

Teachers' beliefs and instructional quality of Linguaging in primary school mathematics

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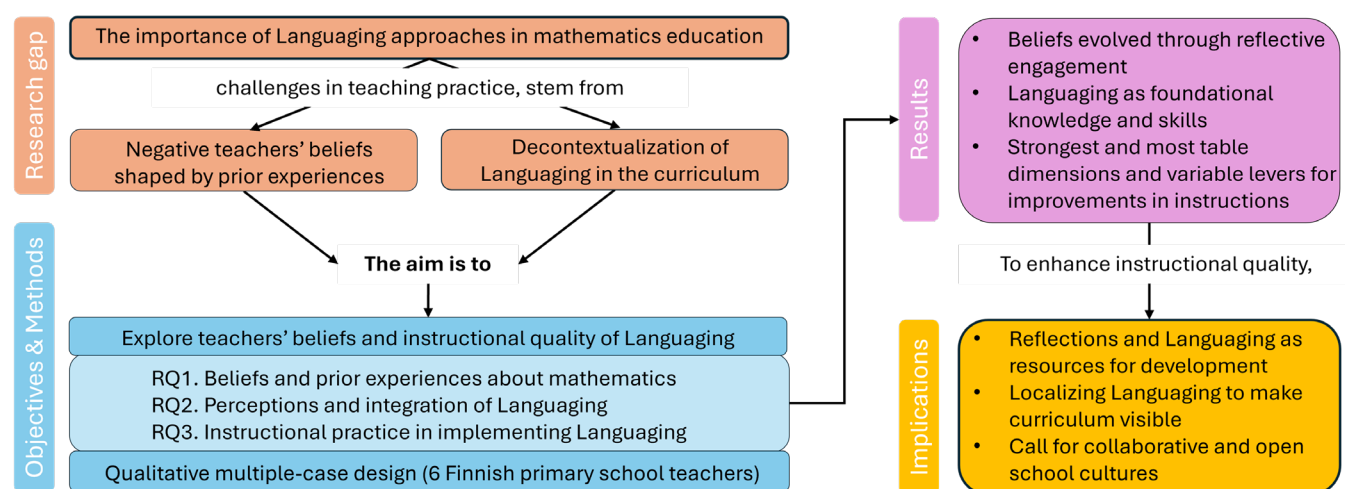
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Abstract: Despite its inclusion in the Finnish national curriculum, the Linguaging approach—teaching mathematics through diverse tools and interactive activities—remains challenging to implement in practice. Teachers' beliefs shaped by prior schooling and the decontextualization of Linguaging in local curricula can hinder interpretation and enactment. This study examined primary teachers' beliefs development, their interpreted curricula around Linguaging, and the enacted curricula as evidenced in classroom instructional quality. Using a qualitative multiple-case design, we purposively sampled six teachers who actively integrate Linguaging; data collection included surveys, interviews, classroom observations with field notes. Qualitative data were analyzed thematically within and across cases; quantitative rubric scores (1–4) were summarized with medians. Findings indicate that teachers' beliefs are shaped by personal learning histories and evolve through reflective engagement with Linguaging. The interpreted curricula commonly emphasized foundational knowledge and skills, as well as the relevance of mathematics in daily life. The enacted curricula showed high and stable performance in functional learning environment, purposeful material use, and teacher-led multiple representations, alongside lower and more variable performance in student-generated representations, sustained conceptual press, and differentiation. The discussion identifies implications for teachers, teacher educators, local curriculum developers, and policymakers.

Keywords: Linguaging, teachers' beliefs, instructional quality, mathematics education, primary school

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1 Introduction

High-quality teaching is crucial for bridging the gap between educational goals and student outcomes in mathematics (Teig et al., 2024). In primary mathematics education, these goals emphasize equipping students with the competence to apply mathematical knowledge and skills to real-life contexts. The Finnish national core curriculum (NCC) for basic education 2014 (Finnish National Agency for Education [EDUFI], 2016) defines mathematical knowledge as conceptual understanding and mathematical skills as the ability to articulate and share mathematical thinking with others. To advance these aims, the *Languaging approach* has been introduced to encourage students to express mathematical thinking through natural, mathematical symbolic, pictorial, and body language within interactive learning contexts (Joutsenlahti & Kulju, 2015; Perkkilä & Joutsenlahti, 2021). Languaging promotes multiliteracy by integrating diverse representational modes to deepen students' conceptual understanding and communication skills (Kalantzis et al., 2012; Perkkilä & Joutsenlahti, 2021).

Despite these pedagogical advancements, notable challenges persist. In Finland, students' interest, knowledge, and skills in mathematics have shown a gradual decline from primary to secondary school, with growing disparities across socioeconomic, cultural, and gender groups (Ministry of Education and Culture, 2023; Ukkola et al., 2025). In response, teacher education programs have prioritized the enhancement of instructional quality. However, achieving meaningful and sustained changes in teaching practices continues to be a considerable challenge (Hart, 2004; Purnomo et al., 2016; Russo et al., 2020). A major obstacle lies in the deeply embedded nature of teachers' beliefs and attitudes toward mathematics teaching and learning, often shaped by their own schooling experiences (Hannula, 2020; Liljedahl & Andrà, 2020). Negative past experiences can lead to low self-efficacy undermining teachers' confidence in their ability to teach mathematics effectively (Artemenko et al., 2021; Kim et al., 2023; Schaeffer et al., 2021). These beliefs can impede the adoption of innovative pedagogical strategies and perpetuate traditional instructional practices.

Another challenge is the limited curricular contextualization of Languaging. The NCC identifies multiliteracy as one of seven key transversal competencies and provides only a general framework for Languaging. A nationwide review of hundreds of local curricula revealed almost no "rational-level" elaboration of this framework, so teachers often reduce Languaging to simply "using language" in class (Palsa & Mertala, 2022). Traditionally, language use in mathematics has emphasized mastery of formal mathematical language, such as numeric, symbolic, and pictorial forms, often confined to textbook-based formats. By contrast, Languaging calls for an active, iterative process in which teachers design, enact, and continuously refine lessons that move flexibly among multiple representations. Without clear curricular anchors and exemplars, teachers may struggle to recognize when and how to orchestrate these representational shifts, to align tasks with multiliteracy goals, and to evaluate students' progress across modes.

This study was prompted by the question: “What can we learn from primary mathematics teachers who actively integrate Languageing in their classrooms?” To address this question, we adopted Teig et al.'s (2024) three-tier framework for teacher practice. First, orientations (*beliefs*) are relatively stable dispositions that guide decisions. Second, instructional planning (*interpreted curriculum*) refers to concrete decisions about what to teach and to what depth. Third, instructional enactment (*implemented curriculum*) comprises observable classroom behaviors, including how content is actually taught and assessed. Improving teaching practice, therefore, requires attention to both the beliefs layer and the practical layer, while also recognizing systemic factors that can decouple intentions from classroom reality.

Building on these insights, it is essential to close the gap between national curriculum goals and teachers' beliefs, planning, and enactment. Using purposive sampling, we invited six Finnish primary teachers who had experience with the Languageing approach and were actively using it in their classrooms. To obtain a rich picture of each teacher's practice, we adopted a qualitative multiple-case design and report descriptive findings.

Accordingly, this study explores the following research questions:

- RQ1. What beliefs about mathematics and prior learning experiences do Finnish primary school teachers hold?
- RQ2. How do they perceive and plan Languageing into the interpreted curricula?
- RQ3. How are their interpreted curricula implemented in their instructional practice involving Languageing?

By addressing these questions, the study demonstrates how experienced Languageing teachers translate pedagogical frameworks into day-to-day practices. The findings will help (a) teacher education programs weave Languageing more explicitly into course work and practica and (b) curriculum designers and school administrators provide clearer examples and supports for bringing Languageing into everyday mathematics lessons.

2 Literature review and conceptual framework

2.1 Languageing in mathematics education

The NCC (EDUFI, 2016) promotes active, self-regulated learning, emphasizing that students construct knowledge both individually and through interaction with others. Learning is supported through positive emotional experiences, multimodal thinking, and diverse instructional methods. Accordingly, learning environments are intentionally varied to facilitate cognitive learning while also engaging students emotionally through multi-sensory experiences, language, and movement.

This pedagogical orientation aligns with two foundational learning theories (Stewart, 2021): (a) constructivism, which emphasizes individual cognitive development (Piaget, 1952), and (b) social-constructivism, which highlights scaffolded learning mediated by

cultural and social interaction (Vygotsky, 1978). These perspectives have long informed key questions in education, such as “What is learning?” and “How does learning occur?” While Piaget focuses on learners internalizing mathematical concepts through active exploration and reflection, Vygotsky emphasizes the importance of communication and collaborative problem-solving in the development of mathematical understanding.

