Argument structure in explaining wave-particle duality of photons in double-slit experiment: A study pre-service teachers' written reports

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We analyze here how pre-service teachers explicate their views about the wave-particle duality of photons and what role it plays in their arguments supporting the quantum nature of light. The data for the analysis is provided by 12 written reports about the double-slit experiment with feeble light. The analysis is based on constructing semantic networks corresponding to pre-service teachers' written texts. Contingency-like associative correlation between word-pairs is used to differentiate between word-pairs, where associations of two terms or words is systematic. Such associations indicate connections, which are significant for key term vocabularies in construction of inferences and arguments. Based on that information of the key vocabulary we then construct the structure of pre-service teachers' argument for the nature of the photon and its wave-particle duality, in the form of directed argument graphs (DAGs). The results show that argument structures in four to six out of 12 cases meet the goals set for pre-service teacher education. In these cases, experimental aspects and wave-particle duality play an important role in the pre-service teachers' argument and its structure.

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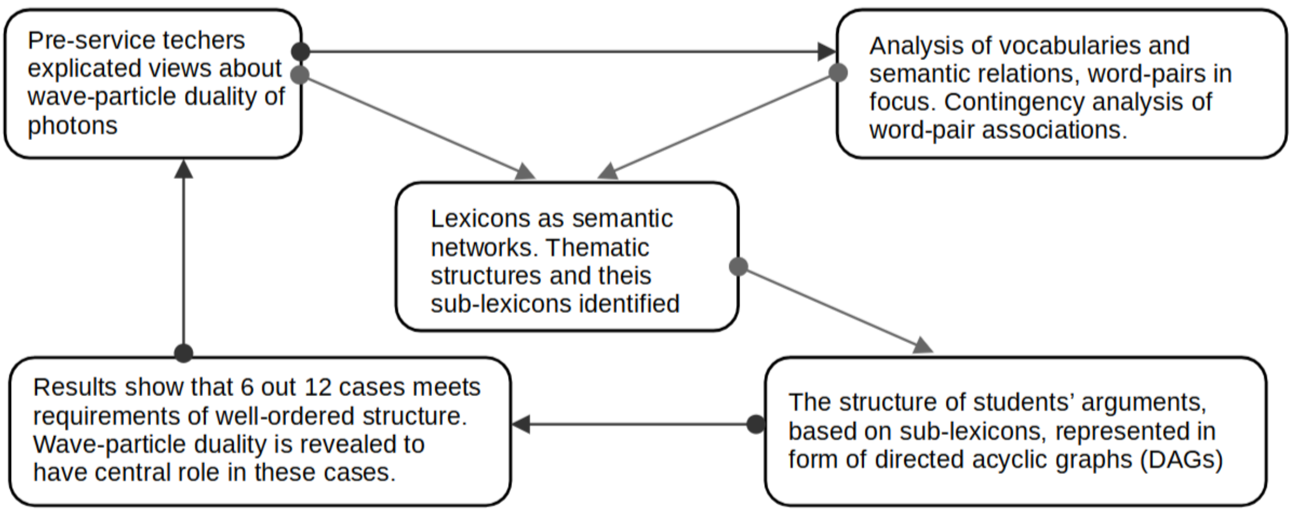

1 Introduction

Science education research has paid a lot of attention to problems related to learning about the wave-particle duality of quantum entities (Ayene et al., 2019; Cheong & Song, 2014; Didiş et al., 2014; Henriksen et al., 2018). Wave-particle duality plays several roles in teaching introductory quantum physics (Cheong & Song, 2014). Sometimes it is seen as a central topic in quantum physics, and thus crucial in teaching quantum physics (see e.g., Ayene et al., 2019; Bøe & Viefers, 2023). However, even when it is noted that such a stance is no longer tenable but rather a remnant from the history of quantum physics (see e.g., Hentschel, 2018), wave-particle duality is nevertheless seen as a suitable didactic starting point for a learner in constructing mental models of a new quantum entity (Bungum et al., 2018; Cheong & Song, 2014; Henriksen et al., 2018). The science education research including wave-particle duality has been comprehensively summarized in many recent studies (Cheong & Song, 2014; Krijtenburg-Lewerissa et al., 2017) whose main findings are not reiterated here. The focus here is on the explicated, declarative knowledge that pre-service teachers use in expressing their views regarding the quantum nature of light, photons, and the role of wave-particle duality in it. The focus is thus on use of language, not in mental models. The importance of language in teaching and learning quantum physics at the introductory level and in teacher education has increasingly been recognized in recent studies (Bøe & Viefers, 2023; Bouchée et al., 2022; Bungum et al., 2018; Henriksen et al., 2018; Vuola et al., 2023).

Here, we continue the analysis of the vocabularies that pre-service teachers use to express their views about photons and wave-particle duality in the context of double slit experiments (Vuola et al., 2023). Attention is now paid not only to vocabularies, but also on the structure of the didactic arguments that pre-service teacher students construct using the vocabularies. Here, the didactic argument is understood as a didactic reconstruction (Duit et al., 2005) of reasoning and inference leading to conclusions that are correct enough from the perspective of contemporary scientific views. The pre-service teachers' ability to explicate the content of teaching in the form of a well-organized didactic argument is of uncontested importance; only then teaching can give students a fair chance to follow and understand the targeted knowledge. The texts written by pre-service teachers provide ample valuable information of such abilities, without a pressing need to ask deeper questions about underlying mental models or conceptual structures. This simple and even self-evident notion motivates a closer look at the vocabularies and structure of didactic arguments used by pre-service teachers in explicating their views; argumentation is a way to communicate coherent and well-ordered teaching. The ability to construct well-organized didactical argumentation is important in teaching (Fischer et al., 2014; Nousiainen & Vuola, 2023).

In a previous study (Vuola et al., 2023) exploring pre-service teachers' use of the term *photon* in written reports about two-slit experiments with photons, frequency-based co-occurrences of words were used as the basis for semantic network (i.e. network of relevant words and terms that students use in their explanations) construction. Here, the same task and empirical data are used. A short summary of the task is provided in Appendix A. The analysis of key terms and vocabulary associated with photons (i.e. the lexicon of photon) was based on communicability centrality (Vuola et al., 2023). This study builds on that previous study, using the vocabularies as a starting point to investigate the argument structure of pre-service teachers' written reports. Because the vocabularies are central to the present study, we provide a summary of the methods used to construct them and the results in Appendix B.

Finding a way to construct (or re-construct) the structure of didactic arguments that pre-service teachers use to support their views (now about the quantum nature of photons and wave-particle duality) is quite a complicated task. One option is to start with some prescribed, view of how a good argument should be constructed (see e.g., Erduran et al., 2004; Sampson et al., 2011). For example, Toulmin's scheme of argument (Toulmin, 2003) has been widely used in science education (Erduran et al., 2004). However, we have opted to avoid adopting any prescribed overall structure. Our approach here is inspired by a pragmatically oriented, context dependent viewpoint on argument (or inference) construction (compare with Brandom, 2009; 2010). The approach we take to pre-service teachers' argument construction is based on the view that, for a good argument or inference, vocabulary is the basic and necessary (if not sufficient) starting point we need to pay attention to. In constructing an argument or inference, in its various thematic parts (here, for example, theory, experiment, or conceptual interpretation), different sets of vocabulary are used. Tracking the overlap of vocabularies from one thematic context to another, it becomes possible to track the information flow, to the degree revealed by vocabularies. This viewpoint and approach shares many similarities with pragmatic semantics, where vocabularies and their relationships in the context of explicating ideas and using the vocabularies are central (Brandom, 2009; 2010).

Following the guiding ideas outlined above, we think that vocabularies and their sub-vocabularies, as they are used in a given context, can be helpful in finding the building blocks of argument structures. Here, by improving and extending the previous study on the two points mentioned above, we perform a re-analysis of 12 cases included in a previous study (Vuola et al., 2023) and made available for the present study. The focus is here on a method of finding the argument structure of pre-service teachers' written reports and how the structure appears from the viewpoint of what is known from contemporary understanding of the same topic (i.e. wave particle duality and single photon interference). The educational implications of the results are discussed briefly, suggesting that care should be taken in considering the usefulness of history-biased views in teaching about the quantum nature of light and the wave-particle duality of photons.

