure 4, a student systematically employs a specific height, positioned on the left, and adds cubes according to the formulation of the sub-question presented by Anna. The generalization that PTs highlight during the oral presentation is experienced incrementally, and both Anna and Bea believe it does not effectively enable the student to solve the problem for an arbitrary number of cubes at that height. Nevertheless, this approach may still serve as a viable method, as it can be applied to towers of varying heights, thus representing a form of generalization.

5.3 Target knowledge 3: The relevance of new mathematics

> Frode: Everything I do I relate to teaching. Or, you know, I try to, anyway. In mathematics, in this course, I put the pupils first. Pupils come first. How will they do the tower problem?

Frode emphasizes the centrality of teaching in his approach: *“Everything I do I relate to teaching. Or, you know, I try to, anyway.* “He underscores the importance of prioritizing students in the learning process, stating *that “In mathematics, in this course, I put the pupils first. Pupils come first”*. This perspective raises an important question: How will students tackle the tower problem? This quote encapsulates the third theme, highlighting the challenges associated with the evolution of the adidactical situation. Pre-service teachers in the study grappled with understanding the relevance of new mathematics in their teaching practices. Many expressed a deep commitment to grasping the significance of this target knowledge, often experiencing *“aha moments”*. For example, Anna's realization of the mathematical content's importance occurs when her students engage with the problem, and she recognizes the need to keep pace with their understanding:

> Anna: [...] in the classroom, when I got the solutions, I checked if the pupils were doing the right thing. Also, during my lesson, when a pupil tried with another height than 4. They tried 6, and one even tried 8, so I had to check quickly. But how?

To ensure that the pupils were on the right path, Anna attempted to foster generalization by presenting The Big Rectangle to one pupil, as it closely resembled his own solution: “At the back of my mind, I had the mathematician’s explanation”, she reflected. “Aha”, the pupil exclaimed. "It was one of those moments, you know... an ‘aha moment’!” Anna’s learning experience highlights the classroom relevance of the targeted knowledge, as she connected her own explanation to the pupil’s reasoning about the problem. Her comments reveal her understanding of algebra in the context of the problem, demonstrating how she masters and applies her new mathematical knowledge to enhance her teaching, thereby boosting her confidence in the mathematical content she discusses with her students.

With the mathematician's guidance, she perceives the mathematics as relevant because she is able to convey the content effectively to her pupils. *"I felt like… our mathematics lecturer is like a mathemagician!"*, she remarked. Anna's reflections on the relevance of the content suggest that she is developing new skills related to problem-solving and the application of mathematical methods.

Similarly, David recounted an instance in which he utilized the target knowledge in his practice, questioning the relevance of the content:

> David: No, but what do we need this for?" he pondered. "Now, y is the total number of cubes. I constantly question everything I do and everything he [the mathematics lecturer] does.

David found motivation in the educators' reminder that PTs *"are going to teach in grades 1-3, not be pupils in grades 1-3."* He emphasized the necessity of knowing how many cubes to use when planning his lessons, which led him to recognize the importance of a direct formula. This reasoning indicates an awareness of the content's relevance as target knowledge. Furthermore, the significance of understanding the purpose of new mathematical content became apparent, affirming the necessity of learning new mathematics. This knowledge is essential for enriching the classroom environment with students' ideas and explanations, thus opening up an adidactical dimension.

After engaging with the problems mathematically, David acknowledges that the process of problem-solving and learning advanced mathematics is relevant and closely tied to his future profession:

> David: I realized that a teacher needs to know much more mathematics than his pupils, to face the challenges around the classroom practice – not merely explaining a problem's solution to the pupils.

Despite struggling with the mathematical content during his oral presentation, David noted that he felt less uncertain than in his previous math course. Through problem-solving and classroom experiences, he began to reflect on the importance of new mathematical knowledge for teachers. All PTs learned new mathematics; however, not everyone agreed that this knowledge automatically benefits the learning environment, as it does not equip them with the skills to evaluate students' solutions. Bea, unlike David, questioned the relevance of all mathematical concepts related to the tower problem, emphasizing the importance of understanding the differences in perspective between teachers and third-grade students.

> Bea: The next problem came, with bricks and all that, it just, it ended!
> My brain sort of stopped, especially when he was talking about multiplying [into the brackets].