Recent research demonstrates that mathematical knowledge is often co-constructed in group settings, particularly when students verbalize reasoning and negotiating solutions (Olsson & Granberg, 2024; Perkkilä & Joutsenlahti, 2021). This interplay between individual cognition and social interaction highlights the value of *Languaging* which bridges both dimensions of learning (Kupari, 2007).

As a multimodal approach that supports multiliteracy, *Languaging* encompasses the following modes (Joutsenlahti & Perkkilä, 2024; Joutsenlahti & Rättyä, 2015):

- Natural language (spoken and written communication)
- Mathematical symbolic language (numerical and algebraic representations)
- Pictorial language (visual models, diagrams, and drawings)
- Body language (concrete manipulatives, digital tools, and physical interactions)

These dimensions are explicitly reflected in the Finnish national curriculum for primary mathematics education (EDUFI, 2016, p. 136):

- Grades 1–2: “Teaching and Learning improve the pupils’ ability to express their mathematical thinking through concrete tools, speech, writing as well as drawing and interpreting images.”
- Grades 3–6: “The instruction supports the development of the pupils’ skills in presenting their mathematical thinking and solutions to others in different ways and with the help of different tools. Solving a wide range of problems independently and in a group and comparing different solutions are important in teaching and learning.”

Beyond cognitive and multimodal goals, the curriculum emphasizes fostering students’ emotional engagement with mathematics. Play-based and game-based activities are encouraged to promote enjoyment, confidence, and a willingness to experiment with mathematical ideas in low-stress, exploratory settings (Kupari, 2007). Implementing *Languaging* thus requires teachers to plan across its multiple modes, organize varied learning environments and working formats, differentiate for diverse learners, assess students’ understanding, and use those data to sequence subsequent lessons.

From a sociocultural perspective, particularly Vygotsky’s, learning is mediated by culturally available tools and signs; while the mechanism is universal, its forms vary. *Languaging* operationalizes this through multiple semiotic modes, whose classroom enactment is shaped by local contexts (e.g., task types, discourse norms, tool availability). This universal–local distinction frames our cross-context implications and shows how Finnish practices can be adapted to diverse linguistic and cultural settings.

2.2 Teachers' roles in curriculum implementation

Curriculum is operationalized through multiple levels. At the national level (Level 1), broad educational objectives and priorities are established. These are adapted regionally through local curricula (Level 2) and further translated into everyday classroom practices through teacher-level implementation (Level 3). Ideally, this multi-level alignment leads to improved student outcomes (Level 4). However, its effectiveness depends largely on how teachers mediate and enact curriculum goals in their instructional decisions (Cai & Hwang, 2021). As key agents of implementation, teachers interpret and translate the curriculum into classroom practices that support equitable and meaningful learning. Yet, variation in curriculum planning across municipalities and schools makes short-term comparisons of student achievement difficult. Moreover, local curricula provide limited guidance for teachers (Palsa & Mertala, 2022). For these reasons, this study focuses on the relationship between Level 1 (national curriculum) and Level 3 (teacher practice), with particular attention to how teachers interpret and enact national goals in mathematics instruction.

Finnish teachers are well-positioned for this role due to their rigorous education and professional autonomy (Krzywacki et al., 2016; Sahlberg & Walker, 2021). Teacher education programs in Finland emphasize research-based methods and individual supports, preparing teachers to design practices aligned with national curriculum goals. While programs vary across universities, several shared principles underpin effective mathematics instruction (Krzywacki et al., 2016): (a) fostering positive emotions, motivation, and self-concept as a mathematics learner; (b) developing conceptual understanding through concrete, didactic, and multimodal approaches; (c) promoting problem-solving, reasoning, and critical thinking for real-world applications; and (d) supporting diverse learners, including those with learning difficulties and from culturally varied backgrounds. Given this pedagogical foundation, it is expected that Finnish primary teachers are familiar with *Languaging* as part of their instructional repertoire.

Despite this foundation, empirical studies suggest that many Finnish classrooms still rely heavily on textbooks and traditional, numerically focused assessments (Hemmi et al., 2018; Krzywacki et al., 2016; Lehtonen, 2022). This ongoing reliance on conventional practices indicates that the curricular flexibility afforded to teachers does not always lead to innovative instruction. As a result, teacher education programs must explicitly address the gap between their preparation and actual classroom implementation of *Languaging*.

2.3 Teachers' beliefs and instructional strategies

A substantial body of research has documented a persistent mismatch between constructivist beliefs and traditional practices. This inconsistency is often attributed to the complex, dynamic nature of beliefs (Kim et al., 2023; Russo et al., 2020; Teig et al., 2024). Within this complexity, certain trends are evident: procedural beliefs tend to align with rule-based, structured instruction, while beliefs centered on conceptual understanding are more

closely associated with active and collaborative learning environments (Pikk et al., 2025).

Historically, early studies approached mathematical beliefs from a behaviorist perspective rooted in social psychology (Thompson, 1992). By the 1970s, research shifted toward examining beliefs as part of the cognitive structure of knowledge, particularly within metacognitive processes related to mathematics learning (Pehkonen, 1993; Thompson, 1992). Initially, teachers were often seen as organizers of fragmented learning experiences; however, later studies emphasized attitudinal and emotional dimensions in translating beliefs into instructional practice.

Pehkonen and Törner (1995) conceptualized belief systems as multidimensional, encompassing views about mathematics, self-perception as a teacher, and beliefs about teaching and learning. Thompson (1992) noted that these systems are malleable and shaped by ongoing experience. Although the link between belief and practice is neither straightforward nor linear, beliefs significantly influence the quality of mathematics instruction (Perkkilä, 2003). Kuhs and Ball (1986) identified four broad teaching orientations. Learner-focused teachers function as facilitators, reflecting a constructivist stance. Content-focused teachers prioritize conceptual understanding, aligned with a Platonist view of mathematics as a system of relationships (Ernest, 1989). Performance-focused teachers emphasize procedural mastery, reflecting an instrumentalist view (Ernest, 1989), while classroom-focused teachers highlight group learning over individualized instruction.

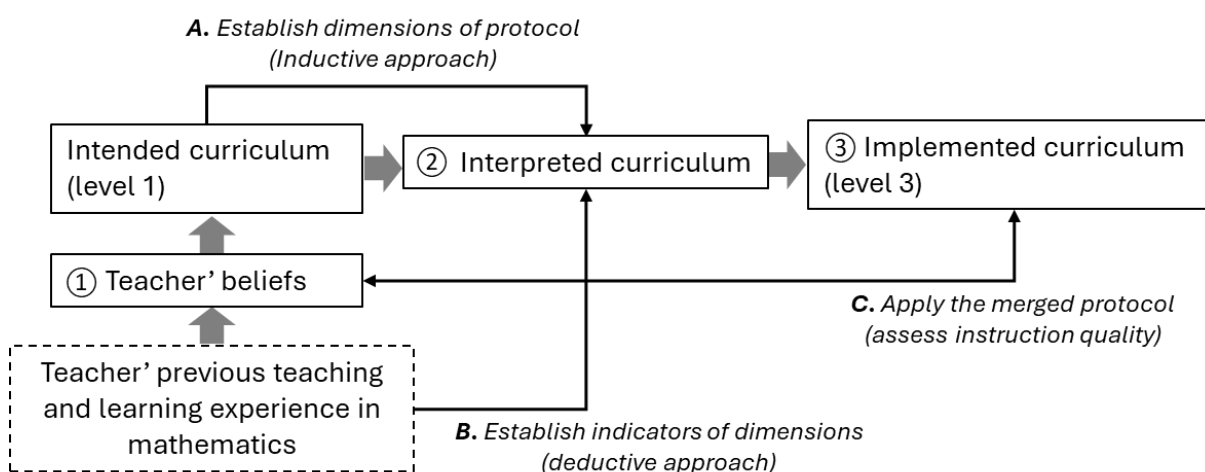
Changes in beliefs are possible but typically require extended experience and structured reflection. For instance, a recent intervention study (Kim, 2024) found that a mathematics course integrating *Languaging* positively influenced pre-service teachers' beliefs. However, those with negative experiences in school mathematics showed fluctuating intentions to implement *Languaging* in their future classrooms—a pattern consistent with prior reports (Bekdemir, 2010; Sánchez Mendías et al., 2020). Thus, while a single course may raise awareness and willingness to adopt new approaches, durable belief change appears to require ongoing support and practice.