2 Materials and methods

We introduce here a method to infer and construct the structure of arguments within pre-service teachers' written texts about the formation of interference fringes from individual detection events in double slit experiments on feeble light. The written texts and the task they are connected to are the same as already reported and analysed in our previous study, focusing on pre-service teachers' vocabularies (Vuola et al., 2023). A summary of the most important details regarding the task is provided in Appendix A. The structure of an argument is constructed on the basis of semantic networks, consisting of the vocabulary of terms used by pre-service teachers to explicate their ideas and views. The basic idea is to use the vocabularies as a proxy for the content of texts in various thematic contexts (i.e. theory, experiments, or interpretations) related to the task. The overlap (i.e. shared parts) between vocabularies across different thematic contexts is taken as a proxy for how information contained in vocabularies (i.e. lexical information) is passed between and across different thematic contexts.

Assuming that a flow of shared information between different pieces of text provides information about the structure of argument is in line with how pragmatic semantics approaches the role of vocabularies and their relation to context of use in making inferences (Brandom, 2009; 2010). It also agrees with views in which the argument is seen primarily in an epistemic role, in explicating and justifying knowledge (Wohlrapp, 2014) rather than as a form of persuasion. Consequently, we try to recover the structure of argument as it emerges from the usage of vocabularies across various relevant contexts. This attempt can be seen as complementary to more traditional approaches on argumentation in the context of learning (see e.g., Erduran et al., 2004; Sampson et al., 2011).

The attempt to construct the structure of an argument as it emerges from pre-service teachers' written texts consists of three steps. We try to avoid pre-fixed ideas of the general form of the argument as much as possible and base our analysis on the usage of key term vocabularies across the varied relevant contexts. Therefore we must rely heavily on semantic analysis of the texts. The analysis consists of three stages, which are construction of: 1) vocabulary; 2) lexicons in the form of semantic networks, based on vocabulary; and 3) argument structure based on lexicons in the form of a directed argument graph (DAG). Mathematical and technical details are provided in Appendices B and C. However, those details are not necessary to capture the basic ideas and results but are nevertheless needed for replication of results and application of the method.

2.1 Empirical setup and sample

In this study, we use as an example an empirical sample reported previously in greater detail (Vuola et al., 2023). The data used here was obtained from the authors of previous study, in fully anonymous form (for data handling, ethical issues, and consent, we refer to the original study). The empirical sample of this study consists of reports (N=12) written by pre-service physics teachers. The length of the pre-service teachers' written reports was usually 1–2 pages. The details of the task and its educational objectives are reported in detail elsewhere (Nousiainen & Koponen, 2020; Vuola et al., 2023) and therefore only a brief summary is provided below, with further details in Appendix A.

In their reports, pre-service teachers describe what happens in a double-slit experiment where an interference pattern builds up in a detector from single recorded hits (data obtained from Rueckner & Titcomb, 1996). In the experiment, feeble (attenuated) laser light is guided through a double slit and intensity is measured by a position sensitive, low light level camera with image intensifiers (Rueckner & Titcomb, 1996). In the experiment, it is possible to observe single, random detection events and accumulation of interference fringes when several events are aggregated. Although the experiment is not a true single photon interference experiment (see e.g., Bhatta, 2021; Hentschel, 2018), it can be used to emulate single photon interference for the purposes of instruction. The task consists of providing arguments for: 1) what one needs to assume about the behavior of photons in a slit-system when only one slit is open or when both slits are open; and 2) how to interpret the gradual formation of interference fringes when several detection events accumulate. In constructing the argument, pre-service teachers are expected to discuss the quantum nature of light, the photon concept in describing the quantum nature of light, photon self-interference, and the impossibility of splitting a single photon. In addition, we expect that wave-particle duality will play a central role in many explanations. It should be noted that while wave-particle duality is not a necessary viewpoint, it has nevertheless been recommended an introduction to more sophisticated views on the quantum nature of light (Bungum et al., 2018; Cheong & Song, 2014; Henriksen et al., 2018).

2.2 Vocabulary and lexicons

The vocabularies used by pre-service teachers in their written explanations are constructed by finding the key terms and words that pre-service teachers use in explicating their views. The construction of vocabulary in the context of double slit experiments has been reported in detail in our previous study (Vuola et al., 2023), and here, with regard to the vocabulary, we build directly on those results. Construction of a vocabulary starts from a pre-selected group of 90 terms and words, on basis of the frequency of their co-occurrence in the sentences of the written reports. From this set of vocabulary items, a semantic network is formed so that nodes correspond to terms or words and links between them are weighted according to the co-occurrence of the words in sentences. A more limited network of about 60 of the most strongly related terms and words is selected, and a semantic network consisting only of these is formed to represent the connected vocabulary of each of the pre-service teachers' written text. This vocabulary is referred to as a lexicon that consist of terms contained in vocabularies (nodes in the network) and the relationships between them (links in the network). In the present study, the co-occurrences of words in sentences is operationalized (quantified) in the form of associative correlations between words, measured by classical φ-factor of contingency (Bonett & Price, 2007). In the previous study, we used a method that was more elaborate than contingency based co-occurrence, but otherwise, the analysis of vocabulary and construction of lexicons of interest follow the approach introduced there (Vuola et al., 2023). Details of the analysis are provided in the previous work (Vuola et al., 2023) and summarized here in Appendix B. In Appendix B, we also provide a brief comparison of the results based on the current method to those obtained previously. The comparison is not essential in what follows but ensures that the current, simplified analysis and the previous, more elaborate one are sufficiently similar for all practical purposes.

The lexicon is a semantic network of connected terms and words (Nousiainen & Koponen, 2020; Vuola et al., 2023). It contains a summary of the information about the most important terms and their mutual connections that students (pre-service teachers) attach to target terms of interest, which is here the term *photon*. Construction of a lexicon starts from links between nodes: connections between word-pairs where the positive values of contingency φ-factors exceed a prescribed threshold value, usually 0.20. These word-pairs are likely to appear more often together than expected on the basis of single word frequencies; co-occurrence has a high statistical significance, for example in case of the pairs *photon* and *light-quanta* or *photon* and *energy-quanta*.

To form a lexicon, nodes (terms and words) are associated with a strength that is related to their global connectivity, i.e. the weighted number of paths connecting them to all other nodes in the network. This strength is the so-called communicability centrality γ_k of a given node k (Estrada, 2012; Estrada et al., 2012) and it describes how well a given node can communicate with all other nodes. Often, but not always, such nodes are the terms and words that also have the highest total contingency factors (sum of all weighted links representing contingency factor-based links). Mathematical details on how the communicability centralities are computed are provided in Appendix B.

The lexicon contains all the information of the vocabulary and its usage that is taken into account in the construction of argument structures in what follows. The lexicon also allows us to find the key terms and words that are most important globally within the entire lexicon. The previous study (Vuola et al., 2023) focused on such key words and lexical profiles consisting of key words. Here, that information, although not of direct interest, nevertheless underpins the construction of argument structures.

2.3 Argument structure

The lexicons are the basic units of analysis that are used to construct the argument structure, in form of directed argument graphs (DAG) that describe how information in pre-service teachers' written reports flows from initial assumptions to final conclusions. To construct a DAG, we first discern the relevant thematic contexts of the task. These we call thematic blocks. Thematic blocks are decided on the basis of what is known about the acceptable arguments and sub-arguments in the case of the given task of double-slit experiments with feeble light (see e.g., Ayene et al., 2019; Cheong & Song, 2014; Rueckner & Titcomb, 1996) and how it has been discussed from the viewpoint of conceptual analysis (Bhatta, 2021; Hentschel, 2018). The research literature about the double-slit experiments and single photon interference thus guides which connections between different thematic blocks we take to be potentially relevant or possible. It should be noted that this does not predetermine the argument structure, which depends on the extent of explicated content of the thematic blocks.

