LUMAT
International Journal on Math, Science and Technology Education

Research article

Addressing complexity for educating the future-ready workforce in STEM fields: MINTco@NRW from a mathematics educational perspective

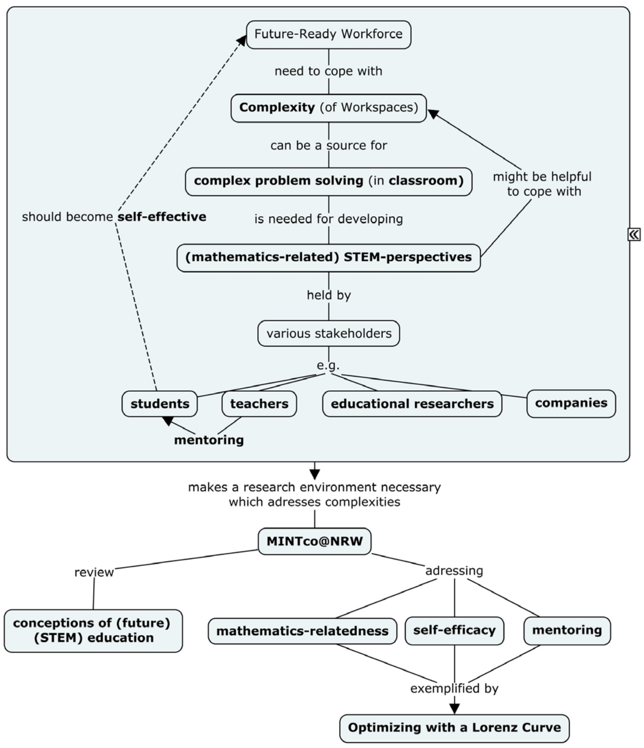
Gero Stoffels*[,1], Julia Schäfer[1], Jacqueline Köster[2] and Ingo Witzke[2]

[1] Universität zu Köln, Germany
[2] Universität Siegen, Germany

Abstract: Educating the future-ready workforce in STEM fields is complex. This is demonstrated by numerous publications in the context of the fourth industrial revolution, the 21st century skills, and the development of integrated models in STEM Education. In this article, this complexity is first addressed on a theoretical level by reviewing developments in the world of work and the associated challenges for General Didactics and Subject Specific Education. A synthesis of the contrasting perspectives shows that empirical research requires sufficiently complex settings. With MINTco@NRW, such a setting is presented, and its complexity characteristics are identified. In addition, insights are provided into the research perspectives associated with the project. They are mathematical-relatedness, students' self-efficacy and the mentoring of STEM problem solving. Research questions that arise are in the contexts of performance assessments in such settings and teacher training to provide the necessary skills to make learners future-ready. The conclusion is that integrated STEM education for a future-ready workforce requires scientific approaches that make fruitful use of the mentioned complexities.

Keywords:
STEM education,
complexity,
future workforce,
long-term problem solving,
project-based learning

Correspondence:
gero.stoffels@uni-koeln.de



Received 27 August 2024 / Accepted 21 June 2025 / Published 22 July 2025
Published by the University of Helsinki, Finland and LUMA Centre Finland.

1 Introduction

Current crises in the environment, health, economy, and politics are raising a lot of challenges. These challenges are not only relevant for professional workers, but they are definitely part of it and need to be ready for the future. Due to rapid technological developments in the fields of Industry 4.0, digitalization, and AI, further complex demands are being placed on the workforce of tomorrow that can hardly be solved in isolation using schematic approaches (Li, 2022).

It is therefore necessary to identify the challenges that will arise in the foreseeable future and what the workplace of the future will look like (1.1). This raises one of the fundamental questions of (mathematical) education, namely to what extent school education can and should meet these challenges, but also, how educational processes can take account of this complexity (1.2). These questions can be addressed from a general or subject-specific perspective. Accordingly, various general concepts, such as 21st century skills (McComas, 2013), 4Cs (National Education Association, 2011), or the OECD Learning Compass (The OECD Future of Education and Skills 2030 Informal Working Group, 2019), and epoch-typical key problems (Klafki, 1985, 1998), which have been developed from social initiatives or research work, are contrasted with subject-specific concepts (Maass et al., 2019; Steiner, 1985) that explore the extent to which mathematics in STEM can help to deal with complexity. This literature-based considerations reveal the need for learning settings to be designed in such a way that learners can gain insights into authentic key problems and thus deepen their (mathematical) skills. Also, corresponding scientific investigation requires correspondingly rich research formats with networked research questions (1.3).

In line with the identified requirements, the MINTco@NRW project is presented as a paradigmatic example of such a complex research setting (2), which enables complex teaching and learning processes in collaboration between schools, companies, and universities, as required by the setting, and also opens up rich research opportunities in the areas of mathematics-relatedness, self-efficacy, and mentoring. To illustrate the complexity and richness of the project, this section provides insights into initial research results on the content, instructional, and reflective levels (3).

Overall, the questions arise as to how the inherent complexity of issues in the field of sustainable cooperation between actors from different institutions and countries can be taken into account to promote a future-ready workforce, considering mathematics-specific and interdisciplinary perspectives, and what further research is needed in this area.

2 Complexity of (future) work and (mathematics) education

Both areas, future work and (mathematics) education are complex in themselves. The first area, as there are diverse professions (Strack et al., 2021), work environments (Fukumura et al., 2021; Grzegorczyk et al., 2021) and work formats (Okenwa-Emegwa et al., 2017; Pan et al., 2023) that change significantly over time. Examples of this changes are the so-called "industrial revolutions" that are currently leading to Industry 4.0 (Balliester & Elsheikhi, 2018; Li, 2022). It is based on a transformation of work and workplaces in the context of digitalization. Educational processes are also complex, which is due to various players in the field of education (González-Pérez & Ramírez-Montoya, 2022; Skott, 2019) different educational goals (Hughson & Wood, 2022; Kennedy & Sundberg, 2020; Weinert, 2002) and the associated formats of teaching and learning (Caratozzolo et al., 2021; Johnson et al., 2021). Additional complexity is added if both areas are to be combined. There are several ways to do this. At this point, two of these possibilities will be outlined to highlight different perspectives for approaching the relationship between education and future work.

One perspective is based on an educational concept that is as general as possible, such as the concept of "Bildung" often found in the German-speaking world, based on Humboldt's educational ideal, which is described by Tyson (2016) in the following excerpt:

> "All schools [. . .] have to support only the development of general human Bildung [general human education]. That which concerns the basic needs of life or the expertise of some individual in his vocation has to be removed and to take place after the completion of general studies. If they are mixed, education becomes impure and one achieves neither complete human beings nor complete citizens. [...] Through general Bildung it is the [...] human being as such that is to be strengthened, refined and regulated. Through special Bildung it is only skills/competences for use that are developed." (Tyson, 2016, p. 235)

This approach assumes that every profession is achievable based on sufficient general education and that profession-specific training should/must only take place later. The future viability of education thus results directly from the fundamental detachment from the requirements of concrete application scenarios of education.

Another possibility is to approach the concept of "future workspaces" and examine afterwards what demands are placed on education. This is then often linked to demands in education policy. Of course, the difficulty here is that it is not possible to predict what exactly the future demands on learners will be, as it is not clear exactly what "future work" and "future workspaces" are (Balliester & Elsheikhi, 2018).

In line with this spectrum of perspectives, various answers have been given in history to the question of what education is, what general education can be, what role specialist and interdisciplinary education plays and what consequences this has for vocational education and training.

In the following contrasting narrative literature review, we want to identify issues on "future work" and "future workspaces" from various areas to elaborate and connect them to different concepts of General Didactics, but also STEM-specific education with a focus on mathematics. The idea behind this approach is to identify complexity as an important topic in the synthesis of future work and education. A result is the necessity of a correspondingly rich research environment, as provided by the MINTco@NRW project, which is then introduced in more detail as a paradigmatic example in the following sections.

2.1 (Future) work & workspaces

As a researcher in the field of Mathematics Education or STEM Education, you are rarely an expert in "future work" or "future workspaces" simply because of your research interests. Various other areas, such as the sociology of work, vocational training, economics, or cyber-physical systems research, must be considered. The following questions are particularly relevant to investigate future work and future workspaces in relation to the question of what STEM education must look like for a future-ready workforce: What competencies and skill requirements are expected from the future workforce in terms of future work? Are there any trends in this regard that are particularly relevant for STEM education? What will the workspaces of the future look like, and what requirements will this place on employees? How are these aspects connected in the "future work"-complex? For answering these questions, it seems to be important to define "future work" or "future workspaces" first. In order to develop a definition, Balliester and Elsheikhi (2018) conducted a literature review. At the beginning of their final report, they state:

> "However, despite a growing body of research in this area, there exists no universally accepted definition of what exactly the "Future of Work" encompasses and what the most relevant drivers are ." (Balliester & Elsheikhi, 2018, p. v)