Building on the multidimensional belief framework of Pehkonen and Törner (1995), the procedural-conceptual distinction of Pikk et al. (2025), and the student-focused orientation of the NCC (EDUFI, 2016), this study examined two interrelated belief objects: teachers' self-perceptions—as former learners and current instructors—and how these shape attitudes toward mathematics teaching and learning; and the alignment between these beliefs and enacted instructional strategies. Because past learning experiences in mathematics are deeply embedded, we adopted a qualitative lens to uncover how teachers negotiate those histories as rich resources within new pedagogical contexts. This approach enables nuanced cross-case comparisons of how beliefs evolve through engagement with *Languaging* and how such evolution relates to advancing curricular goals.

2.4 Conceptual framework for research design

Based on the theoretical foundation, the study developed its conceptual framework through two main procedures: establishing the research questions (RQ1–RQ3) and designing the observation protocol for evaluating instructional quality (Processes A–C).

Figure 1. Conceptual framework guiding the study design



Note. The three main concepts in the framework align with RQ1–RQ3, focusing on teachers' beliefs and experiences, their perceptions of Languageing, and their implementation of the curriculum.

Figure 1 illustrates how teachers interpret and adapt the planned curriculum in light of their beliefs, perceptions, and classroom contexts. This interpretive process shapes the creating of learning environments, the organization of instructional methods, differentiation for diverse learners, and the assessment of understanding. The framework was developed in three steps: (Process A) identify curriculum dimensions from the NCC and prior research; (Process B) refine those dimensions using teachers' perceptions and practical insights; and (Process C) construct a structured observation protocol with scaled indicators (1–4) to evaluate instructional quality across the identified dimensions.

3 Methodology

A qualitative multiple-case study design was adopted to investigate how teachers' beliefs, planning, and classroom enactment interact during Languageing implementation (Yin, 2018). Although the study incorporates descriptive numeric indicators (e.g., scaled observation scores), its primary orientation is qualitative: each teacher constituted an analytic case, and cross-case themes are derived inductively rather than through statistical inference (Creswell & Creswell, 2018). The study proceeded in four sequential phases. Phase 1 used an online survey to gather initial data on teachers' beliefs, perceptions of Languageing, and instructional preferences. Phase 2 expanded on these results through semi-structured

interviews, offering deeper insight into participants' experiences and teaching practices. In Phase 3, classroom observations examined how *Languaging* was enacted in practice. Finally, Phase 4 involved cross-case analysis: qualitative coding and analytic notes established themes within each case, followed by systematic comparison across cases to surface convergent and divergent patterns. This phased, multiple-case strategy enabled triangulation of evidence and a rich, context-sensitive understanding of the relationship between teachers' beliefs and their instructional practice.

3.1 Data collection

3.1.1 Participants

We used purposive sampling to identify teachers who were actively implementing the *Languaging* approach in their mathematics instruction (Creswell & Creswell, 2018). To reach those most likely to have adopted *Languaging*, we contacted participants via email lists distributed in *Languaging*-focused courses, workshops, and in-service training events. Given that over 90% of Finland's primary teachers are women (EDUFI, 2021), and that these recruitment channels were attended almost exclusively by female teachers, our final sample comprised six women teaching in primary schools. Data collection occurred between September and December 2024.

Participants varied in age: three were over 55, one was 46–55, one was 36–45, and one was 26–35 years old. They taught in Central Finland, Northern Finland, and the Uusimaa region. The group comprised five class teachers and one special education teacher. All held master's degrees and were teaching Grades 2–4 or a mixed-grade special education class. Class sizes ranged from 6 to 20 students, and teaching experience spanned 5 to 22 years. To ensure anonymity, pseudonyms were assigned to all participants.

Each teacher's classroom constituted a distinct case, allowing in-depth examination of *Languaging* implementation rather than broad generalization. This focused design enabled us to explore nuances in the interaction between practice and beliefs in authentic instructional settings.

3.1.2 Observation and data collection methods

The survey and interviews were designed to explore teachers' beliefs about mathematics; prior learning experiences; perceptions of *Languaging*; influences of school culture; perceived changes in beliefs over time; and challenges in implementing *Languaging*. The survey included Likert-scale items, multiple-choice questions, and open-ended responses, which were then explored through in-depth interviews.

Classroom observations used an integrated protocol developed via inductive and deductive approaches (see Sect. 2.4 and Figure 1). First, the NCC specifies four dimensions for mathematics teaching—slightly differently for grades: (1) learning environment; (2) working methods; (3) support for diversity; and (4) assessment. For example, functional

learning environments are emphasized in Grades 1–2, with “easy accessibility” added in Grades 3–6. Second, we adapted teaching-related quality dimensions from Schlesinger et al. (2018) to align with the definition, modes, and objectives of Languaging (Joutsenlahti & Perkkilä, 2024; Joutsenlahti & Rättyä, 2015; Perkkilä & Joutsenlahti, 2021), yielding three additional dimensions: (5) foundational knowledge and skills; (6) teacher Languaging; and (7) relevance of mathematics.

Indicators for each of the seven dimensions were drawn from the literature, then refined through pilot observations and teachers’ own perceptions of Languaging. To build and validate our draft protocol, we first conducted a pilot study in a pre-service teacher practice context. Twelve classroom sessions—each co-taught by a pair of a cohort of 24 pre-service teachers—were observed using that draft. Two experienced observers independently applied the protocol during these pilot lessons, then compared notes in debriefing sessions. Through reflective meetings with observers and pre-service teachers immediately after each lesson, we clarified ambiguous wording, confirmed the observability of all items (Cohen’s $\kappa = 0.82$, indicating strong interrater agreement), and refined item descriptions.

Because our study focused on in-service teachers’ enactment of Languaging, we did not field-test the fully revised protocol in the six classrooms. Instead, we finalized the indicators based on participants’ survey responses and interview feedback, preserving flexibility to adapt descriptors as new perspectives emerge. The final protocol (see Appendix A, Table A1) thus reflects both the *interpreted curriculum* (RQ2) and the *implemented curriculum* (RQ3). Each item was rated on a four-point scale (1 = *Does not apply at all*, 4 = *Fully applied*), with unobservable elements marked as n/o.

For classroom observations, teachers selected the lesson topic and content in which they would apply Languaging strategies, without prior discussion with the researchers. We audio-recorded teacher talk using a lapel microphone and backup recorders—“audio recording, rather than video, was chosen to focus analysis on instructional delivery and to address ethical considerations with minors” (Schlesinger et al., 2018). Field notes were taken to capture broader classroom interactions that might not be reflected through structured observation tools (Klette, 2023). Teachers also provided brief post-lesson reflections, which were incorporated into the results. To reduce observer effects, the researcher spent 8–10 lessons in each classroom before formal observations began, allowing participants to settle into natural routines (Creswell & Creswell, 2018).

The full dataset included survey responses from all six participants, interviews lasting 35–75 minutes, and audio recordings of seven mathematics lessons conducted by five participants. Case 6 (Lumi) did not participate in classroom observations because the school principal did not grant research access. Two lessons were observed for Case 2 (Mila) and Case 4 (Elina).

3.2 Data analysis

We analyzed data from surveys, interviews, and classroom observations in four sequential phases to strengthen credibility and minimize bias (Cohen et al., 2002; Creswell & Creswell, 2018). First, we summarized survey responses and thematically analyzed interview transcripts, using descriptive statistics and simple line graphs to visualize patterns and capture each teacher's prior learning histories. Interviews conducted in Finnish were transcribed verbatim, translated into English by bilingual researchers, and reviewed by participants to ensure linguistic accuracy (Creswell & Creswell, 2018). We applied a hybrid coding strategy: deductive parent codes reflected the NCC-aligned dimensions and Languageing modes alongside teaching-related quality, while inductive subcodes were generated from the data. Two researchers double-coded an initial subset of cases to calibrate the codebook; disagreements were resolved in consensus meetings, and all revisions were logged in an audit trail.

Next, observation audio recordings were transcribed and segmented into instructional units based on activity format: whole-class (W), individual tasks (I), and group (G) or peer (P) learning (Luoto et al., 2022). Two researchers independently rated the instructional quality using the observation protocol. Because observation ratings were ordinal, we summarized sub-item and dimension scores using medians (Mdn) and ranges; n/o entries were treated as missing, with per-lesson medians computed at the sub-item level. However, due to the small sample size and limited number of observed lessons per teacher, findings were not subjected to inferential statistical analysis (Schlesinger et al., 2018). Descriptive statistics are presented in Appendix B (see Tables B1 and B2).