Next, we need to operationalize the extent of the thematic block, providing the strength of the given block. In different thematic blocks, pre-service teachers use different parts of the available lexicons to explicate their ideas and the views they found to be relevant (e.g. theoretically, experimentally, or for interpretation). The more extensive and well connected the sub-lexicon, the more information is contained in the thematic block. Consequently, the strength Γ_X of the thematic block is operationalized through the total communicability centrality $\Gamma_X = \sum_{k\in X}\gamma_k$ of the sub-lexicon X. Because communicability centrality as a global measure of connectivity of a node within the network is related to the ability to communicate with all other nodes, such a measure describes how nodes can participate in transporting or feeding information within the structure. The step-by-step mathematical details on how Γ_X is obtained from values of contingency φ-factors are provided in Appendix C.

In the DAG, directionality means that a thematic block Y is preceded by another thematic block X, which acts as support or backing for the flow of information in the argument. Thus, DAG provides a directed flow diagram representing the information flow in the argument from X to Y as a directed link X→Y. Next, we need to take into account the simple notion that if sub-lexicons in X and Y are identical, there is no new information to be passed from X to Y. On the other hand, if the lexicons are entirely different, it is unlikely that block Y can accommodate information from X. Therefore, we take into account the similarity S_{XY} of the blocks X and Y. Sub-lexicons X and Y may be identical $S_{XY} = 1$ and they may be completely different $S_{XY} = 0$. In both cases, we assume that there is no new information passed on from X to Y. To quantify this notion, we attribute to each link X→ Y a weight W_{XY} describing its transfer capability, given by $W_{XY} = \sqrt{\Gamma_X\, S_{XY}(1 - S_{XY})}$, which is a geometric average of terms related to the strength of the node probability of transfer. The mathematical details of the construction of DAGs and how the key quantities Γ_X and S_{XY} are computed from the contingency φ-factor are provided in Appendix C. These details are not necessary to understand the qualitative content and meaning of the results. However, the details in Appendix C are needed for replication of the results or a similar analysis.

The complete DAG is specified by its directed links with weights W_{XY}, their values depending on sub-lexicons attached on each block. The DAG describes how information is fed from each block to another: certain blocks act predominantly as senders of information, other blocks as receivers. Roughly, the blocks with outgoing arrows are senders and the blocks with incoming arrows are receivers. However, the strength of the nodes may be very different, and remembering that blocks with identical lexicons do not exchange information, the role of each block can be very different irrespective of its incoming and outgoing links. The role of each block in the information flow is conveniently described through the strength of incoming or outgoing communicability, again using the total communicability as a measure. Technically this is done similarly as with lexicons, resulting now in total communicability-based strength of incoming $I_{IN}(X)$ and outgoing $I_{OUT}(X)$ information flows for a given thematic block X. The values of I_{IN} and I_{OUT} allows us to track each of the pre-service teachers' argument and how it starts from the explicated initial assumptions (usually sending blocks in DAG) to final conclusions and inferences (usually receiving blocks in DAG).

2.4 Research Questions

The research questions we pose concerning the concept of the photon as it appears in pre-service teachers' written reports about the double-slit experiment concern the construction of lexicons and structure of arguments as they are discernible from the pre-service teachers' written reports. The research question are thus as follows:

RQ1: What are the lexicons characterizing the term photon?
RQ2: What is the argument structure supporting the dual nature of photons?

In a broader view, answers to RQ1 provide information on the terms and vocabulary that pre-service teachers find of importance and have chosen to use in their descriptions of the task, while RQ2 provides information on the pre-service teachers' ability to construct didactic arguments. The research questions are connected: answering RQ2 requires that RQ1 is answered.

3 Results

The 12 written reports analyzed here contain a variable number of (main or subordinate) clauses (from 31 to 177, with an average of 82), and consequently variable number of terms and term-related words. Here we pay attention to how different terms co-occur with the term *photon*. First, we discuss basic statistics that characterize lexicons. Second, we focus on the lexicons and their properties (RQ1). Third and finally, we turn to the question concerning the structure of argument (RQ2).

Frequency statistics of the term occurrence in the 12 texts analyzed are provided in Figure 1, which shows the occurrence (number) frequency distribution *n* of the 24 most frequently occurring attractively associated (i.e. with positive contingency) words (Fig. 1a, at left) and the distribution of *n* of 80 terms and term-like words (Fig 1b, at right) that appear in at least two word-pairs. The distributions are shown as a function of rank of occurrence, the most common term having the highest rank of 1; the larger the rank number, the lower the frequency. The frequency distributions shown in Fig. 1 already contain information on the importance of words in the lexicons and it is interesting to note the exponential decay with increasing rank. However, frequency statistics do not yet provide enough information on the co-occurrence of words in sentences when words that repeatedly co-occur with the term *photon* are of special interest.

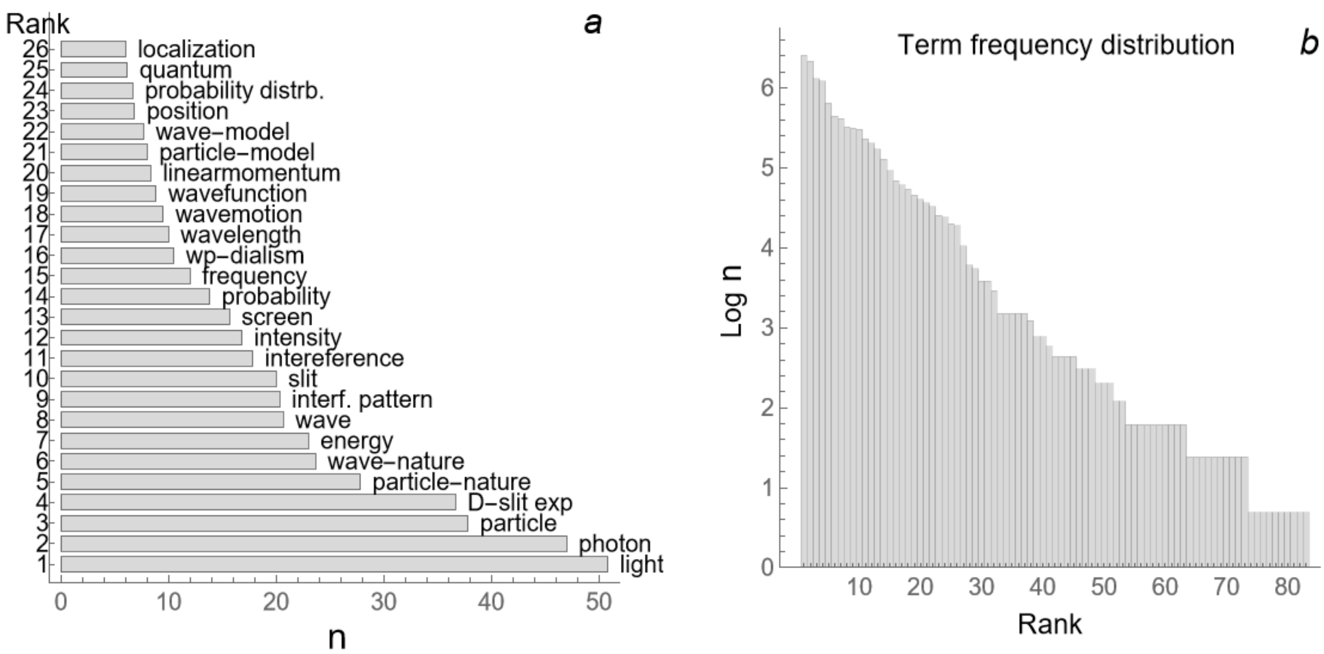

Figure 1. The distributions of occurrence (number) frequency *n* of terms and words in the sample of 12 reports written by pre-service teachers. The 26 most frequently appearing key terms are shown at left (a). At right (b) is shown the distribution of 60 terms appearing at least once (on average) per report.