To frame their literature review, Balliester and Elsheikhi (2018, pp. 1–2) propose five dimensions of future work: number of jobs and their development, job quality, social protection, wage and income inequalities, as well as social dialogue and industrial relations. In order to answer the above questions, it may be helpful to consider the so-called "megatrends" such as technology, climate change, globalization and demographics, in light of these five dimensions. Of these dimensions, job quality as well as social dialogue and industrial relations seem to be particularly relevant to STEM education. Not only the characteristics of future work will change, but also the number of jobs due to new technological innovation (Balliester & Elsheikhi, 2018, p. 8; Li, 2022; Strack et al., 2021). In their literature review Balliester and Elsheikhi (2018) state that little is known about how many new jobs are being created by new technologies, although a large proportion of executives also see these opportunities. Depending on the source used, between 9% and 60% of workers are affected by these changes. Focusing on the changes in activities, it is assumed that 50-70% of workers will be affected, for example by the adaptation of AI technology. As an illustrative example of the change in work, the authors cite an example from France, where half a million jobs have been lost in the last 15 years due to the internet, but where 1.2 million new jobs have been created in the same period. This mega-trend is particularly interesting for STEM education, as many jobs in the STEM sector are located in the area of computer and technology as well as in the green economy. Strack et al. (2021) discuss three labor market developments and their connection to shortages in skilled labor in the STEM-fields, namely in the USA, Germany and Australia, with the conclusion that

> "in many sectors, severe shortages of skilled workers will mean that growth in demand for talent will be unmet. This is particularly true for computer-related occupations and jobs in science, technology, engineering, and math since technology is fueling the rise of automation across all industries. This is why the computer and mathematics job family group is likely to suffer by far the greatest worker deficits in all three countries ." (Strack et al., 2021, p. 11)

This estimate has increased with the accelerated introduction of technology into workplaces during the COVID pandemic (Fukumura et al., 2021; Grzegorczyk et al., 2021).

Trends can also be identified regarding the skills and competencies required by employees. Critical thinking, problem solving, which explicitly includes solving more complex problems, and creativity are still among the top skills that the future-ready workforce should have. In current skills catalogs, a positive attitude towards lifelong learning and employee resilience are also coming into focus due to the experience of the crisis and the ever faster pace of technological development (Li, 2022, p. 2). This required attitude towards lifelong learning is future proof per se, as it enables workers to gain further qualifications later. For this so-called up- and reskilling, Li (2022) identifies both traditional and innovative formats based on various stakeholders, including colleges and companies.

With regard to future workspaces, greater flexibility of work formats and thus also an individualization of workplaces is to be expected, which are to be negotiated between employees and employers (Fukumura et al., 2021; Grzegorczyk et al., 2021; Pan et al., 2023).

With regard to globalization, increased labor migration is to be expected, for which appropriate Actions must be implemented, but at the same time employees must also become global-proof, i.e. be able to deal with different countries and origins in the professional field, which is addressed, for example, by introducing new courses and further training formats (Okenwa-Emegwa et al., 2017).

2.2 Education for Future-Ready Workforce

As outlined in the opening quote, an important aspect of the educational discourse is the extent to which general school education also prepares students for the world of work. A fundamental distinction can be made between two perspectives on education in the classroom: formal and material education (Klafki, 1975, 1985). In formal educational conceptions, the focus is primarily on the individual education of an individual learner in order to train their skills and attitudes as generally and relatively free of purpose as possible without reference to their professional future (Willbergh, 2016). The quote from Humboldt presented at the beginning belongs to this approach according to Tyson (2016). Material education is dominated by (specialist) knowledge, the relationship between learning and knowledge, i.e. the focus is primarily on discipline-specific knowledge or necessary professional knowledge (Willbergh, 2016). Of course, both categories are analytical distinctions. Current educational concepts generally adopt a middle stance in line with the requirements of categorical education (Klafki, 1975). Although with regard to the question of what education for a future-ready workforce should look like, the pendulum has swung mainly in the direction of more formal education and in recent years a turn towards a stronger inclusion of material aspects of education can be seen again (Hughson & Wood, 2022). This also applies to current recommendations for (concrete) reforms of the education system (The OECD Future of Education and Skills 2030 Informal Working Group, 2019; Thornhill-Miller et al., 2023).

In addition to this complex interaction of educational goals with different focal points, the question arises as to what extent general, but also subject-specific perspectives can contribute to how (school) education can contribute to a future-ready workforce, to which, as described in section 1.1 complex requirements are placed on them. Accordingly, examples of current general conceptions and discipline-specific conceptions from the STEM field in relation to the promotion of the "future-ready" workforce are presented below. The selection of the different perspectives is due to the widespread dissemination or greater diversity of approaches.

2.2.1 General conceptions: 21st century skills, 4C's, OECD Learning Compass, and epoch-typical key problems

In this section, we will elaborate four more general concepts which give some perspectives for educating the future-ready workforce. Three of these four different frameworks, 21st century skills (McComas, 2013), 4C’s (National Education Association, 2011) and the OECD Learning Compass (The OECD Future of Education and Skills 2030 Informal Working Group, 2019), are often cited in literature discussing future work (González-Pérez & Ramírez-Montoya, 2022; McComas, 2013; Thornhill-Miller et al., 2023). The origin of these conceptions lay mostly in political or economically oriented associations and groups. In contrast the conception of epoch-typical key problems (Klafki, 1975, 1985; Willbergh, 2016) is an older concept from General Didactical research, which has enjoyed something of a revival in recent years in both English and Scandinavian-speaking countries. This revival can be explained by the fact that it shares similar educational goals as the 21st century skills and the other two frameworks, but also provides a transition to subject specific conceptions (cf. section 2.2.2). All frameworks address the challenge of how education should adapt to accelerated economic and technological developments (Kennedy & Sundberg, 2020).

The concept of 21st century skills has been developed in response to this challenge, which is associated with "digitalization" and "globalization" on the one hand and the specific revolution in the world of work towards "Industry 4.0" on the other. A complex of different skills, also called competencies in some publications, whose possession or training is assumed to enable learners to be successful in the modern world of work (Kennedy & Sundberg, 2020; Thornhill-Miller et al., 2023).

The definitive skills involved are still being discussed from various perspectives. An overview of the various approaches, which cannot be discussed in detail here, is provided by Kennedy and Sundberg (2020) or Thornhill-Miller et al. (2023). The best known framework in the discussion is probably the "Framework for 21st century learning" of the Partnership for 21st Century skills (Battelle for Kids, 2024). It was developed jointly by teachers, education experts and business leaders. The current version of this concrete framework comprises twenty skills, nine key subjects and six 21st century interdisciplinary themes. The skills include: think creatively, work creatively with others, implement innovations, reason effectively, solve problems, communicate clearly, access and evaluate information, use and manage information, analyze media, create media products, apply technology effectively, adapt to change, be flexible, manage goals and time, work independently, interact effectively with others, work effectively in diverse teams, manage projects, guide and lead others and be responsible to others (Battelle for Kids, 2023). Similar skill catalogs have been developed since the 80s (Kennedy & Sundberg, 2020).

All models of 21st century skills are complex, which, according to the developers, is mainly due to the fact that the future (working) world is also very complex (González-Pérez & Ramírez-Montoya, 2022; Kennedy & Sundberg, 2020; Thornhill-Miller et al., 2023; van Laar et al., 2017). The complexity of the model is accompanied by challenges in the concrete implementation and in the empirical evaluation of the acquisition of 21st century skills (Thornhill-Miller et al., 2023). In addition, the problem of definition has not yet been clarified:

> "We are 20 years into the 21st century and we are still struggling to define 21st century skills with precision due to the dynamic nature of the skill sets necessary for success. These definitions will undoubtedly evolve as we move into the 22nd century ." (Kennedy & Sundberg, 2020, p. 491)

Within the 21st century skills, four skills emerge that are rated as particularly relevant in various publications, the so-called 4C's, namely critical thinking, creativity, communication and cooperation (Battelle for Kids, 2024; Thornhill-Miller et al., 2023).

They appear in various catalogs of 21st century skills (Battelle for Kids, 2024; González-Pérez & Ramírez-Montoya, 2022; Li, 2022; The OECD Future of Education and Skills 2030 Informal Working Group, 2019). There are also no universally valid definitions of the 4Cs, which, in addition to the fundamental breadth of the terms is also due to the different perspectives of the disciplines, such as psychology, sociology, didactics and communication sciences, which focus on the respective 4Cs. Nevertheless, quite broad definitions can be given that fit to numerous studies.

Pasquinelli et al. (2021) define: "critical thinking is the capacity of assessing the epistemic quality of available information and–as a consequence of this assessment–of calibrating one's confidence in order to act upon such information". This general definition can be supplemented by six components of critical thinking. These are the ability to interpret information, analyze information, draw conclusions from it, evaluate the quality of the information, provide explanations and regulate oneself in these processes (Thornhill-Miller et al., 2023).

Weick (1993) interprets Bruner's (1983, p. 183) description of learning processes, "'Learning' is most often, figuring out how to use what you already know in order to go beyond what you currently think" as a form of creativity, which can be understood as a broad definition of the latter. The concept of creativity can also be divided into different dimensions, namely the creative process, the product, the characteristics of the creative person and the social and physical environment in which the creation takes place (Thornhill-Miller et al., 2023).

A broad definition of communication is that communication is based on exchanging information in order to change the "epistemic context" of others (Thornhill-Miller et al., 2023, p. 64). A distinction is often made between different dimensions of communication: linguistic, pragmatic and social (Thornhill-Miller et al., 2023). This multidimensionality makes it clear, as with 21st century skills, that different disciplines are necessary to understand communication and that this skill–like critical thinking–is also a complex on its own.