Finally, we performed a cross-case synthesis, comparing themes and instructional patterns across five observed cases to identify both convergent and divergent approaches to integrating Languageing in primary mathematics. Within-case memos linked prior experiences and beliefs (RQ1) to interpreted plans (RQ2) and enacted Languageing (RQ3), followed by a matrix-based cross-case synthesis to compare convergences and divergences (Yin, 2018).

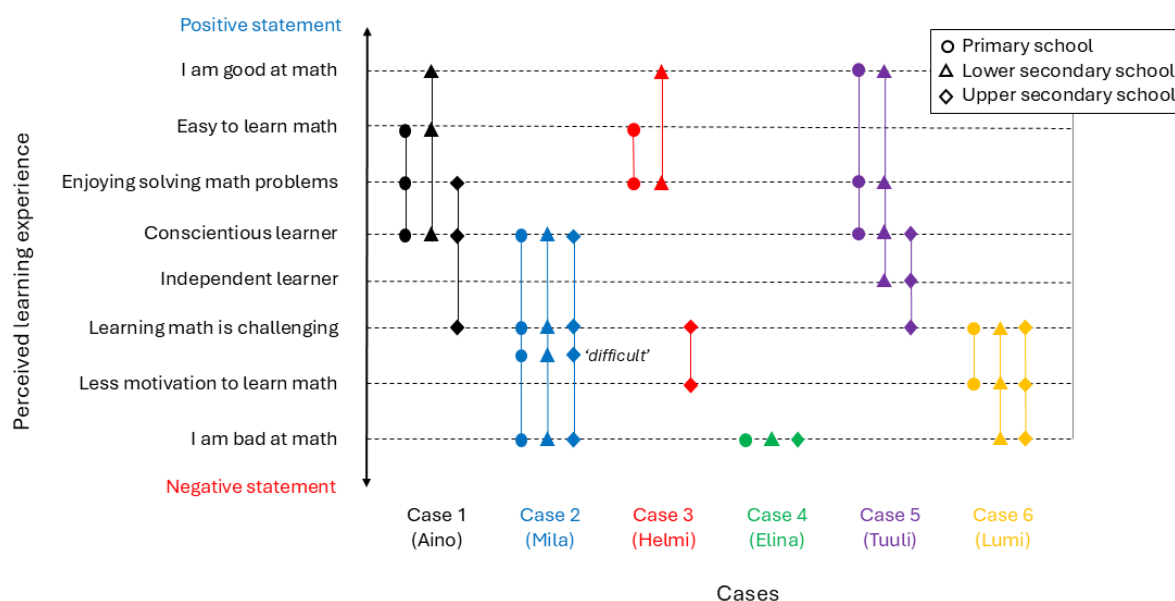
4 Results

4.1 RQ1: Teachers' beliefs about mathematics and prior experiences

In the survey, participants selected multiple statements describing their school experiences in learning mathematics (Figure 2). Three teachers (Aino, Tuuli, Helmi) reported consistently positive or mixed–positive trajectories. Three teachers (Mila, Elina, Lumi) reported consistently negative prior learning experiences. Despite varied pathways, participants converged on two principles of mathematics teaching and learning: (1) building a strong foundation of knowledge and skills for every student, and (2) emphasizing the relevance of learning mathematics in daily life. Examples included “Building a

mathematics house; repetition is important for working-memory” (Aino), “Languaging creates a strong basis for understanding” (Elina), “Bring children’ stories and drawings into mathematics learning” (Lumi), and “It is important to think about where we need mathematics and why” (Helmi). These trajectories reflect movement along the procedural-conceptual continuum (Pikk et al., 2025) and changes in the self-as-learner and self-as-teacher facets of belief systems (Pehkonen & Törner, 1995).

Figure 2. Teachers’ experiences of their own mathematics learning from primary school to upper secondary school



Note. Lines represent responses to multiple-choice items related to learning experiences, including confidence, enjoyment, and personal learning habits in mathematics. The vertical ordering of statements reflects a continuum from positive to negative statement of mathematics learning, based on the content and affective tone of each statement. Symbols indicate the school levels.

Table 1. Thematic categorization of teachers' perceptions on mathematics teaching and learning, aligned with the national curriculum

Dimension	Sub-item	Coding	<i>f</i> (Max 6)
Learning Environment	Functional manner	Priority in functional learning	6
		Easy access to tools	3
	Safe learning atmosphere	Conversational atmosphere	4
		Tolerant and comfortable atmosphere	6
	Critical use of materials	How to use textbook	6
	Positive feedback	n/o	0
Working Methods	Work independently and together	Emphasis on collaborative working	3
	Play and games	Consistent use of play and games	2
	Information or communication technology	Digital materials and practice	1
Support Diversity	Providing enough time to work	Flexible learning	4
	Ensure everyone's opportunity to learn	Diverse tasks simultaneously	1
	Joy of learning and knowledge	Students' positive attitudes	1
	Dealing with heterogeneity	Support for talented students	1
		Using Language tools	3
Assessment	Using multiple representations	Using Language tools	6
	Responding to errors	Observing students' practice	5

Note. "*f*" = number of participants (out of 6) who explicitly referenced the code in survey or interview. Codes were tallied when the idea was named directly; implicit practices (e.g., humor as positive feedback) were not counted unless stated. ICT belongs to working methods (NCC), but the item is regarded as a part of body language in the study.

We coded responses deductively against the NCC dimensions and inductively within each dimension, then tallied codes across participants (see Table 1). To support comprehensive understanding, teachers described student-centered approaches that prioritize conceptual sense-making. For functional learning, examples included "working with hands-on materials while conversing before moving to textbook exercises" (Mila) and "walking around a number sequence while saying the numbers before starting the lesson" (Helmi). To foster a safe climate, teachers used humor, normalized mistakes, and established norms against ridicule. Elina, a special education teacher, underscored the role of strong teacher-student relationships in sustaining engagement.

Perspectives on textbook use varied (critical use of materials). Aino and Helmi used specific sections selectively, whereas Mila projected textbook exercises with a visual presenter and guided students with questions such as "Why?," "What?," and "How?" Elina and Tuuli resisted rigid textbook dependency, noting that no single textbook can comprehensively support learning over an entire school year.

A thematic content analysis of the survey and interview data further explored these belief trajectories, as presented in the following case studies.

Case 1 (Aino) "Life is mathematics—Mathematics is everywhere"

From an early age, Aino viewed the world through a mathematical lens, constantly observing her surroundings with curiosity. Encouraged by supportive teachers, she excelled in learning mathematics, though much of her experience was rooted in mechanical procedures rather than deep conceptual understanding. After working as a preschool teacher for 15 years, she enrolled in an adult education program to qualify as a class teacher. It was during this program that she encountered Languaging, which transformed her perspective on teaching mathematics. Engaging in discussions, drawing, and idea-sharing allowed her to integrate mathematics into meaningful, functional activities. This experience reinforced her belief that mathematics is not just a set of procedures but a tool for understanding and interpreting the world.

Case 2 (Mila) "Mathematics was the painful subject—Now it illuminates my life"

Throughout her 12 years of schooling, Mila found mathematics to be her most challenging subject, struggling to grasp concepts due to reading difficulties. Although her family provided continuous support, unarticulated feelings reinforced her low self-esteem as a learner. She initially pursued a career working with children and later became a school counselor. A turning point came during her teacher education program, where she encountered Languaging that reshaped her beliefs about mathematics. These experiences helped her view mathematics from a new, more accessible perspective. While she still harbors some doubts about her mathematical abilities, she has grown to enjoy teaching the subject and is eager to expand her knowledge.

Case 3 (Helmi) "Difficult to learn, but worth the effort"

In her early years, Helmi found mathematics logical and manageable. However, as the content became more advanced in upper secondary school, she lost motivation and struggled to stay engaged. Even so, the unwavering support from her family played a crucial role in sustaining her learning. She worked as a preschool teacher for 15 years, incorporating play-based learning with concrete tools. While these methods helped children grasp mathematical concepts, she later realized that she did not sufficiently encourage students to express their own thinking. Upon encountering Languaging, she deepened her understanding and became more aware of her own misconceptions when explaining concepts to others. She now deliberately makes mistakes in her teaching to prompt students' questioning and encourage active engagement.

Case 4 (Elina) "Insecurity in childhood, beauty of math is visible in adulthood."