3.1 Lexicons and key terms

We next turn to co-occurrence as measured by the φ-factor of contingency and then on that basis construct the lexicon of the term *photon* in the form of a semantic network in which nodes represent pertinent terms and words in the lexicon. The links between the pairs of words are φ-factors. To construct the lexicons of the term *photon*, we have selected 60 terms and words that have high contingency value to a photon term or to another term that has high contingency to a photon term (i.e., the nearest and next nearest neighbors to a photon term). The 24 terms that appear most frequently attractively associated (i.e. with positive contingency) with the term *photon* are the key terms, and they are listed in Table 1. These 24 terms exhibit a different ranking from the total frequency-based rankings in Figure 1.

Table 1. The 24 most frequently appearing terms attractively associated (i.e. positive contingency) to the term *photon* (Ph). Symbols refer to Fig. 2. The numbering provides the ranking of terms as based on frequency of their appearance. Note the difference from the frequency of appearance shown in Fig. 1.

Term		Term		Term		Term	S
1. Light	L	7. Wave-natr.	wN	13. Probability	pr	19. Wavemotion	
2. D-slit expr.	XD	8. Wave	w	14. Frequency	fr	20. Position	
3. Interf. pattern	FP	9. Interference	IF	15. Wavefunction	Ψ	21. Lin. momentum	
4. Particle	p	10. Intensity	in	16. Wavelength	wl	22. Quantum	Q
5. Slits	Xs	11. Particl. natr.	pN	17. Screen	Xd	23. Particle-model	pM
6. Energy	E	12. WP-duality	D	18. Wave-model	wM	24. Localization	lo

Figure 2 shows lexicons corresponding to the texts of the 12 analyzed reports. The size of the nodes is proportional to their communicability centrality γ within the complete lexicon. Note that only some of the links (100 randomly chosen from a set exceeding the values of 60% of the maximum) are shown. Of these, 24 are the key terms listed in Table 1 (for abbreviations used in Fig. 2. see Table 1). In Fig. 2, all 12 lexicons are projected on the same template of the aggregated lexicon. However, it is nearly impossible to discern relevant similarities or differences by visual inspection, apart from the notion that the lexicons share a limited set of about 10 to 15 terms that appear in most lexicons. All lexicons also contain terms and words that have high communicability centralities but are not, however, relevant to discussions of the physics aspects of the problem. Such terms and words (i.e. those without labels) are often related to certain specific laws, notions augmenting the discussions from the perspective of physics history, etc. The lexicons as shown in Fig. 2 do not yet reveal much about the argument structure but show only the elements from which each argument is composed. Nevertheless, Fig. 2 provides an overall picture of what the lexicons look like, and how much variation there is between the lexicons. The sub-lexicons to be used in the construction of the argument structure are always parts, sub-lexicons of the lexicons shown in Fig. 2.

In what follows, we do not discuss in detail the vocabularies of individual lexicons, already discussed previously (Vuola et al., 2023), providing also a detailed communicability-based analysis of vocabularies and their lexical profiles. In Appendix B, a brief comparison of the results based on the method used here to the results obtained by a more elaborate previous method are compared to show that the results agree and no significant differences are found. This justifies our use of the simplified method.

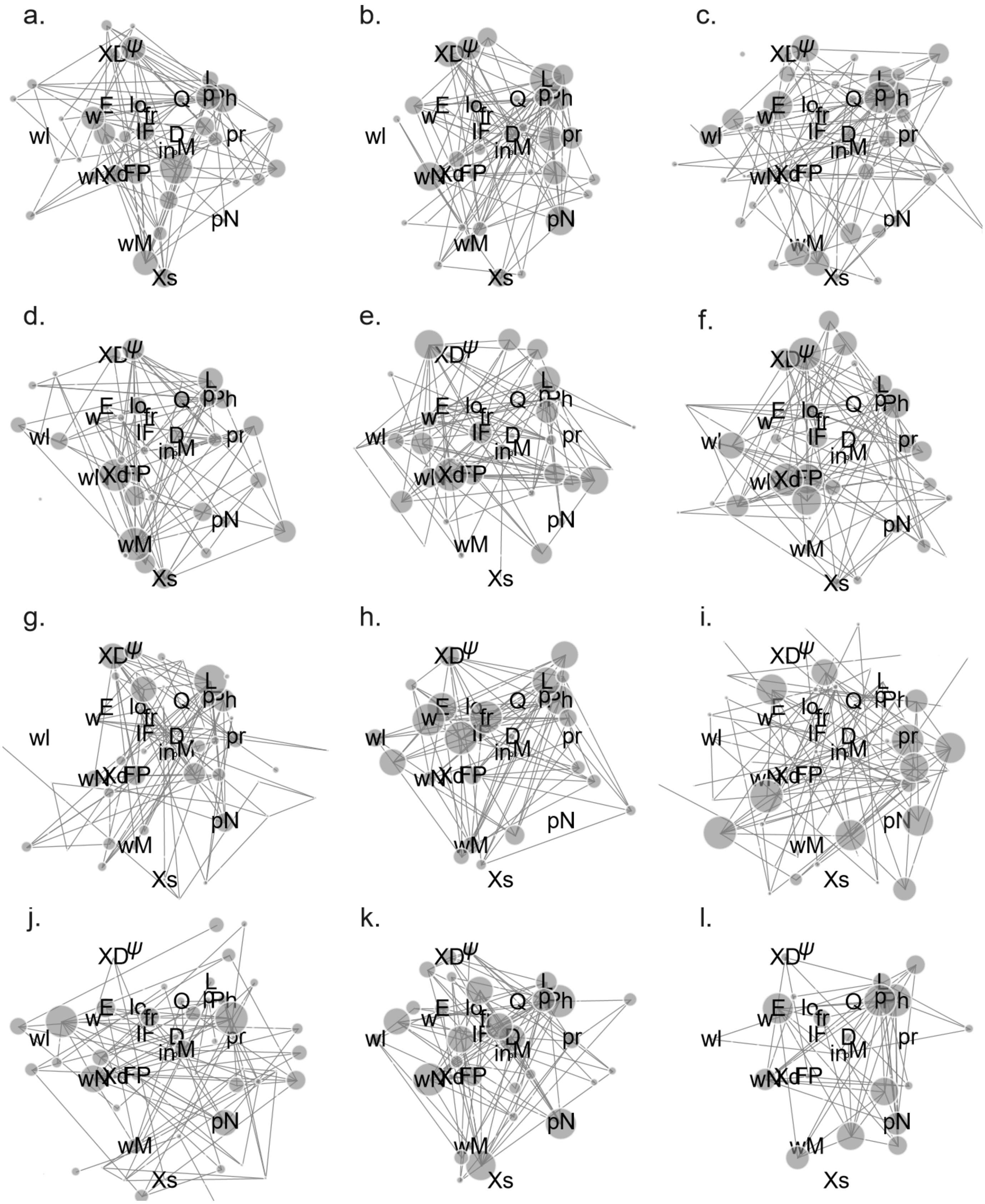

Figure 2. 12 Individual lexicons (a-l) projected on a common aggregated lexicon. Abbreviations identifying terms are explained in Table 1. The sizes of the nodes are proportional to their communicability centrality. Only 100 links (randomly chosen) of the set of the 200 most important links are displayed.

3.2 Argument structure

Next we turn to answering research question RQ2 concerning the argument structure. Argument structure is rendered in the form of directed argument graph DAG as explained in section 2.3. The blocks corresponding to the thematic contexts are provided in Table 2, with information on the most important key words that are associated with the given thematic block. This information is provided in the form of word-pairs (x,y) of sets {x} and {y} of key-words. Note that in recognition of sentences belonging to a given thematic context, it is enough that any pair of key-words x and y appear in a given sentence. The sub-lexicons of each block in DAG consist of key-words thus associated with the given block. The links, i.e. relationships between blocks within the DAG, are pre-fixed on the basis of the known task-structure (i.e. what is known to be adequate reasoning for the double-slit experiment). However, in DAGs corresponding to individual pre-service teachers' texts, links can be zero-weighted and thus non-existent; template DAG provides constraints on possible links but when pre-service teachers' texts are analyzed and sub-lexicons formed, some of the links may turn out to be very weak or zero. The sending blocks of resulting DAGs a-l are shown in Figure 3 for all 12 cases (with total lexicons shown in Fig. 2). The skeletal, pre-fixed structure of the DAG with possible, acceptable links is the same in all cases. The sizes of the nodes correspond to the values I_{OUT}.