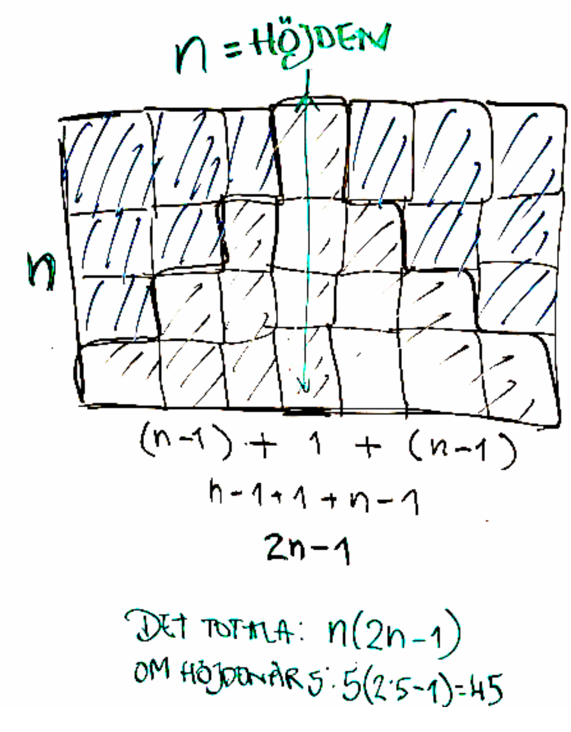
The excerpt illustrates how the algebraic generalization can hinder PTs in this study from coming forward in her own thinking when solving a new problem, describing the mathematics as superfluous and contradictive of her expectations. In the data, it is evident how Bea questions the relevance of the target knowledge in relation to her own teaching.

> Bea: And of course, one pupil asked what would happen if there were 1 million cubes to build from, you know how kids are? [long laughter].

The last part of the sentence, “you know how kids are”, shows her unawareness of the possibility of symbolic actions. Bea is missing out pupils’ ability of generalizing, and the opportunity of the adidactical situation to take a pupil’s solution as the point of departure. Bea struggles when following up a solution when a group of pupils solve the problem for n=100 “*with two rectangles”, that is 99·100 + 99·100 + 100*, dealing with something unexpected in the problem-solving situation.

Some PTs discussed whether the target knowledge should focus on the mathematical content for solving problems or on teaching through problem-solving. Clara stressed the need for teachers to know more mathematics than their pupils. Specific teaching situations, when pupils ask for solutions, could limit opportunities to explore several dimensions of the problem. Additionally, mathematics helped facilitate reflections, demonstrating solutions at the board.

Figure 5. Clara’s written work during her oral presentation.



In Clara’s case, the new mathematics she encountered was the direct formula that establishes a connection between target knowledge 1 and 2. To illustrate her understanding, Clara represented the height of the tower with the variable n (as shown in

Figure 5) and derived the total number of cubes using the formula *n(2n−1)*, specifically calculating the total for a height of 5. Clara's learning experience is intricately linked to her teaching approach, as she guided her pupils to uncover patterns and construct general solutions, adapting her methodology in line with the presentation of the mathematician. Later, during her oral presentation, Clara effectively integrated the direct formula from her mathematics lecture into her own teaching practice. She explained that the total number of cubes in the tower is determined by the formula *n(2n−1)*, which she expressed algebraically based on both the 2D visual representation of the tower and a numerical example for the case when the height is 5.

Although Clara initially simplified the expression almost correctly, she acknowledged a remark from her peers regarding an error in her calculation. As a result, she erased the incorrect computation and made the necessary adjustments, demonstrating her willingness to learn and improve.

6 Discussion

This study aimed to explore the learning experiences of pre-service teachers as they engage with mathematics problem-solving in higher education, guided by the theoretical framework of the Theory of Didactical Situations (Brousseau, 1997). The research focuses on two main questions: first, what target knowledge do pre-service primary teachers identify as challenging in didactical situations related to mathematics problem-solving? Second, what knowledge do these utilize when designing problem-solving situations for their pupils? By addressing these questions, the research seeks to contribute with knowledge about the complexities of mathematical understanding and teaching strategies among teacher educators, ultimately contributing to the enhancement of teacher preparation programs in mathematics education. Despite the efforts, pre-service teachers did not emphasize key aspects of algebraic knowledge, such as pattern generalization (Rivera, 2010), in their own teaching, focusing on direct formulas, reflecting an instrumental (Ambrose, 2004) and self-oriented (Zazkis & Leikin, 2010) approach to learning. Additionally, they did not view pattern construction as a “subjective and constructive activity” (Rivera, 2010, p. 298), missing opportunities for algebraic generalization.