Détienne et al. (2012, p. 197) propose the following definition of collaboration, which they consider to be agreed on: collaboration "involves sharing of goals, resources and representations relating to the joint activity of participants". The "knowledge, behavior, and attitudes" of the collaborators is relevant for productive collaboration (Thornhill-Miller et al., 2023, p. 65).

These four dimensions are also difficult to measure empirically due to their complex structures, which have only been shown here as examples. One advantage for implementation, however, is that only four general skills or competencies need to be focused during implementation, which also enables a broader field of application. Thornhill-Miller et al. (2023) suggest grids for labeling concrete teaching-learning settings of different formats, from curricula to games, with regard to the 4Cs in order to reduce the complexity of evaluation in practice.

Figure 1. OECD Learning Compass (The OECD Future of Education and Skills 2030 Informal Working Group, 2019, p. 15)

The OECD Learning Compass model is similarly complex to the 21st century skills, albeit with additional dimensions such as attitudes and values (The OECD Future of Education and Skills 2030 Informal Working Group, 2019). This framework was developed through collaboration between teachers, experts, businesses, and students. Figure 1. shows the various characteristics of learners summarized in the "compass rose". In addition to skills, their knowledge, attitudes and values also contribute to shaping their future is a part of the core foundations on the basis of which overarching competencies are developed (The OECD Future of Education and Skills 2030 Informal Working Group, 2019). This understanding of competence fits with the common definition of competence, which are

> "the cognitive abilities and skills available to individuals or that can be learned by them to solve certain problems, as well as the associated motivational, volitional and social readiness and abilities to use the problem solutions successfully and responsibly in variable situations." (Weinert, 2002, pp. 27–28)

The next concentric ring contains the transformative competencies, including the competencies creating new value–"new value" in the sense of innovation processes in which the value of the innovation is recognized by the innovators, reconciling tensions and dilemmas–which includes in particular the ability to adopt different points of view and to be resilient, and taking responsibility–which means that the consequences of actions (including ethical ones) should be assessed.

The Anticipation-Action-Reflection cycle is also shown concentrically with the help of arrows in the Learning Compass and is intended to represent the continuous development and learning of learners.

Examining Figure 1. it is noticeable that not only the Learning Compass is part of the OECD framework, but also the student agency. This is defined as

> "the belief that students have the will and the ability to positively influence their own lives and the world around them as well as the capacity to set a goal, reflect and act responsibly to effect change" (The OECD Future of Education and Skills 2030 Informal Working Group, 2019, p. 16)

When collaborating learners act among themselves or with learners in a social context, the actors develop co-agencies in relation to their collaborations. All these aspects should contribute to well-being in 2030. In other words, making the world a better place (The OECD Future of Education and Skills 2030 Informal Working Group, 2019).

The 4Cs can also be found in the OECD Learning Compass, or in the associated notes (The OECD Future of Education and Skills 2030 Informal Working Group, 2019, 38,57). However, there is also a discussion of various areas of knowledge, including disciplinary, interdisciplinary, epistemic, and procedural knowledge. The relationship between disciplinary and interdisciplinary knowledge is of course particularly interesting for STEM Education. Hughson and Wood (2022) come to the conclusion with their analysis of the Learning Compass that disciplinary knowledge is only used instrumentally to learn further skills or professionally relevant knowledge. This means that disciplinary knowledge does not come to the fore in terms of its value for a single discipline. Regarding the individual components of the Learning Compass, similar challenges arise for both empirical investigation and practical implementation as already discussed for the 21st century skills and the 4C framework, as the Learning Compass encompasses these.

To conclude the presentation of these modern frameworks for cultivating a future-ready workforce, the aim of their presentation should be explained once again. All three concepts aim to provide answers as to which skills or competencies learners should acquire to be part of a future-ready workforce and well prepared for their future lives. Based on the assumption that the future is becoming more complex and that the challenges are constantly changing due to technological and natural developments, complex models are proposed for good reasons.

Interesting to note, this multi-perspective view of complex future problems is not genuinely new, which is why a classic approach is now experiencing a renaissance in English-speaking and Scandinavian countries. It starts from general educational issues and not only, but also, takes the future world of work into consideration.

Klafki (2000) in addition to the already discussed idea of categorical education, i.e. the education of learners in interaction with learning materials, also formulates the idea of epoch-typical key problems that learners should deal with in problem-based learning and project-oriented lessons. His recommendation to devote teaching time to interdisciplinary work to such comprehensive problems is based on his educational question, which fits in perfectly with the OECD Learning Compass:

> "What knowledge, skills and attitudes do young people need today and for their future in order to be able to deal productively with those universal developments and problems and gradually become capable to judge, co-determine and co-design ?" (Klafki, 1998, p. 2)

According to Klafki (1985, 1998), in line with his concept of categorical education, these competencies can only be developed by students dealing with these epoch-typical key problems. In retrospect, these also include the key issues of our society today, such as peace, environmental issues (which encompasses more than just climate protection (the 2030 Agenda for Sustainable Development, 2015)), socially produced inequality, challenges posed by digitalization. In this debate, Klafki sees particular opportunities to stimulate the skills and abilities that can be subsumed under the aspects of critical thinking, collaboration, communication, creativity and agency (Klafki, 1998). These key problems are exemplary in that they enable structural insights that go beyond the concrete problems and enable different (inter-)disciplinary perspectives due to their complexity. This balance of knowledge and learning with a direct future-oriented relevance of the content to be learned goes beyond the mere training of skills (Willbergh, 2016) and can show that (inter)disciplinary perspectives are relevant for complex problem solving and are not just arbitrarily interchangeable topics that have no relevance for the professional or non-professional future.

2.2.2 Domain specific conception: Contribution of mathematics as an example for STEM dealing with complexity

In this section, we will extend our view with how STEM perspectives, in this article in particular mathematical perspectives, can contribute to the development of a future-ready workforce and how these differ from or complement the general concepts.

What STEM is, is received differently in various publications, although the publications agree that there is no fixed definition for this term and that STEM is located in the spectrum between a mere bundle of individual disciplines to the integrated view of science, technology, engineering and mathematics (Basham et al., 2010; Falloon et al., 2020; González-Pérez & Ramírez-Montoya, 2022; Kennedy & Sundberg, 2020; Maass et al., 2019; Playton et al., 2024). This broad spectrum of definitions not only leads to difficulties in practical implementation, e.g. it is unclear what specialist's background the supervising teacher should have (Maass et al., 2019) or whether STEM can only be productive in co-teaching (Basham et al., 2010; Falloon et al., 2020). But also in scientific investigations, e.g. in the identification of data sources (Johnson et al., 2021). There is empirical evidence that science and mathematics is not clearly differentiated by some students (Playton et al., 2024, p. 178).

Multiple answers are also given to the question of why STEM is important to learn. STEM knowledge and skills are becoming more relevant for all citizens, both in terms of their future work and their everyday lives (Basham et al., 2010). Other justifications include the lack of future workers in this field, which also results in corresponding funding for research projects (Daily & Eugene, 2013; Maass et al., 2019; Playton et al., 2024). The goals of STEM teaching also range from the acquisition of (inter-)disciplinary knowledge to 21st century skills (Camilli & Hira, 2019; Maass et al., 2019). This complexity becomes also evident in models that seek to incorporate all aspects (Falloon et al., 2020).

For a contribution to this complex, the question can be raised as to what role a discipline-specific perspective can play in STEM learning (Falloon et al., 2020). Maass et al. (2019) pose this question in relation to mathematics and provide three answers. In mathematics there is a "source of evidence" to support 21st century skills like critical thinking, problem solving and analytical skills (Maass et al., 2019, p. 871). Mathematics helps to understand and make quantized prediction, e.g. constructing mathematical models and applying them on real contexts (Maass et al., 2019, p. 871). Mathematics is important for responsible global citizens (Maass et al., 2019, p. 871). Especially with regard to the second answer, there is great potential to combine mathematics with the other STEM disciplines in an interdisciplinary way, as mathematical models can only be used to structure the world through knowledge in the disciplines, depending on the context (Uhden, 2012). One further question is then, to what extent the different disciplines are perceived as separate at all. In the factor analysis for the STEM Future-CIS instrument, no separate factors could be identified for mathematics and science (Playton et al., 2024). These considerations also show that the question to what extent a mathematical or at least mathematics-based perspective contributes to a future-ready workforce in STEM education is not a one-dimensional problem but is also complex. In addition, mathematics education research is also complex per se.

> "Mathematics education is a field whose domains of reference and action are characterized by an extreme complexity: the complex phenomenon "mathematics" in its historical and actual development and its interrelation with other sciences, areas of practice, technology and culture; the complex structure of teaching and schooling within our society; the highly differentiated conditions and factors in the learner's individual cognitive and social development, etc." (Steiner, 1985, p. 12)

These complexities in mathematics education research can be analyzed in relation to the relationship of mathematics in STEM through three discourses, one theoretical, one historical and one pragmatic (Lerman, 2015). The theoretical discourse deals with the question of how different systems interact, e.g., interactions of mathematics teachers with science teachers in relation to different perceptions of the respective matters of their own and other subjects. In terms of historical discourse, learners could experience how the development of different disciplines depend on each other and have evolved. In the pragmatic discourse, questions can be clarified as to how complex teaching-learning settings can be designed, also regarding the complexity of collaboration processes, such as agile formats at learner and teacher level.