Figure 4 shows the receiving blocks, with the size of nodes now corresponding to the values I_{IN} characterizing the incoming information flow. When sending blocks remain small, it is mainly due to the low strength of the lexicons, but when receiving blocks remain small, it is due to the strength of sending lexicons and the similarity between the lexicons of the sending and receiving blocks. The DAG with associated strengths I_{OUT} and I_{IN} now describes the structure and information flow of the argument as it can be extracted from pre-service teachers' written text, on the basis of semantic analysis of text. We next discuss the relevant characteristic features of the DAGs.

In the case of the sender blocks shown in Figure 3, the DAGs corresponding to lexicons a-d and i have strong thematic blocks related to experimental details (XD, Xs and Xd) of double-slit experiments, at least two of them being strongly featured. In addition, at least one of the blocks IF and FP, related to interference, is strong in these cases. It is noteworthy that blocks IF and FP appear strong as sending nodes but also as receiving nodes; they are thus important nodes in transmitting information in argumentation as represented in the form of DAGs. This is in good agreement with how the double slit experiment of feeble light is discussed in research literature (see e.g., Bhatta, 2021; Rueckner & Titcomb, 1996), where many discussions focus on interference (IF) and fringe patterns (FP). It is also expected that if IF or FP receive attention, experimental aspects XD, Xs and Xd should also be important. DAGs e and f are interesting, because they are very similar although the corresponding total lexicons are clearly different. In these cases FP is hugely central and XD also strongly featured.

Table 2. The blocks in directed argument graph (DAG), their acronyms (Acr.) and the word pairs (x,y) to identify them (in the middle column) consisting of words x and y (in the last column, numbers referring to pair (x,y) in the middle column). Note that certain terms that would be multiple words in English appear as single compound words in Finnish.

Block in DAG	**Acr.**	**word pairs (x,y)**	**Sets {x} and {y} of key-words in pair (x,y)**
Quantum light source	QLS	(1,10); (2,5); (8,11)	1: light, energy, energy quantum, source 2: d-slit expr, screen, detector, position, localization 5: probability, observation, prob. wave, prob. density 10: photon, coherent light, single photon 11: quantum, quantum theory, quantum light, boson
Photon, coherent light	cPh	(1,9); (12,9)	9: light, chaotic light, regular light, coherent light, monochromatic 12: light, interference, chaotic light, regular light
Photon, quantum light	qPh	(8,11); (11,15)	8: single photon, half-photon, non-split, light quantum 15: photon, single photon
Single photon	sPh	(8,11); (8,12)	
Non-splitting (photon)	NS	(7,8)	7: identifiable, path, path-differc., anticorrel
Self-interference	SI	(8,13)	13: interference-pattern, fringe-pattern, interference
Interference, fringes	FP	(3,7); (13,14)	3: slit, slit-system, double-slit 14: intensity, intensity distribution
Interference, intensity	IF	(11,14); (12,14); (13,14)	
Double-slit experiment	XD	(3,6); (4,9)	4: d-slit expr., wave, classical wave 6: interference-pattern, fringe-pattern
Slit system in XD	Xs	(3,5); (5,7); (7,8)	
Screen/detector in XD	Xd	(2,7); (2,5)	
Particle characteristics	Pc	(15,20)	20: particle, particle-nature, classical particle
Particle analogy	Pa	(15,21); (15,22)	21: particle-model, particle-analog(y) 22: particle-like, non-wavelike
Wave characteristics	Wc	(15,23)	23: wave, wave-nature, classical wave
Wave analogy	Wa	(15,19); (15,24); (15,25)	19: behav(e)/(ior) 24: wave, wave-like, wave-model, wave motion 25: wave-model, wave-analog(y), nonlocalized
Duality, complementarity	Dc	(2,17); (3,17); (16,19)	17: complementar(y)/(ity), complementarity principle 16: wp-duality, wp-feature, duali(ty)/(ism)
Duality, analogy	Da	(3,18); (16:18);(22;25)	18: model, analog, analogical claim

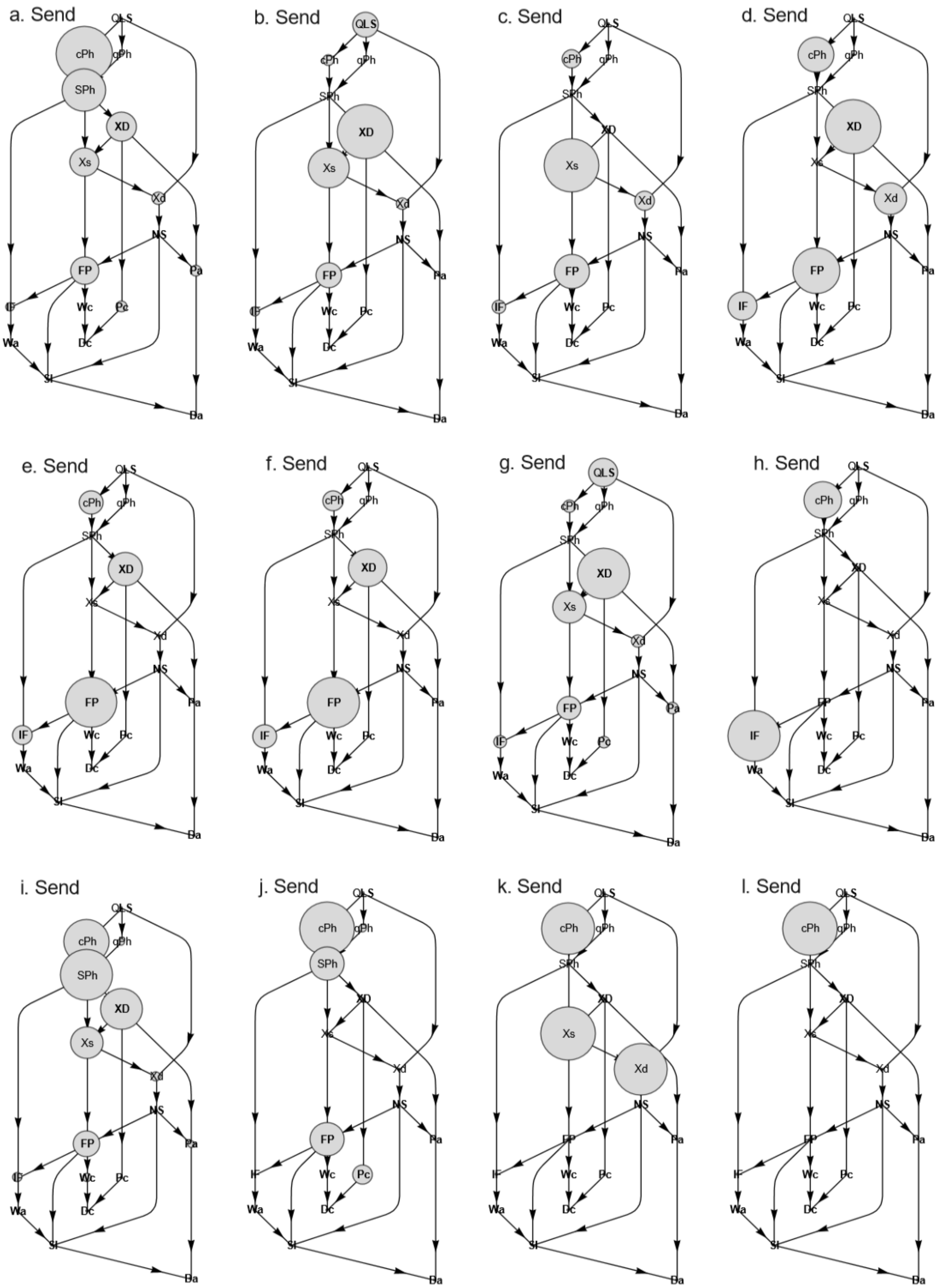

Figure 3. Sender blocks (nodes) in directed argument graphs (DAG) corresponding to lexicons a-l in Fig.2. The size of the node is proportional to the communicability centrality of the sending node.