Elina's school experiences with mathematics were marked by insecurity and anxiety. Her teachers relied heavily on memorization-based instruction, offering little to no conceptual support. Despite receiving good grades, she often felt disconnected from the subject,

reinforcing her belief that she did not belong to the so-called “math-head” or “talented” category. These early struggles deeply shaped her self-image as weak and incompetent in mathematics. A decisive shift occurred when she pursued a career in special education, motivated by a desire to make mathematics accessible to all learners. Through years of teaching, she gradually built confidence in her ability to teach mathematics effectively. Today, she is committed to fostering a learning culture where every student feels they belong in mathematics.

Case 5 (Tuuli) "Enjoys challenging in mathematics—no shortcuts to learning"

Tuuli was an independent learner who enjoyed tackling challenging mathematical problems, either by solving them on her own or by seeking help without hesitation. Encouragement from her teachers and her older brother's pursuit of advanced mathematics strongly motivated her to study harder. However, during her initial teaching practice, she struggled to address the diverse learning needs of her students and found it difficult to support those who did not grasp mathematical concepts. Learning about various concrete teaching tools helped her approach foundational skills more effectively, implementing them step by step. A turning point in her development was collaborating with an enthusiastic colleague who shared her passion for teaching mathematics.

Case 6 (Lumi) "Now, I can understand why learning was always difficult"

Lumi's journey with mathematics was marked by frustration and a lack of motivation. Throughout her schooling, she found the subject difficult and meaningless, as it was taught primarily through formulas and rules without visual or hands-on support. She never saw its relevance to her life, which led to a deep dislike of the subject. However, her perspective changed in university when she was introduced to visualization tools. This experience helped her understand why mathematics had always been challenging for her and sparked a desire to improve her teaching so that her students would not develop the same negative feelings. Although her core beliefs about teaching have remained stable over her six-year career, her understanding of the learning process and skill development has deepened. She now feels more confident in structuring her lessons and aligning them with curriculum goals to ensure that students experience mathematics in meaningful and accessible ways.

Consequently, teachers' learner histories powerfully shaped present beliefs, yet most trajectories show movement toward conceptual, student-centered orientations aligned with the NCC, often catalyzed by Languageing. These orientations frame the next results sections: how beliefs informed interpreted plans (RQ2) and how they were enacted during lessons (RQ3).

4.2 RQ2: Teachers' perceptions and integration of Languageing

Teachers frequently integrated multiple Languageing modes—primarily spoken language, but also drawing, physical movements, hands-on materials, and mathematical symbols. Since they taught Grades 2–4, where numerical grades are not mandatory, they prioritized observational assessment. This included listening to student talk, examining work with manipulatives, and reviewing drawings and written tasks to diagnose misconceptions and guide the next steps. Although positive feedback was not explicitly mentioned in surveys or interviews (see Table 1), encouragement of individual and team efforts was consistently visible in classrooms and contributed to a psychologically safe climate.

Table 2 outlines the teachers' implementation plans by the three Languageing-specific dimensions of the observation protocol (Foundational knowledge and skills; Teacher Languageing; Relevance of mathematics). These plans were not always articulated explicitly in the NCC terminology, yet teachers' belief in the practical significance of mathematics was evident in how they framed tasks and talk. For instance, Aino frequently posed spontaneous, everyday problems, “How many pairs of cards do we need so that everyone has a partner? What should we do if someone is left without a partner?,” and Lumi contextualized operations through short stories (e.g., a monkey-and-banana scenario). When Lumi incorporated digital materials for small-group practice and formative assessment, we coded these uses within the relevant Language modes (typically body language for interactive use) rather than a separate technology category.

Table 2. Thematic categorization of teachers' implementation plans for Languageing

Dimension	Sub-item	Coding	<i>f</i> (Max 6)
Foundational knowledge and skills	Checking previous knowledge	Cognitive activation for recalling	1
	Deliberate practice	Cognitive activation for remembering	3
Teacher Languageing	Mathematical thinking	Exploring diverse ways and answers	4
	Multiple representations of the teacher	Combining Languageing strategies for students' understanding	4
Relevance of mathematics	Relevance of mathematics for students	Everyday math related to life	5

Note. “*f*” = number of participants (out of 6) whose survey/interview data included that code.

Classes typically included at least four students with special educational needs and multilingual backgrounds. Teachers reported collaborating with special education teachers and school assistants and often used pictorial representations to scaffold access. However, explicit strategies for supporting talented students were rarely mentioned. Instead, teachers offered optional activities during their free time or additional tasks for students seeking more advanced challenges.

These themes fed directly into the observation indicators (Appendix A): “Foundational knowledge and skills” captured cognitive activation and practice; “Teacher Language” captured how teachers model and translate across modes; and “Relevance of mathematics” captured the everyday framing of tasks. In this way, RQ2 operationalizes the *interpreted curriculum* (what teachers planned and valued) and provides the bridge to RQ3’s analysis of the *implemented curriculum* (what was enacted in lessons).

4.3 RQ3: Instructional quality in the context of Language

Overall, across the seven observed lessons taught by five teachers, median ratings (1–4) was strongest for the learning environment ($Mdn = 3.50$) and working methods ($Mdn = 3.50$), with similarly high performance for teacher Language ($Mdn = 3.45$) and foundational knowledge and skills ($Mdn = 3.30$). Assessment ($Mdn = 3.10$) support for diversity ($Mdn = 3.00$), and relevance of mathematics ($Mdn = 3.00$) were solid but comparatively lower. Indicator coverage was 88.9% (112/126 observable ratings), with a small share of items marked n/o due to lesson focus or brevity (see Appendix B).

Classrooms were orderly and emotionally safe (L2: $Mdn = 3.3$), with purposeful selection and integration of materials (L3: $Mdn = 3.5$). Teachers’ use of positive feedback varied more widely (L4: $Mdn = 3.0$, range = 2.6–4.0), ranging from explicit praise for effort to quieter monitoring. Working formats balanced whole-class, pair, and individual activity (W1: $Mdn = 3.3$), and play/game features appeared often but not universally (W2: $Mdn = 3.6$; two lessons n/o).

Regarding Language modes in use, spoken language was ubiquitous, with frequent prompts for explanations and peer talk. Teachers’ multiple representations (T2) were consistently strong ($Mdn = 3.7$). Students’ own production of multiple forms (A1) was more variable ($Mdn = 3.0$, range = 2.1–3.3), suggesting that while teachers modeled richly, eliciting student-generated representations at scale was less even. Body language (e.g., number walks, card/line placements, manipulatives) was used across several lessons.

Support for diversity showed low medians with wider spread (S1 Time to work: $Mdn = 3.0$; range = 2.0–3.7; S2 Equitable opportunity: $Mdn = 3.0$, range = 2.5–3.6; S4 Dealing with heterogeneity: $Mdn = 3.0$; range = 1.0–4.0; two lessons n/o). This indicates uneven enactment of pacing flexibility and tailored scaffolds (e.g., for multilingual and high-attaining learners). Conceptual pressing via teacher questioning (T1 Mathematical thinking) also varied ($Mdn = 3.0$; range = 2.0–3.5): many exchanges sought reasons and comparisons, but procedural rehearsal sometimes dominated. Foundational routines were steady (F1 Previous knowledge: $Mdn = 3.4$; F2 Deliberate practice: $Mdn = 3.3$). Relevance-making was present but concise (R1: $Mdn = 3.0$; range = 2.0–3.3; one lesson n/o). Table 3 highlights the item-level of instructional quality.

Table 3. Item-level highlights of instructional quality in Language (higher- vs. lower-performing sub-items)

Higher-performing (strong and stable)	Lower-performing (low and variable)
L1 Functional manner	T1 Mathematical thinking (conceptual press)
T2 Teachers' multiple representations	S4 Dealing with heterogeneity
L3 Critical use of materials	S1 Providing enough time to work
F1 Checking previous knowledge	A1 Students' multiple representations

Note. "Strong and stable" denotes higher medians with narrow ranges; "Low and variable" denotes lower medians with wider ranges.

Recognizing that the medians and ranges provide only a descriptive result, the brief case vignettes (teaching grade; lesson topic) that follow contextualize these patterns by showing how each teacher enacted Language in the classrooms.

Case 1: Aino (4th grade; Geometric concepts)

Consistent with Aino's long-standing view that "life is mathematics," this lesson pressed for conceptual justification while maintaining clear routines. She primarily used natural and mathematical symbolic language, encouraging students to verbalize their thinking and take notes. She consistently prompted conceptual understanding by asking students to justify their reasoning, for example, "Why does it say line AB rather than line BA here?" and "Is there any other way? Again, you will notice many different ways in mathematics." She monitored learning with positive feedback and sticker rewards, then provided targeted support to a student with a multilingual background. Fast finishers were offered optional extension activities.