To conclude the role of mathematics in the STEM subjects, something specific to Germany should be mentioned at this point, namely the concept of "fundamental experiences mathematics teaching and learning contributes to Allgemeinbildung" of Winter (1996) which are listed in all preambles to the current curricula for mathematics in Germany. Two of these fundamental experiences explicitly go beyond what can be labelled structural mathematics.

The first fundamental experience is about an application perspective of mathematics. Mathematics lessons should offer fundamental experiences of "perceiving and understanding phenomena of the world around us, which concern or should concern us all, from nature, society and culture, in a specific way." (Winter, 1996, p. 35). Such phenomena include, on the one hand, scientific phenomena that can be modeled accordingly, e.g., motions of moving objects. But also, normative models such as economic models in the context of interest calculation. The third fundamental experience is about "acquiring problem-solving skills that go beyond mathematics (heuristic skills) in dealing with tasks." (Winter, 1996, p. 35). When discussing this third category of fundamental experiences, this process does not necessarily occur automatically but is stimulated by suitable didactic interventions. Nonetheless, this fundamental experience clearly resonates with the general competencies discussed in section 2.2.1 in terms of critical thinking and problem solving.

2.3 Synthesis of general and domain specific conceptions: A need for complex research environments

> "If we as adults, and especially as educators, want to act responsible for the next generation, we must, in my opinion, tackle the theoretical and practical aspects of opening up the key problems of our historical epoch for them." (Klafki, 1998, p. 8)

The given excerpt by Klafki (1998) addresses the responsibility of educators for introducing learners to the key problems of our historical epoch. This is the common theme of all discussed frameworks with a focus on the connection of education and the world of work. Aspects such as environmental protection are highly relevant for many companies today and create further STEM jobs. At the same time, the question that Klafki (1998) himself asks in his work is to what extent these problems can be solved in the classroom. The other general concepts for a future-ready workforce also require complex phenomena, such as those we have identified as fruitful in the STEM field, to develop the skills and competencies mentioned. Adopting a subject specific perspective–also in educational research–leads to a reduction in complexity but does not eliminate it and can thus lead to a perspective on the question of how STEM education can cultivate a workforce for a future that is certainly complex.

This leads to a need for complex research environments that are rich enough but can also do justice to this complexity in terms of methodology. In this way, the following more specific research questions formulated by Li (2022) on the basis of general educational perspectives can be investigated:

- In which ways sustainable collaborations between universities, companies and other partners can be successfully organized?
- In which ways different countries deal with such concepts?
- In which ways programs to promote the skills and competencies of a future-ready workforce can be designed and evaluated in the above sense?

Considering the discussion of the role of mathematics in interdisciplinary STEM education by Maass et al. (2019) we can add more discipline specific questions to Li's (2022) list:

- In which ways the focus on the individual disciplines can be maintained within the framework of an interdisciplinary problem-oriented approach?
- How should scaffolding elements that support such learning processes look like?
- What can multi-perspective teaching with reference to all disciplines look like, and how must the training and further education of teachers be designed to achieve these goals?

These questions arose by evaluating that mathematics plays an understated role in (interdisciplinary) STEM-Settings, contrary to the typical view that mathematics is an important foundation for all STEM disciplines (Maass et al., 2019). We want to focus in the following sections on mathematics educational perspectives and therefore are not able to discuss all connections between STEM and mathematics in more detail. As authors we see the beneficial connections between mathematics and the other STEM disciplines, especially in discussing authentic applications in the mathematics classroom addressing the different STEM disciplines explicit (Dilling et al., 2022; Stoffels, 2017), discussing discipline-specific commonalities and differences (Stoffels, 2024; Witzke et al., 2024) and therefore work together interdisciplinary with different STEM-educational researchers (Stoffels & Hohmann, 2022; Stoffels et al., 2022).

3 Adressing complexity for cultivating future-ready workforce

Following this theoretical analysis, this section discusses in which ways such a rich research environment can be designed. Therefore, the MINTco@NRW project is presented regarding its general idea, the variety of stakeholders and the complex problem-solving processes that are to be stimulated by the project. The second part of this section then provides an overview of the research perspectives taken in the project.

3.1 MINTco@NRW: Complex environment for mathematics educational research

MINTco@NRW is a cooperation project between the University of Siegen and the University of Cologne, which aims to investigate school-based STEM learning for the future. The acronym "MINTco@NRW" is composed of the components "MINT", which is the German equivalent of STEM, but in the order of mathematics, engineering, natural sciences, and technology; "co", which refers to the focus on collaboration both in the sense of collaborative learning and the collaboration of various stakeholders in the project (learners, teachers, companies, universities, and other partners); and finally "NRW", which is the acronym for the federal state of North Rhine-Westphalia, where most of the German schools involved in the project are located. Spread across NRW, the University of Cologne and University of Siegen support teachers at ten schools in the development and implementation of authentic and problem-oriented STEM learning settings. In solver teams, students solve real unsolved STEM problems from companies in NRW in the areas of optimization, digitalization, big data, modelling, and AI. The clue is that these solver teams–embedded in various formats of regular school lessons–work together with counterparts from the USA over a period of 3-4 months in international hybrid settings and thus experience problem-solving processes in a digitalized and globalized world. In doing so, we draw on experiences of the extracurricular predecessors MINT-Pro²Digi (Stoffels & Holten, 2022) and Authentic-STEM (Stoffels, 2023). These have had a positive effect on the skills development and self-efficacy expectations of the students involved and were also characterized by a high level of voluntarily female participants (consistently over 50%).

Teachers take on a mentoring role in MINTco@NRW, which enables them to identify and elementarize suitable authentic STEM problems with external business partners and accompany students in the independent problem-solving process through targeted training elements at the participating universities. The pilot phase in an extracurricular setting was completed in July 2024 and will then be continued in the context of regular interdisciplinary lessons in lower secondary level and upper secondary level (students age, 14–17 years).

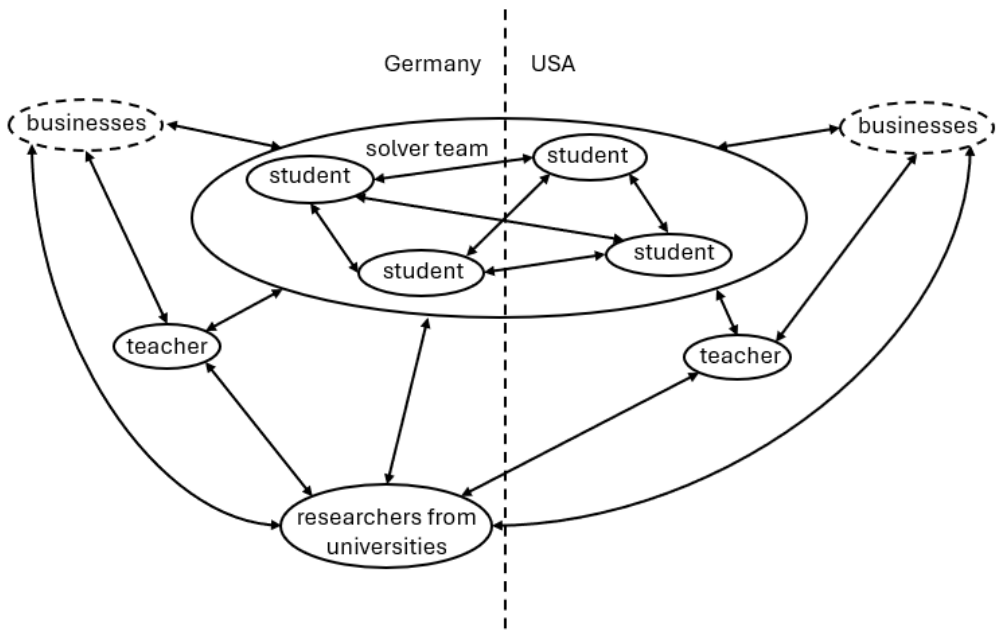
Teachers use in regular classes various forms of performance assessments (grades for participation, long-term portfolio work, final presentations by students as part of the "Forum of Innovation"). One practical aim of research is to develop sustainable formats for learning, teaching, and assessment from the jointly developed and evaluated best-practice materials for regular lessons. To this end, questions relating to mathematics-relatedness, the students' experience of self-efficacy and the mentoring of teachers are taken into consideration.

The complexity and richness of this project to investigate the cultivating of a future-ready workforce in the STEM fields results from the variety of stakeholders and the core of the project, the long-term and complex problem-solving processes.

3.1.1 Variety of stakeholders

Figure 2. shows the various stakeholders in the project who interact during the problem-solving process. This includes a business from the USA or Germany that, together with the teachers, identifies suitable ongoing problems for one to three solver teams in the classroom. A solver team consists of about 3-5 students from Germany and the USA who work together to solve the business' problems. This gives students insights into the respective business and the opportunity to apply their STEM skills and competencies in an authentic context.