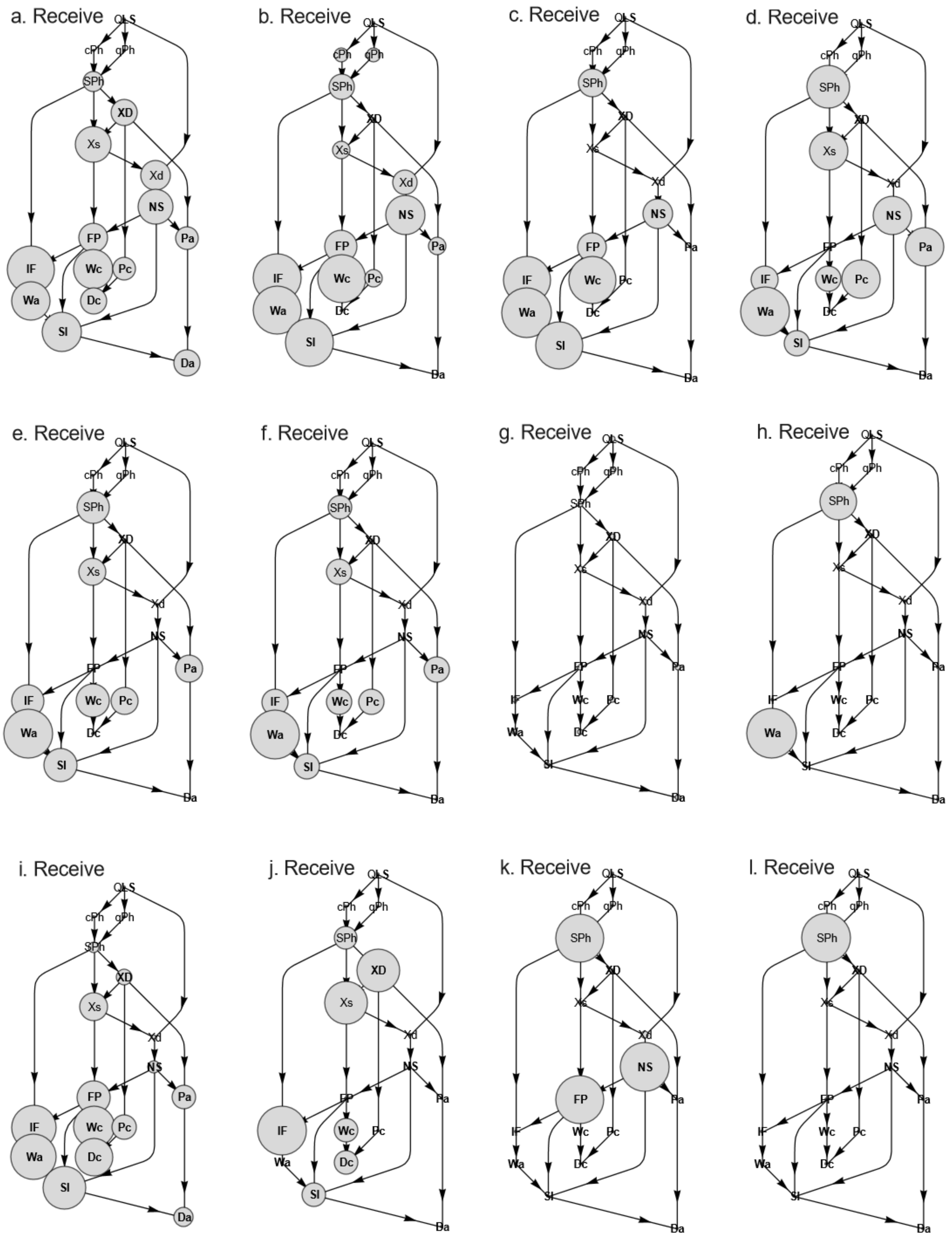

Figure 4. Receiver blocks (nodes) in directed argument graphs (DAG) corresponding to lexicons a-l in Fig.2. The size of the node is proportional to the communicability centrality of the sending node.

The notions made of above mean that sub-lexicons related to FP dominate, sub-lexicons related to XD are also important, and furthermore, both are relatively similar despite other differences in the lexicons. In DAGs a and I, the sender blocks related to photons connected to classical, coherent light (cPh) and single photons (sPh) are strong and thus central to the starting point of argument construction. Consequently, all these cases a-d and I, as well as e and f, are candidates for DAGs where initial information appears to be well covered and central as the starting point of the argument.

Given the results that in DAGs a-d, e, f, and I, the blocks acting as senders are well aligned with expectations of good argument, it is not surprising to find that the final inferences represented as the receiving block and shown in Figure 4 to be related to wave and particle characteristics are strong ones. Of these nodes, Wc and Pc refer to wave and particle characteristics or behavior of photons in experiments or experimental outcomes, paralleling the views contained in the complementarity view of wave-particle duality (Dc). The nodes Wa and Pa capture more moderated ways where different analogies or analogical features of photons or their behavior to waves or particles (Wa and Pa, respectively) are mentioned. It should be noted that while wave-particle duality is not necessarily central to a contemporary understanding of the single photon experiments and the interference phenomena encountered in them (Bhatta, 2021), it is assumed to be central to introductory discussions of photons (Bøe & Viefers, 2023; Bungum et al., 2018). Taken together, the results in Figures 3 and 4 suggest that DAGs a-d, e, f and i represent good and coherent arguments, where the flow of information is as could be expected from didactically recommendable argument structure.

Interestingly, however, of the all DAGS discussed above, only a and i refer explicitly to duality and how it is discussed in complementarity (represented by node Dc) or explicitly mention analogies or analogical models or principles as a starting point for understanding duality (Da). This runs somewhat contrary to expectations based on the historical picture of wave-particle duality, when duality (and complementarity) were centrally connected to wave-particle duality. In these cases, however, receiving nodes (see Fig. 4) corresponding to photon self-interference (SI) and non-splitting (NS) are strong and thus important thematic blocks in all cases a-d and i, thus also including cases b, c, and d, where duality is not explicitly present (strengths of Dc and Dc are nearly zero). These notions are in agreement with recent results based on the analysis of single particle photon interference experiments (Bhatta, 2021), where it is noted that duality and complementarity are not necessarily useful or even tenable underpinnings to approach wave-particle duality, and analogies that are built around interference patterns and the notion of single detection events (i.e. aspects contained in blocks SI and NS) might be more adequate.

On the other hand, many science educators have emphasized the role of wave-particle duality as a didactic starting point for building an understanding of photons as quantum entities (Bøe & Viefers, 2023; Bungum et al., 2018). From this perspective, DAGs a, e, and i, which fall a bit short in regard of sending nodes related to the nature of light (cPh and sPh for photons associated with coherent light and single photons, respectively) and the relevance of experiments (XD for double-slit experiment, Xs, and Xd for source and detector, respectively), but emphasize duality (Dc for complementarity or Da analogy based, or both), appear quite satisfactory and may be feasible as didactic arguments, if duality is considered important. Here we cannot dwell on deeper discussions of the implications of these results for teaching of the topic, but only note that a weak or non-existent emphasis on duality (either Dc or Da) and a strong emphasis on SI and NS is well in line with modern conceptions of advanced experiments with single photon interference (see Bhatta, 2021). Nevertheless, the wave-particle duality retains its centrality and didactic uses, but on the basis of analogical thinking rather than historically motivated views that adopt duality (or complementarity) as a starting point. On the other, emphasizing Dc and Da may also be viable, and lead to acceptable arguments.

The rest of the DAGS (g, h, j, k, and l) are more or less unsatisfactory as representations of argument structure. The arguments fail to lead anywhere or terminate in blocks that are neither inferences or conclusions concerning photons, nor about their duality, but instead related to experimental evidence (e.g. blocks FP for interference fringe patterns, IF interference or XD for double-slit experiment), which should be starting or intermediate steps in the construction of an argument, not terminal points. In these cases, the argument stops at explaining different findings in the experiments but does not proceed to make inferences about the quantum nature of light, the wave-particle duality of photons, or the features that photons display in different experiments. Consequently, the argument structure in this group remains deficient.