Also, this study reveals a disconnect between the mathematics teachers' learning preferences and the practical applications of mathematics in the classroom. Pre-service teachers relied on direct formulas for problem-solving, but realized their limited applicability in teaching contexts, deeming them inappropriate for their pupils’ age levels. Critically, this focus on direct formulas may hinder pre-service teachers from engaging with the adidactical dimensions of teaching, which can facilitate example-related generalization (Lynch et al., 2022). To enhance instructional design, there is a pressing need to create learning experiences that enable pre-service teachers to engage with patterning strategies more effectively. This can be achieved by allowing them to visually explore the regularities within complex problems (Rivera, 2013), such as the tower problem involving bricks in this study, which not only draws on numerical experiences but also encourages following a logical sequence. By fostering an environment where PTs can investigate various pattern structures before resorting to the more advanced strategies typically reserved for algebraic generalizations, strategies that primary school pupils may find challenging, they can build a more robust foundation in mathematics. Ultimately, by prioritizing visual and experiential learning in the exploration of mathematical patterns, pre-service teachers will not only improve their own understanding but also be better equipped to teach these concepts to their pupils. This shift in focus from immediate solutions to a more exploratory method of understanding patterns will likely lead to a richer problem-solving experience, fostering greater mathematical insight and creativity.

Despite the dominance of an instrumental focus (Ambrose, 2004) in pre-service teachers' learning experiences, the relevance of the target knowledge in relation to teaching is considered challenging, as is evident in the third theme, as well as in previous research (Weber et al., 2023), a challenge on a higher psychological level (Leikin et al., 2017), as the reflections on the different nature of the target knowledge involve aspects of both learning new mathematics and learning how to teach mathematics. The fixation on the relevance of mathematics can hinder the developing the adidactical situations. The findings suggest a potential risk of an instrumental focus in university-level mathematics courses; however, this study does not investigate the underlying factors contributing to this risk. An instrumental approach may suggest a superficial engagement with mathematical concepts; however, the specific contextual factors that contribute to this behavior remain unexamined. Furthermore, although pre-service primary teachers may replicate solutions demonstrated by their educators, the implications of this imitation are nuanced and warrant deeper investigation. To further understand these behaviors, it is essential to explore the motives behind them, the pedagogical frameworks employed by educators, and the impact of institutional expectations. Conducting further research on instructional development could yield valuable insights that enhance teacher education practices and improve mathematics instruction overall. Thus, additional studies are crucial for advancing our understanding in this domain. For example, conducting focus group interviews with pre-service teachers as they transition into in-service roles, combined with observations of their teaching practices during problem-solving sessions, would represent a compelling avenue for future research.

Teaching via problem-solving (Shroeder & Lester, 1989) was not evident as a target knowledge in this study, since none of the informants discussed problem-solving as a way of teaching mathematics or contributing to considering mathematical content in a new way. This is not surprising, since "teacher telling" is easier than "teaching through problem-solving" (Masingila, Olanoff & Kimani, 2018). The second theme, which explores challenges related to the generalization of the problem, highlights an instrumental approach to problem-solving. In contrast, the third theme addresses the difficulties of creating adidactical situations, which are closely linked to pupils engaging in independent problem-solving. This suggests that PTs may conceptualise teaching primarily in terms of their classroom activities (Ambrose, 2004), neglecting the focusing on working autonomously to find solutions to problems without predefined methods. A tendency among PTs seems to be a rush towards an algebraic generalization, instead of following the process between abductive-inductive action on objects and symbolic actions, described by Rivera (2010).

Previous research stresses that the target knowledge of the mathematical content and teaching approaches are often intertwined (Ball, Thames & Phelps 2008; Murphy, 2012; Masingila et al., 2018), as they are in the third target knowledge. Awareness of moving away from the dominance of the direct formula is useful in teacher education, since being uncertain of the mathematical solutions to a problem can limit teachers' actions in the didactical situation (Ball, 2009).

This study has several methodological limitations. Firstly, it lacks classroom data from educators, providing only one example of integrating mathematical content into school-related problems. The findings are based on retrospective accounts rather than direct evidence of classroom practices. However, these perspectives are valuable, helping mitigate researchers' overinterpretation (Cohen et al., 2018), reflecting on what pre-service teachers see as important in their teaching. This approach captures the essence of their experiences in detail. The sample size limits generalizability, offering insights into pre-service teachers' challenges that can inform both future large-scale studies and teaching education practice. Further, the dual role of the researcher as both educator and data collector must be acknowledged. The author made concerted efforts to balance these roles, being mindful of their closeness to the research subjects (Cohen et al., 2018). Future research could explore how teacher educators select target knowledge for pre-service teachers. Although Creswell and Guetterman (2019) caution against generalizing from descriptive themes, the target knowledge identified in this study can be communicatively validated by readers.