Figure 2. Variety and interactions of stakeholders



In the first half of the school year, students learn about the various roles in the project, different project organization formats and problem-solving strategies in a classroom setting. In the second half of the school year, a company representative presents the problem to the international solver-teams as part of a "kick-off". This is followed by a project work phase lasting around 3 months, during which the solver-teams exchange information and work on their solution on a weekly basis using a video collaboration tool. During this period, the business is also visited at least once with the student group, which is located near the business, as it has been shown that STEM projects with local relevance are particularly effective in promoting learner ownership (Falloon et al., 2020). Also, the business representative is open for students' question. The final event is the so-called "Forum of Innovation" in which the students present their solution to the business, their classmates, and the public.

As researchers, we act as catalysts to bring businesses and schools together, develop concepts for project-based learning together with teachers and pursue the research perspectives described in more detail in section 3.2. An essential aspect of all research perspectives is the various interactions that have not yet been part of everyday students' and teachers' life, particularly in cooperation with businesses and universities.

3.1.2 Complex problem solving

The core of the project and the work of the solver teams is the problem of the company. The companies come from various sectors, they can be from the IT industry, but also from the manufacturing industry. In the past companies proposed often problems from the field of Industry 4.0, i.e. digitalization, automation or similar. Accordingly, different disciplines are often stimulated by the problem.

In this article we follow the definition of a complex problem according to Priemer et al. (2020): "A complex problem can be defined as a non-routine situation in which a person tries to follow aims in a partially non-transparent, dynamic, and polytelic environment". The problem-solving process is usually described as a multi-step process in which one or more hurdles must be overcome (Roberts et al., 2022; Rott et al., 2021). However, such problem-solving processes do not only occur in mathematical problem solving, but vary depending on the STEM discipline. Priemer et al. (2020) developed an integrated problem-solving cycle with a total of 13 steps on the basis of a theoretical review, which combines different subject-specific perspectives and their models of problem-solving cycles from the fields of science, mathematics, engineering and technology, as well as computer science. In the MINTco@NRW project, we use this integrated cycle as a template and examine to what extent this integrated cycle is suitable for describing the students' problem-solving processes and which steps the students take. From this, we hope to draw conclusions about the students' subject-specific working methods.

Another aspect that is interesting in the context of complex problems and has potential for further investigation in the project is the joint problem posing process (Baumanns & Rott, 2022) between business representatives and teachers. This will be investigated in the next project cycle.

3.2 Research perspectives in MINTco@NRW

Due to the complex research setting, numerous studies are conceivable. In the upcoming cycle, the research perspectives of mathematical-relatedness, self-efficacy of students and mentoring by regular (STE)M teachers will be examined. Initial insights into these perspectives can be found in section 3.

3.2.1 Mathematics-relatedness

One research perspective within the MINTco@NRW project is the investigation of mathematics-related (Stoffels, 2024) activities (basic mathematical activities, problem solving, modeling, structuring of solution procedures, systematic presentation of solutions, etc.) (Käpnick, 1998; Neumann et al., 2018) of learners working on authentic, unsolved STEM problems from businesses. The starting point of the study is the problem, which is first identified by the business representatives and teachers. It is then analyzed to what extent the learning environments based on this problem have the potential to be mathematics-related and in which ways it stimulates mathematical activities.

Methodologically, a didactic analysis of these problems is first carried out (Hußmann et al., 2016; Kirsch & Scherk, 2012). The extent to which these potential mathematical activities are also implemented by the learners and what existing knowledge they can recognize and activate during solving them is afterwards examined. This is done by video graphing the problem-solving processes of the solver-teams. It is interesting to see whether mathematical activities are revealed that are not to be expected from didactic analysis and how these are compatible with the problems. In addition, the extent to which the problem-solving process as such is compatible with mathematical problem-solving models such as Rott et al. (2021).

In addition to the observations, the mathematical activities that the learners reflect on after the lesson and document in portfolios are examined. These entries offer the possibility of reconstructing perceptions of mathematics in relation to the learners' setting and can be triangulated with the observations.

One aim of this research perspective is to develop a categorization system for students' mathematical activities in complex problem situations in STEM. Also, the relationship between the problem-based learning environment and students' perceptions of mathematics and their mathematical activities are reconstructed. This is expected to provide insights into the influence of authentic, problem-based learning environments on students' mathematical perceptions and activities. Due to the long-term problem-solving phase of over four months, it is possible in this setting to determine changes in the fundamentally stable beliefs.

3.2.2 Self-Efficacy of students

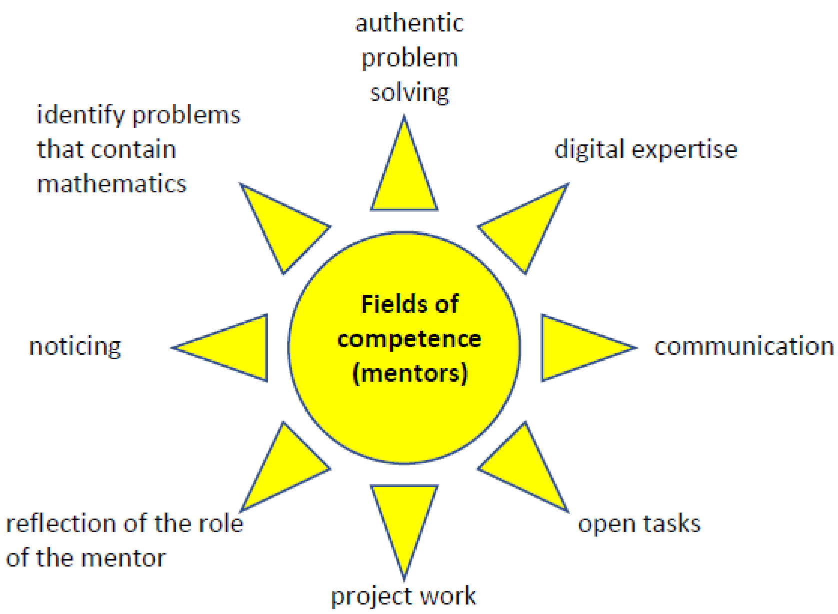
Another research perspective in the MINTco@NRW project focuses on learners' self-efficacy expectations. With the help of an intensive qualitative case study (Yin, 2014) to investigate the relationship between self-efficacy expectations, mathematical skills and the motivation of learners in dealing with the unsolved problem. Here, reference is made to the findings of Bandura (1997, p. 215) as a starting point that "efficacy beliefs predicted interest in, and positive attitudes toward, mathematics, whereas actual mathematical ability did not."

The previous projects have shown that some of the learners experienced their own self-efficacy for the first time in a vocationally oriented setting. Comparable results are in the data (cf. 3.2).

3.2.3 Mentoring by regular (STE)M teachers

Mentoring of the solver teams by the accompanying teachers is an important success factor for supporting complex STEM problem-solving processes (Falloon et al., 2020; Maass et al., 2019). In addition to the challenge of STEM teaching, which, depending on the problem, requires at least interdisciplinary work (Peterßen, 2000) the project setting also requires intercultural communication and coordination with the companies. Accordingly, the research question focuses on how suitable mentoring can be specifically designed by the teachers and which support options are helpful for the teachers.

Figure 3. Fields of competence for mentors in an international project-based learning STEM context



Accordingly, a competence model for mentors was developed on the basis of the predecessor project "Authentic-STEM", which is shown in Figure 3. (Marx & Stoffels, 2023). In an instrumental multiple case study (Yin, 2014) the competence model is used for training of teachers and also serves as a constant source of reflection for the teachers in the project. Through its use and further development, we hope to gain insights into the personal development of the teachers over the course of the project and develop a tool for the sustainable transfer of the project into regular lessons, especially in the starting phase.

4 A complex example: Solving logistical problems with a Lorenz-curve

The following section uses a specific example from the project to illustrate the complexity of the problem and the associated problem-solving processes. Initial insights into the data on the respective research perspectives are provided exemplary.

In the problem, the students should conduct an assortment adjustment of articles that are rented out and solve further problems based on their analysis. The assortment adjustment has a direct influence on the stock of articles held, but also on the organization of a sorting station in which the articles periodically arrive after being rented out. The students have solved the further problem of the sorting station.