4 Discussion and conclusion

We have here scrutinized how pre-service teachers address the wave-particle duality of photons in the context of double-slit experiments. This particular topic of science education has been the focus of attention in several studies, with varying views of its importance in learning about the quantum nature of light (Bøe & Viefers, 2023; Bungum et al., 2018). This has motivated our continued interest to discover how pre-service teachers explicate their views about the duality of photons and what role it plays in their arguments supporting the quantum nature of light. Previously, we explored the vocabularies used by students (pre-service teachers) in explicating their views about double-slit experiments (Vuola et al., 2023). Here, continuing the analysis, we have focused on lexicons (connected vocabularies) and the argument structures (in the form of directed argument graphs, DAGs) in which the lexicons are used.

The results presented here show that in pre-service teachers' reports, experiments and details related to the experiments are indeed very important in constructing arguments about the quantum nature of light and in relating the concept of the photon to wave-particle duality. This is a very satisfactory finding, because deeper understanding of concepts of quantum physics is deeply rooted in experimental situations, preparations of experiments, and the workings of the devices that produce the phenomena under investigation. In particular, such connections are of fundamental importance in understanding the elusive quantum nature of light (see e.g. Bhatta, 2021 for a discussion from the viewpoint of contemporary physics and physics history). Maintaining, supporting, and facilitating further formation of close connections between theoretical concepts and description and experiments is clearly a desirable goal of physics teaching and instruction.

The DAGs provide a detailed picture of how pre-service teachers discuss the role of experiments and wave-particle duality in expressing their views about the quantum nature of light and photons. In a sample of 12 texts, we found five cases (a, b, c, d, and i in Figs. 3-4) where experiments and experimental results (sender blocks in DAGs) were used centrally as a starting point to discuss the quantum nature of light, providing a sound empirical grounding and thus a satisfactory didactic argument. In four (a-d) of these five cases, the experiments were also closely connected to notions of self-interference and non-splitting of photons, both notions that can be warranted and argued in acceptable ways in the context of modern single photon interference experiments (see e.g., Bhatta, 2021; Hentschel, 2018). In these same four cases (a-d), wave-particle duality is also strongly featured as an outcome and end result of the argument, either emphasizing wave-particle duality as a characteristic property of photons that is revealed in various experiments (thus resembling the idea of complementarity) or, alternatively, as a model to characterize the features of experimental outcomes. In all four cases, the arguments of the role of wave-particle duality can be supported but are not crucial to any other key steps in the arguments; wave-particle duality remains an additional feature. This notion supports the view that wave-particle duality, though not needed in our modern understanding of the quantum nature of light, may yet serve useful didactic purposes.

Concerning the role of wave-particle duality, the DAGs reveal that it may play a significant role as the final terminal point (receiving node) of an argument even in the absence of relevance of notions of self-interference and non-splitting, and furthermore, being very weakly supported by experimental data. Two such cases (e and f in Figures 3–4) can be found among the 12 texts. In these cases, wave-particle duality is most probably related to philosophical views and musings rather than concrete experimental findings. Such features are not necessarily desirable outcomes of physics instruction aiming to understand the quantum nature of light. Such views may reflect an uncritical emphasis on historical, long since outdated conceptions of early quantum physics (see Bhatta, 2021; Hentschel, 2018).

Aside from the seven cases discussed above, the five remaining cases (g, h, j-l) are too fragmented and disorganized to allow further discussion. However, even in these cases the vocabularies and lexicons appear satisfactory enough to allow better and more thorough argument constructions. Taking into account how DAGs are constructed, the most obvious explanation for this unexpected finding is that in these five cases, the texts use adequately extensive vocabularies but are by nature disorganized lists or unrelated or weakly related statements of facts.

The most important message finding with regard to practical teaching is that while wave and particle features are sometimes connected to experimental outcomes and then associated with the photon as its characteristic properties, in some cases wave-particle duality is directly and explicitly connected to philosophical and historical ideas of duality and complementarity. Now, while the argument structure in these latter cases may be quite satisfactory, such an increased emphasis on duality as a principle (see Bhatta, 2021) is not necessarily an advantageous feature. Instead, it may signal overemphasis on either naïve particle-wave agglomerative conceptions of the quantum nature of photons or focusing too much on historical views about corpuscular light-quanta (c.f. critical discussions by Bhatta (2021) from viewpoint of philosophy of science). Such a historically oriented viewpoint may not be helpful if a goal of physics instruction is to achieve an improved understanding of the quantum nature of light and the concept *photon* in discussing it (compare with Cheong & Song, 2014).

Of course, understanding the historical views is important in grasping the bigger picture, for example of the difficulties and problems overcome in achieving the current understanding. However, such an understanding becomes useful when views better aligned with contemporary, physics-based understanding are achieved to at least some degree. Following historical routes does not necessary serve that purpose. Our results and their interpretations appear to align with the conclusions drawn by Cheong & Song (2014). Given the results presented here, it is evident that much still needs to be improved in pre-service teachers' education aiming to provide a good ability to construct sound and coherent didactic arguments for teaching modern quantum physics.

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Appendix A The task: Double slit experiment

The empirical sample of 12 reports investigated here was obtained from a study concerning written reports about double slit experiments, written by pre-service physics teachers who will obtain a license to teach physics at the upper secondary level. The study was carried out at a large research-intensive university in Finland. The participants were in their third or fourth year of university studies and they all had a background in basic physics studies, including quantum physics. The data was collected as part of the physics teacher preparation course focusing on introductory quantum physics, from the viewpoint of teaching it at upper secondary school level.

The data came from a task, in which pre-service physics teachers were asked to express their understanding of the double-slit experiment with attenuated laser light (Rueckner & Titcomb, 1996), which for practical purposes for the intended level can be interpreted as consisting of single photons (although on closer inspection, the experiment used as an example is strictly speaking not caused by single photons, see e.g. Hentschel, 2018). The pre-service teachers did not carry out the experiments themselves but were familiarized with the authentic results of the experiment as they appeared in the original research report. They were then asked to give written explanations of the phenomena observed in the experiment. The task was to produce a written report (augmented by a concept map) to explain the formation of interference fringes out of the individual, discrete observation events. The instructions for completing the task were designed so that the pre-service teachers were required to write down an explanation for the basic purpose of the experiment, the findings, and the argumentation to support the findings. The length of the report was usually 1–2 pages. The task itself was designed so that it would encourage the pre-service teachers to express a multifaceted view of how the double-slit experiments can be interpreted.

As background material, the pre-service teachers read a research article that suggests how photons can be interpreted as field quanta in the context of the double-slit experiment (Hobson, 2005). We thus assumed that the explanations of the behavior of photons in the double-slit experiment should contain similarities, especially regarding the quantum terminology.

Other details of data collection, data handling, and issues related to anonymity are as reported in the previous, original research (Vuola et al., 2023). In this study, no additional empirical sample was collected and only the completely anonymized data provided by earlier works by Vuola et al. (2023) has been utilized.

Appendix B Vocabularies and lexicons.

Lexicons are based on vocabularies, and vocabularies are extracted from the text, analyzing its sentences sentence-by-sentence. In what follows, we first describe how vocabulary is extracted from pre-service teachers' texts and how lexicons are obtained from vocabularies.

B.1 Co-occurrence and contingency

The first step in transforming the written texts into semantic networks, in the form of lexicons of terms, consists of splitting the text into sentences. After that, nouns and within them term-like words are picked up. A set of the 70 most relevant terms is chosen for closer attention. The second step consists of carrying out a co-occurrence analysis of the 70 selected terms to obtain a contingency φ-factor (Bonett & Price, 2007) measuring associative co-occurrence to estimate the deviations from random co-occurrence. To obtain the φ-factor describing association between the words A and B in a text, we count four different frequencies of co-occurrence of words A and B in a given sentence: the frequencies n_{11} and n_{00} that A and B occur together or neither of them occurs, respectively; n_{10} and n_{01} that only A or B occurs, respectively. The φ-factor measuring the association is then given as (Bonett & Price, 2007)

$$\varphi = \frac{(n_{00}n_{11} - n_{01}n_{10})}{N}, \tag{B.1}$$

where normalization factor N is chosen so that the values fall within the range $-1 \leq \varphi \leq 1$. The φ-factor measures the associative correlation of co-occurrence of words A and B so that positive values indicate preference to predominant (more frequent than expected) joint co-occurrence (attractive association) while negative values indicate less frequent than expected association (repulsive association). Value φ=0 corresponds to randomly shuffled but otherwise similar text. Here, we are interested only in the attractive associations (φ > 0) because they are obviously the most important for forming the semantic meaning of words as they occur in sentences within a given text.