This lesson primarily reviewed concepts introduced the previous day. Students explored key geometric terms—*piste* (point), *puolisuora* (half-line), *suora* (straight line), and *jana* (line segment)—through drawing activities (pictorial language), coordinated with notation (symbolic). The following day, she reinforced these concepts using a digital textbook for visualization, which served primarily as a pictorial representation. Her teaching illustrated a deliberate conceptual press (T1) alongside purposeful selection of materials (L3) and multiple teacher representations (T2). In her reflection, Aino noted that it was well-structured for geometry learning but lacked opportunities for pair discussion, and that multilingual learners faced linguistic barriers when using the textbook.

Case 2: Mila (4th grade; Place value)

In line with Mila's self-described aim to make mathematics "accessible and enjoyable for everyone," the lesson orchestrated multiple representations within a clearly segmented sequence. Seven concise phases flowed with smooth transitions, and students, familiar

with routines, followed directions and engaged readily. Mila integrated natural language (brief explanations and checks for understanding), pictorial language (displays), mathematical symbolic language (place-value notation), and body language (e.g., measuring the hallway with a rolling ruler and weighing sugar, rice, and pasta). The concept was reinforced using ten blocks (Dienes blocks) and connecting clips in both pair and whole-group work. She aimed to deepen students' understanding by linking basic place values to operations such as multiplication, division, and comparing large numbers.

Explicit praise was comparatively sparse (L4), yet Mila's tone and interaction style nonetheless created a conversational, supportive atmosphere. Conceptual press (T1) was moderated by frequent closed questions, which efficiently checked accuracy but left fewer openings for extended student reasoning. Differentiation for heterogeneity (S4) was not observable in this lesson segment. Overall, Mila's enactment showed strong use of teacher multiple representations (T2) and functional, hands-on consolidation of place-value ideas, with room to expand student-generated explanations and representation-making.

Case 3: Helmi (2nd grade; Number sequences and two-digit concepts)

Echoing Helmi's reported stance, "difficult to learn, but worth the effort," the lesson was logically structured into five phases using play- and game-based activities (W2), with mixed working methods (W1). Language was central throughout, particularly natural language (students verbalizing numbers) and body language (card-matching and movement to place values or positions).

Helmi frequently used spoken language to scaffold students' thinking. She asked questions like: "Who will explain why this one is placed here?" and "What does the one in 13 represent?" These prompts, along with hands-on tools, encouraged students to articulate their reasoning indicating active cognitive engagement. Although the lesson included both pair work and whole-class work, individual tasks were always embedded in these structures and were not independent. Helmi ensured that all students had access to materials, but the instructional content remained consistent across working types (S4).

In her reflection, Helmi noted the challenges of supporting students with special educational needs. One student demonstrated strong mathematical interest and ability, yet individualizing instruction within a whole-class format was difficult.

Case 4: Elina (4th grade special education class; Multiplication)

Reflecting Elina's stated orientation toward language-supported access and careful scaffolding, the session blended multiple Language modes to meet diverse needs. Six phases and incorporated three play-based activities were designed to reinforce multiplication concepts through engaging and interactive methods. She effectively integrated Language tools (T2), combining body language (hands-on materials and small movement), natural language (peer discussion and storytelling), pictorial language (visual representations), and mathematical symbolic language (formulas).

The lesson was tailored to accommodate students with diverse learning needs (S4). This was reflected in differentiated textbook assignments and homework, with students working on different pages according to their individual progress (L3). She actively engaged students by asking how they felt about their tasks to identify areas of difficulty and provide targeted support. She introduced multiple solution strategies and encouraged students to reason through their answers. However, her questioning remained within students' current skill levels and did not challenge them to think critically beyond comprehension.

In reflection, Elina voiced concern about widening the learning gap, particularly for the weakest students. She noted that declining literacy skills, including reading and writing difficulties, negatively affected students' ability to learn mathematics. She strongly believed that proficiency in the native language was linked to mathematical thinking.

Case 5: Tuuli (2nd grade; Subtraction)

In keeping with Tuuli's practice-first orientation, the lesson foregrounded worked examples and guided rehearsal of subtraction with regrouping. Instruction moved in a deliberate sequence (e.g., from $60 - 5$ to $45 - 7$), with Tuuli modeling across natural language ("How did you do that? Could you explain it again? Yes, it's a bit confusing. Maybe we'll do it again, but it's good. Just explain to me how you calculated it."), pictorial language (teacher drawings and worked examples), and mathematical symbolic language.

During individual and pair work, Tuuli monitored learning by providing positive feedback and guiding questions to help students connect previous knowledge with new concepts (L4, F1). She also asked how students felt about problem-solving tasks to gauge confidence and engagement. In pair work, she encouraged peer verification of answers. Materials were distributed individually to ensure equal access. The lesson focused mainly on reviewing and practicing subtraction procedures (F2), specifically breaking down a ten-unit into ten ones, without incorporating open-ended or cognitively demanding questions.

A challenge arose in the absence of a special education co-teacher. Tuuli provided additional support to learners with attention and numeracy needs. One student, for example, decomposed $57 - 9$ by splitting 9 into 7 and 2, computed $57 - 7 = 50$, and then incorrectly concluded $50 - 2 = 84$. When the same quantity was framed contextually ("If you eat 2 out of 50 chocolates, how many are left?"), the student immediately answered 48. This contrast revealed that while abstract computational strategies remained fragile, students demonstrated strong situational reasoning when tasks were contextualized.

Discussion

The study examined how the national curriculum's emphasis on Languageing connects to teachers' instructional quality, using a qualitative multiple-case design with Finnish primary school teachers. By reflecting on their own school experiences in mathematics, teachers' beliefs about mathematics teaching and learning were examined. These beliefs

evolved over time, shaped by their diverse pathways into teaching and a growing recognition of the importance of Languageing. Their perspectives aligned closely with the instructional dimensions and their own interpreted curriculum. We refined these belief themes into an observation protocol and used it to analyze classroom enactment. Integrating these meaningful connections among teachers' beliefs, the curriculum, and their instructional practices reveals three key areas for discussion.

Reflecting on personal histories and Languageing as resources for professional development

Consistent with prior work, teachers' beliefs about mathematics are deeply influenced by prior experiences but can undergo dynamic change over time (Russo et al., 2020; Teig et al., 2024; Thompson, 1992). While negative school experiences have been linked to low confidence levels (Artemenko et al., 2021; Kim et al., 2023; Schaeffer et al., 2021), participants in this study demonstrated that a growth mindset contributed to their passion for teaching mathematics in ways that contrasted with their own schooling. Beyond mindset, Languageing as a pedagogical approach played a key role in reinforcing or transforming teachers' perspectives. When teachers planned for explicit use of multiple modes—natural language, pictorial language, mathematical symbolic language, and body language—the result was a more dialogic, communicative climate and clearer opportunities for students to “show what they know” in diverse ways. This aligns with evidence that structured programs, courses, and peer collaboration help teachers (re)contextualize abstract curriculum aims into concrete classroom moves (Palsa & Mertala, 2022). In our data, those supports appeared to strengthen language awareness and normalize practices such as prompting explanations, inviting student-generated representation, and cycling between teacher models and student talk.

These patterns suggest practical directions for professional learning. First, treat pre-service and in-service teachers' mathematics autobiographies as assets by using guided reflections linked to video of their own lessons or practicum. Such work can surface how personal histories map onto current Languageing choices (e.g., frequent T2 demonstrations but less A1 student production). Second, design tasks that expand student voice—For example, brief “represent-then-compare” routines in which students produce both pictorial and mathematical symbolic language, or short body language sequences that lead to natural language justifications. Finally, embed reflective practices in teacher education so that pre-service teachers understand themselves both as learners and as future educators, while cultivating sensitivity to diverse learning needs. By integrating diverse modes and social interaction activities into preparation, candidate gain authentic experience with the four Languageing modes, better positioning them to apply these strategies in practice.

Localizing Language: Making curriculum visible through strengths and gaps

Across cases, teachers positioned Languageing as a central pedagogy—consistent with the objectives of the NCC for functional learning, multiple representations, and joy in mathematics (EDUFI, 2016) and grounded in classic learning theories that links action, language, and concept formation (Piaget, 1952; Vygotsky, 1978). In practice, lessons routinely blended four Languageing modes within coherent activity sequences, mirroring research on the value of multimodal expression for mathematical meaning-making (Joutsenlahti & Perkkilä, 2024; Rinneheimö & Suhonen, 2022). Our observations, however, showed a consistent pattern: teacher modeling of multiple representations and purposeful material use were strong, while student-generated representation, sustained conceptual press, and differentiation were more variable. To make Languageing locally actionable, we suggest the following design moves.