Figure 4. Lorenz-curve - Material of an Excel workshop for the Solver-Teams during the solving phase

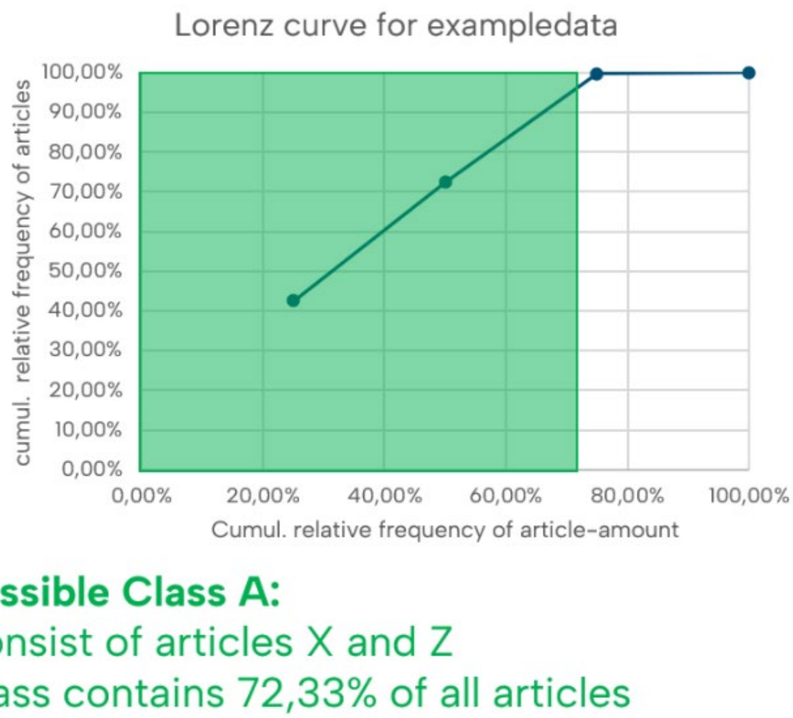
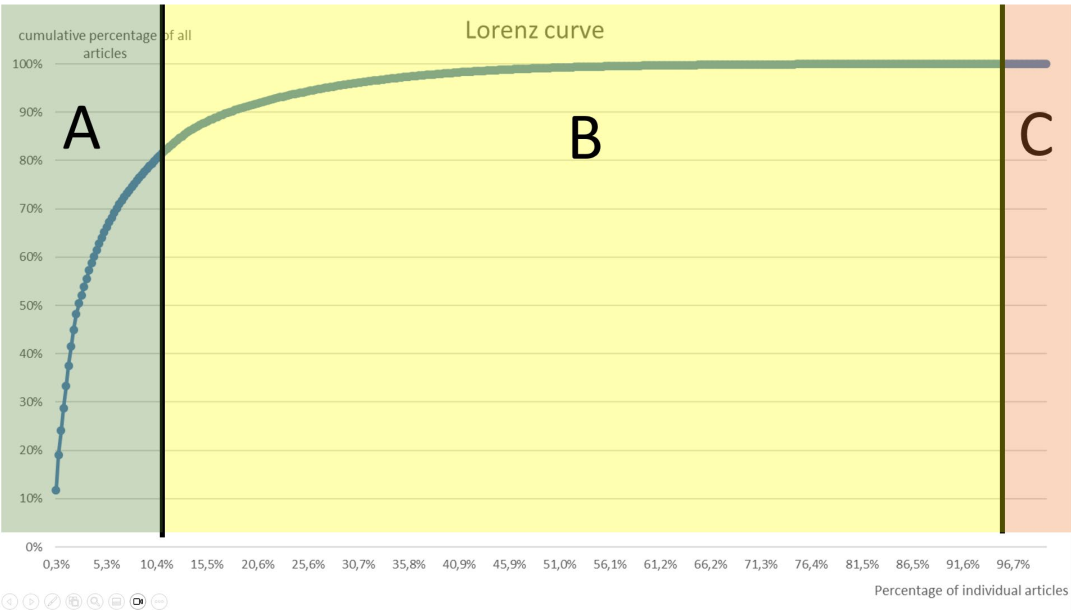


Figure 5. Students' Lorenz-curve



When adjusting a storage, the data must first be analyzed. This is carried out by the company–as it is customary in business management–by graphing a Lorenz-curve and subsequent ABC-analysis (Utsch, 2002; Wächter, 2023). The aim of this analysis is to determine which items are rented out particularly frequently or rarely, to develop a descriptive representation using the Lorenz-curve. The curve is based on the relative frequencies of the items in relation to the total stock. Afterwards an ABC classification can be done, which is often optimal for a pareto ratio of 80:20. Using a very simple example data set, a representation and classification is shown in Figure 4.

After an Excel workshop on pivot tables and charts, the learners then analyzed the company's original data. The data comprised around 170,000 data points associated with 394 article numbers. The results of their analysis and the redesign of the sorting station based on it are shown in Figure 5. and Figure 6.

Figure 6. Students' reorganization of the storage

4.1 (STE)M perspectives on the problem

The Lorenz-curve was developed by the US statistician and economist Max Otto Lorenz in the 20th century and makes inequality distributions visible. In the shown Lorenz-curve most of the articles are not in the majority of article numbers, which explains the upward belly of the Lorenz-curve. As Winter (1996) suggests, economic or logistical problems are very productive for mathematical work. In this case, the statistical analysis led to a geometric analysis.

To conclude this research perspective, an excerpt from an interview (Table 1. Translated by G.S.) from the group's final evaluation is given

Table 1. Transcript on the topic of mathematics-relatedness.

Speaker	
I:	I have one more question about the problems, and you know that I'm a mathematics education researcher, and that's why I'm also interested in the following question: To what extent did you find that there was math in the problem?
S1:	Personally, I didn't think it was very bad.
S2:	Me neither...
S1:	I uhh... I like math umm... I'm not such a big math hater, you know?

Interestingly, the interviewer asked about mathematical-relatedness and not about their emotion of to which mathematics was bad or less bad in the project. Accordingly, mathematics-relatedness also needs to be expanded to include an emotional dimension.

4.2 Self-Efficacy

The students were proud to present their solutions in the "Forum of Innovation". It was particularly self-efficacious that the company representative was so happy about the students' solution that he takes their solution to the next optimization meeting. The students who were interviewed are pupils from lower secondary level one year before graduation (Table 2. Translated by G.S.).

Table 2. Transcript on the topic of self-efficacy.

Speaker	
I:	...what were your highlights?
S1:	What I found particularly good about the project was that we worked together with complete strangers. That we learned once again how we can present our ideas better. .. So... When I had an idea in my head, I explained to the others how it made sense for all of us. I would say that was a lot of fun.

4.3 Mentoring

In the topic of mentoring, there are many aspects that could be discussed in line with the broad fields of competence identified. At this point, we will limit ourselves to two aspects. Firstly, the use of the representation of the mentors' fields of competence as an opportunity for reflection (Figure 3.). For this purpose, the conversation of the mentoring teachers, who should rank the fields of competence from the most important to the least important competence field is given (Table 3. Translated by G.S.).

Table 3. Transcript on discussing the notion of competence fields

Speaker	
T1:	I thought. I understand the authentic [problem solving] now in such a way that you also approach this within the borders of your own, as a mentor, your own competencies.
T2:	That you get the problem solved, correct?
T1:	Uh... yes
T2:	That would be especially important, of course. Yes, that's right.
T1:	Not just get it solved, but solve it the way *you* do, so that you would solve the same problem differently than I do, based on my own abilities and you on your own abilities. And also in relation to the students, if they have to solve it. That *you* would approach the problem differently than *I* would approach it with them.

Here you can see that the teachers are encouraged not only to compare their own skills and experiences with each other, but also to address the effects of differences on their teaching.

Another aspect that will be relevant for the upcoming implementation of the project in regular lessons from August 2024 on is the possibility of transferring the project to regular school lessons. A major issue from the teacher's perspective is assessing students. The teacher in the Interview supervised another solver team that has worked on a different problem (Table 4. Translated by G.S.).

Table 4. Transcript on discussing possibilities of transfer with a focus on assessing students

Speaker	
I:	Where do you see obstacles to implementing this in the classroom?
T3:	In principle, I do believe that this can be implemented. I think a big difficulty will be in the evaluation of the whole thing, because to what extent–now related to the project–so how do I evaluate the proposed solution. So how do I evaluate it. I think that will be exceedingly difficult and to make that evaluation transparent to the students, what you expect. It's not about working out an idea to the end, but rather about this connected thinking, that I think about... how my solution fits in with the other variables in the company, and things like that. So how many thoughts are actually involved. I imagine it's difficult. On the other hand, these are things where I can't just ask ChatGPT somehow or math isn't about applying a stupid formula that I've just memorized. Because I really have to come up with creative ideas myself, work.

5 Conclusions and outlook

The aim of this article was to highlight and benefit from the complexity of STEM Education for a future-ready workforce. This complexity is rooted in various aspects. The world of work is (once again) undergoing major changes and the future is of course unpredictable. This is expressed in the world of work by the terms "Industry 4.0" or 4th Revolution. Accordingly, various concepts have been developed to optimally prepare learners for this uncertain future. Even if some epoch-typical key problems have remained highly relevant for the last 30 years! Also, since STEM education is still a relatively young field of practice and research, it offers further complexities that are difficult to implement in terms of discipline-specificity, necessary collaboration, subject-specific didactical models and the organizational structure, depending on different national formats of STEM learning.

There are many ways of dealing with such complexes, one of which is to break them down. However, this would make interdisciplinary or transdisciplinary STEM activities difficult. Accordingly, research designs and developmental work are needed to do justice to this complexity and connect different stakeholders in society so that–even if only occasionally–there are real applications of STEM that are relevant to the students and that they can experience their relevance in contributing to their solutions. In our view, MINTco@NRW is an example of such a complex setting in terms of regular and long-term intensive cooperation between schools and international partners in education and the working world.

The next step in our project is to investigate sustainable transfer, including suitable materials, for which there is already a wide selection, but also concrete further research on how to measure performance in such complex STEM settings with a long-term duration. Another perspective is, how to prepare teachers to enjoy working competently in such contexts and at the same time keep an eye on the self-efficacy of their students.

Research ethics

Author contributions

G.S.: conceptualization, data curation, funding acquisition, investigation, methodology, project administration, supervision, funding acquisition, visualization, writing—original draft preparation, writing—review and editing
J.S.: data curation, investigation, methodology, visualization, writing—original draft preparation, writing—review and editing,
J.K.: data curation, investigation, methodology, project administration, writing—original draft preparation, writing—review and editing
I.W.: conceptualization, funding acquisition, supervision, writing—review and editing.