Conclusions and outlook

In conclusion, this study highlights the challenges of navigating mathematics problem-solving instruction for pre-service teachers in teacher education. While the problems such as the Tower Problem are often seen as highly relevant to pupils, pre-service primary teachers may find themselves mimicking the solutions provided by their university instructors instead of engaging in authentic problem-solving processes. As Bousseau (1997) shows, if the solution of a problem relies too heavily on the educator, genuine mathematical learning cannot occur, and the findings of this study serve as a crucial reminder of the need to strike a balance in teaching approaches, ensuring that students focus on developing their problem-solving skills rather than merely relying on direct formulas as the primary knowledge. Thus, this study underscores implications for teacher education, particularly in comprehending pre-service teachers' perspectives as learners. The reliance on predetermined solutions highlights the need for ongoing dialogue concerning essential mathematical knowledge (Ambrose, 2004; Weber et al., 2023). Collaborative efforts between mathematicians and educators are crucial (Leikin, Zazkis, & Meller, 2017; Livy, Herbert, & Vale, 2018) and should prioritize the alignment of problem selection to enhance the relevance of content in teacher education.

While prior research indicates that problems deemed relevant for specific school grades can also serve as valuable resources in teacher education (e.g., Weber et al., 2023), thereby contributing to pre-service teachers' understanding of mathematics (Livy, Herbert, & Vale, 2018), the findings of this study raise concerns. Specifically, promoting such problems may inadvertently reinforce an emphasis on instrumental learning, potentially detracting from deeper conceptual understanding. To address these challenges, further studies can go deeper into the learning opportunities presented through problem-solving for pre-service teachers. Research can further explore innovative approaches that encourage critical thinking and foster a more profound engagement with mathematical concepts. Investigating the interplay between problem relevance and pedagogical strategies will be essential for developing frameworks that support meaningful learning experiences. Further, longitudinal studies could provide insights into how pre-service teachers evolve in their understanding of mathematics over time, particularly in relation to their experiences with problem-solving in teacher education programs. Ultimately, a comprehensive exploration of these areas will not only enhance the quality of teacher education but also equip future teachers with the skills necessary to inspire their own pupils' mathematical journeys.

This study highlights the necessity for innovative pedagogical strategies that promote deeper cognitive engagement among pre-service teachers. By shifting the emphasis from rote memorisation of solutions to a more exploratory and inquiry-based approach, teacher education programs can create an environment in which pre-service teachers actively construct their understanding of mathematical concepts. This can be accomplished by integrating real-world problems that demand critical thinking and creative problem-solving, encouraging students to analyze, discuss, and collaborate on a variety of mathematical scenarios. Furthermore, mentorship programs that pair pre-service teachers with experienced educators can offer invaluable insights into effective instructional practices, fostering a more nuanced understanding of how to facilitate authentic problem-solving experiences.

In summary, this study identifies challenges pre-service teachers face in teaching mathematical problem-solving. While individual learning is important, overemphasis on it can obscure critical adidactical dimensions of the learning environment. These challenges extend beyond individual student learning and may overlook the collaborative nature of the process. The research underscores the importance of adidactical situations in problem-solving. Erkan and Kar (2021) note that without this focus, teaching may devolve into mere information transmission, detracting from more effective problem-solving approaches. Addressing these challenges is essential to create engaging learning opportunities in mathematics teacher education. As the educational landscape continues to evolve, it is imperative that teacher education frameworks adapt to prioritize dynamic learning opportunities, ultimately equipping future educators with the skills and confidence needed to inspire their own students in the complexities of mathematics. Hopefully, by doing so, not only teacher preparedness is enhanced, but also contributes to a more profound and lasting appreciation of mathematics among future teachers.

Research ethics

Artificial intelligence

Artificial intelligence has been used in the writing of the article. ChatGPT Edu was used for language editing, including grammar check, spelling and correction of typographical errors. It did not contribute to the scientific content of the article.

Informed consent statement

Informed consent was obtained from all research participants.

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Conflicts of Interest

The authors declare no conflicts of interest.

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