First, define student-generated outputs at task level, for instance, “Students produce a diagram and a statement in mathematical symbolic language, then they explain in natural language how the two match.” Provide low-threshold or high-ceiling task frames (e.g., *represent-then-compare*, *express-justify-revise*) with sample success criteria and annotated samples. Second, embed brief “conceptual thinking” routines in every unit. Add micro-routines prompts that push for explanation and comparison (e.g., Why this and not that? Another way? What ways the same, what changes?). Third, build differentiation into task anatomy, not just pacing. To support multilingual learners, students with learning difficulties, and high-attaining students, attach “mode-based” scaffolds and extensions to each core task. Fourth, clarify expectations for material access and purposeful use. Distinguish “available” from “instructionally assigned” materials and specify when students may choose tools versus when tools are required for learning goals—by grade band if helpful. Fifth, align assessment with multimodal goals. Use short formative checks where evidence must appear in more than one mode (e.g., *Draw it, write it, say it*) and rubrics that value how clearly students map between modes, not only correctness within a single mode. Finally, make planning choices explicit. Provide a planning matrix linking (a) objectives, (b) primary/secondary Languageing modes, (c) the chosen conceptual-press routines, and (d) a targeted scaffold/extension for diversity.

Variability and the case for collaborative, open school cultures

The variability we documented—strong teacher modeling of multiple representations alongside uneven student-generated representations, conceptual press, and differentiation—points to where collaboration could matter most. Co-planning with special educators and language specialists can build “mode-based” supports into task design (e.g., sentence stems for natural language, manipulatives for body language sequences, visual frames for pictorial language), which is especially important as multilingual and special educational needs populations grow.

As Krzywacki et al. (2016) pointed out, Finland’s strong emphasis on teacher autonomy can sometimes lead to professional isolation. While participants in the study

expressed high confidence in their teaching, many were reluctant to share their instructional approaches with colleagues. Most teachers underestimated the influence of school culture on their teaching, except Elina who had worked in several schools. Lumi's case is instructive. She piloted digital tools for small-group practice and quick checks but those moves fell outside our scored observations. In an open culture, such innovations would be made visible—shared, tested, and refined with peers—rather than remaining idiosyncratic and hard to spread.

One in all, collaboration needs time and tools. Protecting regular time and shared routines for co-planning, peer observation, and task moderation can turn isolated innovations into collective capacity, narrowing variability where it matters most.

Limitations and future studies

While this study provides valuable insights into how Finnish primary school teachers integrate Languaging into mathematics instruction, several limitations should be acknowledged. First, the small sample size limits the generalizability and may not fully capture the diversity instructional practices among Finnish primary teachers. Second, the study did not systematically account for background factors—such as teaching experience or prior professional development—that could influence beliefs and practices. Third, although the observation protocol supported reliability, additional validity evidence is needed to strengthen the robustness of the findings.

To address these limitations, future research should examine larger and more diverse samples and make greater use of video-based analysis. Employ mixed method, multilevel designs to trace connections between teachers' beliefs and classroom practice. Further research should also validate the observation protocol cross-culturally (translation/back-translation; cognitive interviews with teachers), examine measurement invariance of the rubric across languages and systems, and conduct comparative multiple-case studies (e.g., Nordic–East Asian contrasts) to trace how discourse norms and materials shape Languaging. Linking these observations to student outcomes (reasoning, participation, representations) would strengthen external relevance.

Conclusions

This multiple-case study connected primary teachers' personal histories and beliefs with their classroom enactment of Languaging. Across seven lessons, we observed consistently strong learning environments, purposeful material use, and rich teacher-led multiple representations. At the same time, variability in student-generated representations, sustained conceptual press, and differentiation suggests where support is most needed. Taken together, the findings argue for localizing the national curriculum through concrete, mode-specific task designs and for strengthening collaborative school cultures that make such designs visible, shareable, and improvable.

While Finland was the empirical setting, the study offers transferable, mode-specific design moves and a practical observation lens grounded in sociocultural theory that other systems can adapt to their curricular and linguistic ecologies. Moving forward, professional development initiatives that foster reflective teaching, peer collaboration, and purposeful Languageing strategies can further enhance instructional quality. Ultimately, this study reinforces the significance of Languageing as a pedagogical approach for fostering meaningful, multimodal, and engaging mathematics learning experiences.

Research ethics

Author contributions

E.K.: Conceptualization, investigation, methodology, project administration, observation, visualization, writing—original draft preparation, writing—review and editing

P.P.: Supervision, validation, observation, formal analysis, writing—review and editing.

J.J.: Supervision, validation, formal analysis, writing—review and editing.

All authors have read and approved the final version of the manuscript.

Artificial intelligence

ChatGPT by OpenAI was used to assist with proofread spelling and grammatical errors during the preparation of this article. All AI-assisted edits were reviewed and approved by the authors to ensure accuracy and alignment with the research content.

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Institutional review board statement

The study complied with the Finnish National Board on Research Integrity (TENK, 2019) guidelines and received approval from the Research board of City (Nro 177/2024; § 28/2024). Special care was taken to anonymize all voice recordings and protect participant confidentiality throughout the study. Teachers, students, and guardians were fully informed about the study's purpose, procedures, potential risks, and benefits. They were assured that their participation was voluntary, and all personal identifiers were removed or coded to maintain anonymity.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

Audio recordings and transcriptions cannot be shared due to privacy or ethical restrictions upon reasonable request.

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Conflicts of interest

The authors declare no conflicts of interest.

Appendix A

Table A1. Observation protocol for evaluating mathematics instructional quality: A focus on Language strategies and pedagogical practices

Dimension	Code/Sub-item	Indicator
(L) Learning environment	L1 Functional manner	(Grades 1–2) The teacher provides various tools. (Grades 3–6) The tools are easily accessible for students. The teacher emphasizes functional learning using Language strategies.
	L2 Safe learning atmosphere	The teacher establishes a safe learning environment through classroom rules and encouragement. The teacher fosters a conversational atmosphere using humor, storytelling, or acknowledging mistakes. The teacher effectively balances active engagement with a respectful and tolerant atmosphere.
	L3 Critical use of materials	The teacher selects and integrates various instructional materials effectively. The teacher demonstrates skillful use of textbooks and supplementary resources.
	L4 Positive feedback	The teacher provides constructive feedback on students' effort, strengths, and active engagement in group work.
(W) Working methods	W1 Work independently and together	(Grades 1–2) The teacher provides various working types. (Grades 3–6) The students shall choose various working types. The teacher facilitates and integrates collaborative learning methods effectively.
	W2 Play and games	The teacher incorporates play-based and game-based activities in a pedagogically meaningful way.
	W3 Information or communication technology	The teacher provides opportunities for students to use technology, including digital tools or calculators, to enhance learning.
(S) Support diversity	S1 Providing enough time to work	The teacher ensures students have adequate time to develop their skills, continuously monitoring their progress. The teacher fosters a flexible learning environment that supports diverse learners.
	S2 Ensure everyone's opportunity to learn	The teacher ensures every student receives sufficient practice opportunities. The teacher offers diverse tasks either simultaneously or sequentially to support learning.
	S3 Joy of learning and knowledge	The teacher creates a positive and enjoyable learning environment, fostering students' enthusiasm for mathematics.
	S4 Dealing with heterogeneity	The teacher provides tailored support for talented students, multilingual learners, and students with learning difficulties.
(A) Assessment	A1 Multiple representations* (students)	The teacher encourages students to express mathematical thinking through varied representations (NL, SL, PL, BL). The teacher assesses students' conceptual understanding through different representations.
	A2 Responding to errors	The teacher evaluates students' fluency and accuracy in problem-solving. The teacher identifies misconceptions and guides students independently correcting errors. The teacher actively observes students' learning processes.
(F) Foundational knowledge and skills	F1 Checking previous knowledge	The teacher activates students' previous knowledge and connects it to new learning content. The teacher emphasizes connections between prior and new topic.
	F2 Deliberate practice	The teacher explains the importance of exercises. The teacher provides opportunities for exploring and reflection, and self-differentiating. The teacher promotes cognitive activation for deep learning.
(T) Teacher Language	T1 Mathematical thinking	The teacher provokes students to think about the reason for their answers. The teacher prompts students to compare multiple solution methods. The teacher uses open-ended questions to encourage deeper mathematical thinking.
	T2 Multiple representations* (teacher)	The teacher uses varied representations (NL, SL, PL, BL) to support conceptual understanding. The teacher effectively integrates different Language strategies to aid comprehension.
(R) Relevance of mathematics	R1 Relevance of mathematics for students	The teacher connects mathematics to students' everyday life. The teacher provides relevant examples of mathematics in lessons. The teacher may encourage students to share personal experiences and interests.