All authors have read and agreed to the published version of the manuscript.

Artificial intelligence

To translate parts of this manuscript we used DeepL Translate. After the translation the whole paper was fully reviewed by the authors.

Funding

The project MINTco@NRW is funded by the Ministerium für Schule und Bildung des Landes Nordrhein-Westfalen.

Institutional review board statement

Not applicable.

Informed consent statement

Informed consent was obtained from all research participants.

Data availability statement

Data available on request due to privacy restrictions.

Acknowledgements

We want to thank all participating schools and businesses in Germany and the U.S. of the MINTco@NRW Spring 2024 cycle to open their doors for our project.

Conflicts of Interest

The authors declare no conflicts of interest.

References

Balliester, T., & Elsheikhi, A. (March 2018). *The future of work: a literature review* (Research Department Working Paper No. 29). International Labour Office. https://englishbulletin.adapt.it/wp-content/uploads/2018/07/wcms_625866.pdf

Bandura, A. (1997). *Self-Efficacy: the exercise of control*. W. H. Freeman and Company.

Basham, J. D., Israel, M., & Maynard, K. (2010). An Ecological Model of STEM Education: Operationalizing STEM for All. *Journal of Special Education Technology*, *25*(3), 9–19. https://doi.org/10.1177/016264341002500303

Battelle for Kids. (2023). *Framework for 21st century learning: Definitions*. https://www.battelleforkids.org/wp-content/uploads/2023/11/P21_Framework_DefinitionsBFK.pdf

Battelle for Kids. (2024, January 8). *P21 Resources | 21st Century Learning Resources | Battelle for Kids*. https://www.battelleforkids.org/insights/p21-resources/

Baumanns, L., & Rott, B. (2022). The process of problem posing: Development of a descriptive phase model of problem posing. *Educational Studies in Mathematics*, *110*(2), 251–269. https://doi.org/10.1007/s10649-021-10136-y

Bruner, J. S. (1983). *In search of mind: Essays in autobiography*. Harpercollins. https://pure.mpg.de/pubman/faces/viewitemoverviewpage.jsp?itemid=item_2301309

Camilli, G., & Hira, R. (2019). Introduction to Special Issue—STEM Workforce: STEM Education and the Post-Scientific Society. *Journal of Science Education and Technology*, *28*(1), 1–8. https://doi.org/10.1007/s10956-018-9759-8

Caratozzolo, P., Bravo, E., Garay-Rondero, C., & Membrillo-Hernandez, J. (2021). Educational Innovation: Focusing on enhancing the skills of Generation Z workforce in STEM. In *2021 World Engineering Education Forum/Global Engineering Deans Council (WEEF/GEDC)*. IEEE. https://doi.org/10.1109/weef/gedc53299.2021.9657304

Daily, S. B., & Eugene, W. (2013). Preparing the Future STEM Workforce for Diverse Environments. *Urban Education*, *48*(5), 682–704. https://doi.org/10.1177/0042085913490554

Détienne, F., Baker, M., & Burkhardt, J.-M. (2012). Perspectives on quality of collaboration in design. *CoDesign*, *8*(4), 197–199. https://doi.org/10.1080/15710882.2012.742350

Dilling, F., Rott, B., & Witzke, I. (2022). Mathe in physikalischen Kontexten unterrichten: Chancen nutzen, Herausforderungen begegnen [Teach mathematics in physics contexts: Seize opportunities, meet challenges]. *Mathematik Lehren*(231), 2–6.

Falloon, G., Hatzigianni, M., Bower, M., Forbes, A., & Stevenson, M. (2020). Understanding K-12 STEM Education: A Framework for Developing STEM Literacy. *Journal of Science Education and Technology*, *29*(3), 369–385. https://doi.org/10.1007/s10956-020-09823-x

Fukumura, Y. E., Gray, J. M., Lucas, G. M., Becerik-Gerber, B., & Roll, S. C. (2021). Worker Perspectives on Incorporating Artificial Intelligence into Office Workspaces: Implications for the Future of Office Work. *International Journal of Environmental Research and Public Health*, *18*(4), 1690. https://doi.org/10.3390/ijerph18041690

González-Pérez, L. I., & Ramírez-Montoya, M. S. (2022). Components of Education 4.0 in 21st Century Skills Frameworks: Systematic Review. *Sustainability*, *14*(3), 1493. https://doi.org/10.3390/su14031493

Grzegorczyk, M., Mariniello, M., Nurski, L., & Schraepen, T. (2021). *Blending the physical and virtual: A hybrid model for the future of work* (Bruegel Policy Contribution 14/2021). Brussels: Bruegel. https://www.econstor.eu/handle/10419/251067

Hughson, T. A., & Wood, B. E. (2022). The OECD Learning Compass 2030 and the future of disciplinary learning: A Bernsteinian critique. *Journal of Education Policy*, *37*(4), 634–654. https://doi.org/10.1080/02680939.2020.1865573

Hußmann, S., Rezat, S., & Sträßer, R. (2016). Subject Matter Didactics in Mathematics Education. *Journal Für Mathematik-Didaktik*, *37*(S1), 1–9. https://doi.org/10.1007/s13138-016-0105-5

Johnson, C. C., Peters-Burton, E. E., & Moore, T. J. (2021). *Stem road map 2.0: A framework for integrated STEM education in the innovation age* (C. C. Johnson, E. E. Peters-Burton, & T. J. Moore, Eds.) (2nd edition). Routledge, Taylor & Francis Group. https://doi.org/10.4324/9781003034902

Käpnick, F. (1998). *Mathematisch begabte Kinder: Modelle, empirische Studien und Förderungsprojekte für das Grundschulalter [Mathematically gifted children: models, empirical studies, and support projects for primary students]*. Peter Lang.

Kennedy, T. J., & Sundberg, C. W. (2020). 21st Century Skills. *Science Education in Theory and Practice*, 479–496. https://doi.org/10.1007/978-3-030-43620-9_32

Kirsch, A., & Scherk, J. (2012). Aspects of Simplification in Mathematics Teaching. In *Teaching As A Reflective Practice* (pp. 267–284). Routledge. https://doi.org/10.4324/9780203357781-21

Klafki, W. (1975). *Studien zur Bildungstheorie und Didaktik [Studies in educational theory and didactics]*. Beltz Verlag.

Klafki, W. (1985). *Neue Studien zur Bildungstheorie und Didaktik [New Studies in educational theory and didactics]: Beiträge zur kritisch-konstruktiven Didaktik*. Beltz.

Klafki, W. (1998). Zentralprobleme der modernen Welt und die Aufgaben der Schule: Grundzüge internationaler Erziehung [Central problems of the modern world and the duties of schools: Basic principles of international education]. https://archiv.ub.uni-marburg.de/sonst/1998/0003/k06.html

Klafki, W. (2000). The Significance of Classical Theories of Bildung for a Contemporary Concept of Allgemeinbildung. In I. Westbury, S. Hopmann, & K. Riquarts (Eds.), *Studies in curriculum theory. Teaching as a reflective practice: The German Didaktik tradition*. L. Erlbaum Associates.

Lerman, S. (Ed.). (2015). *Encyclopedia of mathematics education* [Enhanced Credo edition]. Springer; Credo Reference.

Li, L. (2022). Reskilling and Upskilling the Future-ready Workforce for Industry 4.0 and Beyond. *Information Systems Frontiers*, 1–16. https://doi.org/10.1007/s10796-022-10308-y

Maass, K., Geiger, V., Ariza, M. R., & Goos, M. (2019). The Role of Mathematics in interdisciplinary STEM education. *ZDM – Mathematics Education*, *51*(6), 869–884. https://doi.org/10.1007/s11858-019-01100-5

Marx, B., & Stoffels, G. (2023). *Authentic-STEM: Mit Mentoren offene und langfristige mathematikhaltige Projektarbeit begleiten [Authentic STEM: Supporting open-ended, long-term math-containing project work with mentors]*. https://doi.org/10.17877/DE290R-23381

McComas, W. F. (2013). "21st-Century Skills". In W. F. McComas (Ed.), *The language of science education: An expanded glossary of key terms and concepts in science teaching and learning* (p. 1). Sense Publishers. https://doi.org/10.1007/978-94-6209-497-0_1

National Education Association. (2011). *Preparing 21st century students for a global society: An educator's guide to" the four Cs"*.

Neumann, I., Pigge, C., & Heinze, A. (2018). Mathematische Lernvoraussetzungen für MINT-Studiengänge aus Sicht der Hochschulen [Mathematical prerequisites for STEM degree programs from the perspective of universities]. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*(105), 22–26. https://ojs.didaktik-der-mathematik.de/index.php/mgdm/article/view/820/0

The OECD Future of Education and Skills 2030 Informal Working Group. (2019). *OECD Future of Education and Skills 2030: OECD Learning Compass 2030*. A Series of Concept Notes. OECD.

Okenwa-Emegwa, L., Paillard-Borg, S., Tinghög, P., Saboonchi, F., & Strauss, E. von (2017). A global workspace is the emerging reality for future public health workforce. *Socialmedicinsk Tidskrift*, *94*(3), 132–140. https://www.diva-portal.org/smash/record.jsf?pid=diva2:1139856

Pan, J., Cho, T. Y., Sun, M., Debnath, R., Lonsdale, N., Wilcox, C., & Bardhan, R. (2023). Future workspace needs flexibility and diversity: A machine learning-driven behavioural analysis of co-working space. *PLOS ONE*, *18*(10), e0292370. https://doi.org/10.1371/journal.pone.0292370

Pasquinelli, E., Farina, M., Bedel, A., & Casati, R. (2021). Naturalizing Critical Thinking: Consequences for Education, Blueprint for Future Research in Cognitive Science. *Mind, Brain, and Education*, *15*(2), 168–176. https://doi.org/10.1111/mbe.12286

Peterßen, W. H. (2000). *Fächerverbindender Unterricht: Begriff-Konzept-Planung-Beispiele [Interdisciplinary teaching: conception, conceptualization, planning, examples]*. ein Lehrbuch. Oldenbourg. https://ixtheo.de/record/317922912

Playton, S. C., Childers, G. M., & Hite, R. L. (2024). Measuring STEM Career Awareness and Interest in Middle Childhood STEM Learners: Validation of the STEM Future-Career Interest Survey (STEM Future-CIS). *Research in Science Education*, *54*(2), 167–184. https://doi.org/10.1007/s11165-023-10131-8

Priemer, B., Eilerts, K., Filler, A., Pinkwart, N., Rösken-Winter, B., Tiemann, R., & Belzen, A. U. zu (2020). A framework to foster problem-solving in STEM and computing education. *Research in Science & Technological Education*, *38*(1), 105–130. https://doi.org/10.1080/02635143.2019.1600490

Roberts, T., Maiorca, C., Jackson, C., & Mohr-Schroeder, M. (2022). Integrated STEM as Problem-Solving Practices. *Investigations in Mathematics Learning*, *14*(1), 1–13. https://doi.org/10.1080/19477503.2021.2024721

Rott, B., Specht, B., & Knipping, C. (2021). A descriptive phase model of problem-solving processes. *ZDM – Mathematics Education*, *53*(4), 737–752. https://doi.org/10.1007/s11858-021-01244-3

Skott, J. (2019). Understanding mathematics teaching and learning "in their full complexity". *Journal of Mathematics Teacher Education*, *22*(5), 427–431. https://doi.org/10.1007/s10857-019-09446-z

Steiner, H.-G. (1985). Theory of Mathematics Education (TME): An Introduction. *For the Learning of Mathematics*, *5*(2), 11–17.

Stoffels, G. (2017). Tomaten erzeugen Parabeln: Analyse einer Aufgabe mit nicht authentischem Realkontext aus authentischen Perspektiven [Tomatoes create parabolas: analyzing a task with a non-authentic real-life context from authentic perspectives]. *Der Mathematikunterricht*, *63*(5), 3–11.

Stoffels, G. (2023). Authentic-STEM: Opening long-term domains of experience for fostering students' and mentors' self-efficacy through mathematics. In Alfréd Rényi Institute of Mathematics (Chair), *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*. Symposium conducted at the meeting of Eötvös Loránd University of Budapest, Budapest, Hungary. https://hal.science/hal-04420539/

Stoffels, G. (2024). „Flat Earthers", Ernsthaft? – Weltbilder mit mathematischen Methoden interdisziplinär betrachten ["Flat Earthers", seriously? – Taking an interdisciplinary perspective on worldviews using mathematical methods]. In I. Witzke, K. Holten, & F. Dilling (Eds.), *MINTUS - Beiträge zur mathematisch-naturwissenschaftlichen Bildung. Interdisziplinäres Forschen und Lehren in den MINT-Didaktiken: Mathematik mit Informatik, Naturwissenschaften und Technik in der Bildungsforschung vernetzt denken [Interdisciplinary research and teaching in STEM didactics: Thinking about mathematics, computer science, natural sciences, and technology in a networked way in educational research]* (1. Auflage 2024, pp. 9–33). Springer Fachmedien Wiesbaden GmbH; Springer Spektrum. https://doi.org/10.1007/978-3-658-43873-9_2

Stoffels, G., & Hohmann, S. (2022). Comparison: Stochastics with a Focus on Probability Theory. In F. Dilling & S. F. Kraus (Eds.), *Research. Comparison of mathematics and physics education* (pp. 277–297). Springer Spektrum. https://doi.org/10.1007/978-3-658-36415-1_20

Stoffels, G., & Holten, K. (2022). MINT-Pro2Digi: Authentisches projektorientiertes mathematisches Problemlösen in außerunterrichtlichen digitalen Kontexten [MINT-Pro2Digi: Authentic project-based mathematical problem solving in extracurricular digital contexts]. In F. Dilling, F. Pielsticker, & I. Witzke (Eds.), *MINTUS – Beiträge zur mathematisch-naturwissenschaftlichen Bildung. NEUE PERSPEKTIVEN AUF MATHEMATISCHE LEHR LERNPROZESSE MIT DIGITALEN MEDIEN* (pp. 47–71). Springer. https://doi.org/10.1007/978-3-658-36764-0_3

Stoffels, G., Witzke, I., & Holten, K. (2022). Comparison: Differential Calculus Through Applications. In F. Dilling & S. F. Kraus (Eds.), *Research. Comparison of mathematics and physics education* (pp. 227–242). Springer Spektrum. https://doi.org/10.1007/978-3-658-36415-1_17

Strack, R., Carrasco, M., Kolo, P., Nouri, N., Priddis, M., & George, R. (March 2021). *The Future of Jobs in the Era of AI*. Faethm AI; Boston Consulting group. https://web-assets.bcg.com/f5/e7/9aa9f81a446198ac5402aaf97a87/bcg-the-future-of-jobs-in-the-era-of-ai-mar-2021-r-r.pdf

Thornhill-Miller, B., Camarda, A., Mercier, M., Burkhardt, J.-M., Morisseau, T., Bourgeois-Bougrine, S., Vinchon, F., El Hayek, S., Augereau-Landais, M., Mourey, F., Feybesse, C., Sundquist, D., & Lubart, T. (2023). Creativity, Critical Thinking, Communication, and Collaboration: Assessment, Certification, and Promotion of 21st Century Skills for the Future of Work and Education. *Journal of Intelligence*, *11*(3), 54. https://doi.org/10.3390/jintelligence11030054

Tyson, R. (2016). What Would Humboldt Say: A Case of General Bildung in Vocational Education? *International Journal for Research in Vocational Education and Training*, *3*(3), 230–249. https://doi.org/10.13152/IJRVET.3.3.4

Uhden, O. (2012). *Mathematisches Denken Im Physikunterricht [Mathematical thinking in physics lessons]: Theorieentwicklung und Problemanalyse. Studien Zum Physik- und Chemielernen Ser: v. 133*. Logos Verlag Berlin. https://ebookcentral.proquest.com/lib/kxp/detail.action?docID=5223915

Transforming our world, September 25, 2015.

Utsch, A. (2002). *Proof of Pareto's 80/20 law and precise limits for ABC-analysis*. Technical Report (c). University of Marburg.

van Laar, E., van Deursen, A. J., van Dijk, J. A., & Haan, J. de (2017). The relation between 21st-century skills and digital skills: A systematic literature review. *Computers in Human Behavior*, *72*, 577–588. https://doi.org/10.1016/j.chb.2017.03.010

Wächter, L. (2023). Logistik [Logistics]. In L. Wächter (Ed.), *Lehrbuch. Groß- und Außenhandel: Einführung in die Handelsbetriebslehre mit historischen und praktischen Bezügen [Wholesale and foreign trade: Introduction to business administration with historical and practical references]* (pp. 329–385). Springer Gabler. https://doi.org/10.1007/978-3-658-39993-1_10

Weick, K. E. (1993). The Collapse of Sensemaking in Organizations: The Mann Gulch Disaster. *Administrative Science Quarterly*, *38*(4), 628. https://doi.org/10.2307/2393339

Weinert, F. E. (2002). *Leistungsmessungen in Schulen [Performance assessments in schools]* (2. Aufl.). Beltz. https://ixtheo.de/record/1620303078

Willbergh, I. (2016). *Bringing teaching back in: The Norwegian NOU The school of the future in light of the Allgemeine Didaktik theory of Wolfgang Klafki*. https://uia.brage.unit.no/uia-xmlui/bitstream/handle/11250/2599414/ilmi%2bw.pdf?sequence=2

Winter, H. (1996). Mathematikunterricht und Allgemeinbildung [Mathematics education and basic knowledge]. *Mitteilungen Der Deutschen Mathematiker-Vereinigung*, *4*(2). https://doi.org/10.1515/dmvm-1996-0214

Witzke, I., Holten, K., & Dilling, F. (Eds.). (2024). *MINTUS - Beiträge zur mathematisch-naturwissenschaftlichen Bildung. Interdisziplinäres Forschen und Lehren in den MINT-Didaktiken: Mathematik mit Informatik, Naturwissenschaften und Technik in der Bildungsforschung vernetzt denken [Interdisciplinary research and teaching in STEM didactics: Thinking about mathematics, computer science, natural sciences, and technology in a networked way in educational research]* (1. Auflage 2024). Springer Fachmedien Wiesbaden GmbH; Springer Spektrum. https://doi.org/10.1007/978-3-658-43873-9

Yin, R. K. (2014). *Case Study Research* (5th ed.). SAGE Publications.