Appendix B

Table B1. Descriptive statistics of audio analysis across cases 1-5

Case 1–5	Aino	Mila		Helmi		Elina	Tuuli	
Dimension / Sub-Item		Lesson 1	Lesson 2	Lesson 1	Lesson 2			Mdn
(L) Learning Environment								
L1. Functional manner	n/o	3.9	3.5	3.8	3.5	3.7	3.7	3.7
L2. Safe learning atmosphere	3.8	3.3	3.3	3.3	3.1	3.4	3.4	3.3
L3. Critical use of materials	3.5	4.0	3.4	3.5	3.6	3.8	3.3	3.5
L4. Positive feedback	4.0	2.8	2.6	3.0	3.0	3.3	3.5	3.0
(W) Working Methods								
W1. Work independently and together	3.0	3.9	3.5	3.7	3.3	3.2	3.0	3.3
W2. Play and games	3.0	3.5	n/o	3.6	4.0	4.0	n/o	3.6
W3. Information or communication technology	n/o	n/o	n/o	n/o	n/o	n/o	n/o	-
(S) Support Diversity								
S1. Providing enough time to work	2.5	2.0	3.0	2.3	3.0	3.7	3.3	3.0
S2. Ensure everyone’s opportunity to learn	3.0	3.0	2.5	2.5	2.9	3.6	3.0	3.0
S3. Joy of learning and knowledge	3.7	3.0	n/o	3.0	3.0	3.0	3.0	3.0
S4. Dealing with heterogeneity	3.0	n/o	n/o	3.0	1.0	4.0	2.0	3.0
(A) Assessment								
A1. Multiple representations* (students)	3.0	2.1	2.3	3.3	3.3	3.0	3.0	3.0
A2. Responding to errors	3.0	3.3	3.3	3.2	3.2	3.7	3.0	3.2
(F) Foundational knowledge and skills								
F1. Checking previous knowledge	3.8	4.0	3.3	3.5	3.0	3.3	3.4	3.4
F2. Deliberate practice	3.8	3.3	3.3	2.7	2.5	2.7	3.5	3.3
(T) Teacher Linguaging								
T1. Mathematical thinking	3.3	2.0	3.0	3.4	3.5	2.2	2.5	3.0
T2. Multiple representations* (teacher)	3.2	4.0	3.8	3.5	3.5	3.9	3.7	3.7
(R) Relevance of mathematics								
R1. Relevance of mathematics for students	3.3	3.0	3.0	2.0	n/o	2.8	3.0	3.0

Note. The first author participated in all classroom observations, while the second and third authors each independently observed half of the lessons. The median of the two authors' ratings was calculated for each sub-item (scale: 1 = *Does not apply at all*; 4 = *Fully applied*; n/o = *Unobservable*). Final ratings were determined through joint discussions. The Mdn columns summarize central tendency and calculations excluded n/o values. These statistics are descriptive and complement qualitative multiple-case analysis; they are not used for inferential claims.

Table B2. Evaluation of mathematics instructional quality based on observation protocol

Case 1 (Aino)			L				W			S				A		F		T		R
P	WT	Time	L1	L2	L3	L4	W1	W2	W3	S1	S2	S3	S4	A1	A2	F1	F2	T1	T2	R1
P1	W	4'00"	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P2	W	5'17"	-	4	-	-	-	-	-	-	-	3	-	-	-	4	4	4	4	3,5
P3	W	6'47"	-	3,7	4	4	-	-	-	-	-	4	-	3	2	4	4	3	3,5	3
P4	1, P	25'49"	-	3,7	3	4	3	3	-	2,5	3	4	3	3	4	3,5	3,3	3	2	-
Case 2 (Mila 1)			L				W			S				A		F		T		R
P	WT	Time	L1	L2	L3	L4	W1	W2	W3	S1	S2	S3	S4	A1	A2	F1	F2	T1	T2	R1
P1	W	1'28"	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
P2	P	9'11"	4	3	-	3	4	3	-	2	3	-	-	2,5	3	-	4	-	-	-
P3	W	3'12"	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P4	P,G	2'57"	3,5	4	4	3	3,7	4	-	-	3	3	-	2	3	-	3	-	4	-
P5	W	6'40"	4	-	4	3	4	4	-	-	-	-	-	2	4	-	3	-	-	-
P6	W	7'07"	4	3	4	2	-	-	-	-	-	-	-	2	3	4	3	2	4	-
P7	W	14'17"	-	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Case 2 (Mila 2)			L				W			S				A		F		T		R
P	WT	Time	L1	L2	L3	L4	W1	W2	W3	S1	S2	S3	S4	A1	A2	F1	F2	T1	T2	R1
P1	W	3'20"	-	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
P2	I	2'10"	3,5	3	-	3	-	-	-	-	-	-	-	3	3,7	-	-	3	-	-
P3	W	9'33"	-	3	4	3	-	-	-	3	3	-	-	2	3	4	3	3	4	3
P4	I	2'35"	-	-	-	3	4	-	-	-	-	-	-	-	3	3	-	-	-	-
P5	W	2'51"	-	-	-	2	-	-	-	-	-	-	-	-	3	-	-	-	-	-
P6	P,G	7'02"	4	4	3,5	2	3	-	-	-	2	-	-	-	3	-	-	-	4	3
P7	W	13'39"	3	-	3	-	-	-	-	-	-	-	-	2	4	3	-	3	3,5	-
Case 3 (Helmi 1)			L				W			S				A		F		T		R
P	WT	Time	L1	L2	L3	L4	W1	W2	W3	S1	S2	S3	S4	A1	A2	F1	F2	T1	T2	R1
P1	W	5'57"	4	3	4	3	-	3	-	-	-	3	-	3,5	3,5	-	-	4	3,5	-
P2	W	3'33"	3	3	-	-	-	3	-	-	-	3	-	-	3	-	-	-	3,5	-
P3	P,I,W	18'28"	4	3,3	3	3	3	4	-	-	-	3	-	3	3	-	2,5	3,3	3	-
P4	I,W	13'40"	4	3,7	4	3	4	4	-	2,5	2,5	3	3	3	3,3	3	3	3,3	4	2
P5	I,W	4'16"	4	3,3	3	3	4	4	-	2	2,5	3	-	3,5	3	4	2,5	3	3,5	-
Case 3 (Helmi 2)			L				W			S				A		F		T		R
P	WT	Time	L1	L2	L3	L4	W1	W2	W3	S1	S2	S3	S4	A1	A2	F1	F2	T1	T2	R1
P1	I	8'32"	3,7	3	3	3	3	-	-	3	3	-	-	-	-	-	2	-	-	-
P2	I,G	7'49"	3,7	3	4	3	3	4	-	-	2,5	3	1	3	3	-	-	3	3,5	-
P3	W	3'29"	3	3,3	4	-	-	4	-	-	3	-	-	4	3	-	-	-	-	-
P4	I,W	10'04"	4	3	4	3	4	4	-	3	3	-	-	3	3,5	3	-	-	3,5	-
P5	W	5'29"	3	3	3	-	-	-	-	-	-	3	-	-	-	-	3	4	-	-
Case 4 (Elina)			L				W			S				A		F		T		R
P	WT	Time	L1	L2	L3	L4	W1	W2	W3	S1	S2	S3	S4	A1	A2	F1	F2	T1	T2	R1
P1	W	4'28"	3,5	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
P2	P,W	7'30"	4	3,5	4	4	3	-	-	4	4	-	4	3,5	3,7	4	3	3	4	2,7
P3	P	8'43"	4	3,3	4	3	3	4	-	3	2,5	3	-	3,5	4	-	2,5	1	4	-
P4	W	5'43"	3	3	-	-	-	-	-	-	-	-	-	2	-	3	2,7	2,5	3,5	-
P5	W	2'45"	4	3,3	4	3	3,5	4	-	-	4	3	-	3	3	3	-	-	4	-
P6	I	7'26"	-	-	3	3	-	-	-	4	4	-	4	3	4	-	-	-	-	-

Note. L = Learning environment; W = Working methods; S = Support diversity; A = Assessment; F = Foundational knowledge and skills; T = Teacher's Language; R = Relevance of mathematics.

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