B.2 From contingency factors to a lexicon

We are interested in the association of all pairs of terms i and j in a set of 70 selected terms of interest. Each pair is then characterized $\varphi(i,j) = \varphi(j,i)$ as defined by Eq. (1) for each pair. Obtaining all (positive) φ-factors $\varphi(i,j)$ for each pre-service teacher, we can construct a lexicon of all pairs i and j of interest. These pairwise values are used to form a weighted adjacency matrix $\boldsymbol{\phi}$ with elements $[\boldsymbol{\Phi}]_{ij} = \varphi(i,j)$ to construct a lexicon whose items are related through the φ-factors.

The importance of the terms contained in the lexicon is operationalized using the so-called communicability centrality (Benzi et al., 2013; Estrada, 2012; Estrada et al., 2012) as a measure of the term's (i.e. node's) global connectivity in a network: this measure is well-suited to characterizing the key terms and words in the lexicon (Koponen, 2020; Koponen & Nousiainen, 2018; Nousiainen & Koponen, 2020). The communicability centrality γ_k of the node k is obtained by using an exponential matrix transformation of the weighted adjacency matrix $\boldsymbol{\phi}$ and is given in the form (Benzi et al., 2013; Estrada, 2012; Estrada et al., 2012)

$$\gamma_k = Z^{-1} \sum_{i \neq k} \exp[\beta \boldsymbol{\Phi}]_{ik} \, , \tag{B.2}$$

with a weight factor $\beta > 1$ and normalization $Z = \mathrm{Tr}\exp[\beta\boldsymbol{\phi}]$, where Tr[.] is a matrix trace. The details of the derivations are not important in what follows and are provided elsewhere (Koponen, 2020; Nousiainen & Koponen, 2020).

Communicability centrality is here used to define the importance ranking of nodes, i.e., the key nodes. The higher the communicability centrality, the more important the given node. In addition, to construct lexical profiles and sub-lexicons and analyzing argument structures, we need the total communicability Γ_X of sub-vocabularies containing a sub-set X of total vocabulary, obtained as a partial sum

$$\Gamma_X = \sum_{k \in X} \gamma_k \, , \tag{B.3}$$

The communicability centrality and total communicability are thus the key quantities in the construction of lexicons and argument structures.

B.3 Lexical profiles

Starting from the lexical networks, we construct a representation of the lexical information attached to a target term T (which is here the term *photon*). To extract the relevant lexicon for T, the terms of interest in the lexical networks are classified into nine different categories P, corresponding to nine different properties, each consisting of key terms $p \in P$. We use here the same thematic categories P1-P9 and key terms (at most seven for each category) as previously (Vuola et al., 2023), summarized in Table I. The information contained in lexicons for *photon* is condensed into a nine-dimensional vector consisting of the total communicability centrality of all key terms in categories P1-P9. The nine-dimensional vector is normalized to have a norm of unity and is called the lexical profile of the term *photon* (see Vuola et al., 2023).

We compare the results for lexical profiles based on the present analysis to the profiles reported in (Vuola et al., 2023). The lexical profiles display the total communicability of words and terms that belong to a given dimension of the nine dimensions P1-P9, listed in Table 3, together with some of the most important key terms used to define the dimensions (see Vuola et al., 2023).

Table 3. The nine thematic dimensions P1–P9. Some examples of central terms (key terms) describing each of the dimensions are provided.

	Profile category	**Example terms**
P1	Classical field, radiation	Magnetic field, electromagnetic radiation, light
P2	Classical energy, intensity	Conservation of energy, intensity maximum, kinetic energy, power
P3	Classical wave model	Interference, diffraction, wavelength, scattering, frequency
P4	Classical particle model	Elementary charge, mass, trajectory
P5	Quantum mechanics	Elementary particle, electron field, quantum of energy, photon model, state
P6	Stochastics	Predictable, random, statistical, probability distribution
P7	Duality	Wave nature, particle nature, de Broglie hypothesis, dual(istic)
P8	Localization, single hits	Place of occurrence, observation point, local, hit, single, individual
P9	Double-slit experiment	Diffraction experiment, electron beam, double-slit system, shutter speed

Note that in constructing the lexical profiles, we have used the same key terms as previously (Vuola et al., 2023). The lexical profiles based on analysis using present contingency φ-factor are in most cases essentially similar to the results previously reported. In Table 4 are summarized Kendall's τ and Spearman's ρ rank correlation coefficients for the communicability centrality values of terms associated with the term *photon* as they are contained in profiles P1-P9.

Table 4. Kendall's τ and Spearman's ρ rank correlation coefficients for communicability centrality of terms contained in profiles P1-P9 as obtained by current and previous analyses, respectively. The p-values lower than 0.05 and 0.005 and 0.001 are indicated by *, ** and *** respectively.

	Lexicon											
	a	b	c	d	e	f	g	h	i	j	k	l
τ	0.56*	0.78**	0.56*	0.61*	0.56*	0.5	0.28	0.61*	0.56	0.78**	0.42	0.42
ρ	0.70*	0.90***	0.75*	0.70*	0.72*	0.63	0.50	0.78*	0.60	0.90***	0.46	0.56

The results in Table 4 show that the profile weights P1-P9 based on communicability centralities obtained in both analyses correlate well in most cases, and with such low p-values that results can be taken as statistically significant. However, there are also cases (i, g, k, and l) where the results are clearly different. These lexicons are rather sparse and small changes can have large outcomes. Although the results in Table 4 should be understood as indicative trends rather than strictly of statistical significance, we can conclude that although the different analysis method provides different results in some cases, the discrepancies are moderate. This notion supports the view that simple frequency-based counting of co-occurrence provides information on lexicons, which is reliable enough for most practical purposes. It also encourages us to think that analysis of vocabulary and lexicons provide safe enough grounds to attempt the construction of the argument structure of pre-service teachers' reports.

Appendix C Directed argument graph (DAG)

The key quantities to construct the DAG are now strengths $\Gamma(x)$ and similarities $S_{XY.}$ They are defined through key terms in a lexicon and their communicability centralities. The argument structure of texts is made visible by using lexicons as a starting point. This attempt resembles the so-called topic construction on the basis of co-word mapping, widely used in automatic analysis of text corpora (Leydesdorff & Nerghes, 2017; Leydesdorff & Welbers, 2011). Thematic contexts can usually be recognized by sectioning the text into paragraphs, but here the texts analyzed are too disorganized and lack clear structure or sectioning into paragraphs. Therefore, thematic blocks are here decided on the basis of what is known of the relevant contexts from the research literature, as explained in the main text. The sub-lexicons that can be associated with each thematic block X are then discerned through the appearance of certain keywords in the given block, as explained in the main text. The sub-lexicons are then always parts of complete lexicons shown in Fig. 2.

In constructing the DAG, we need an operationalization to describe the information flow. First, the total communicability Γ_X of the sub-lexicons in each of the thematic blocks is needed. This is given simply as the sum of communicabilities γk of all terms and words k that belong to the sublexicon,

$$\Gamma_X = \sum\nolimits_{k \in X} \gamma_k \,. \tag{C.1}$$

Second, as explained in section 2.3 of the main text, the information flow from block X to block Y is assumed to be related to the availability of information from the source and the capability to receive the information. This leads to transfer capability transfer capability,

$$\mathrm{W_{XY}} = \sqrt{\Gamma_\mathrm{X}\,\mathrm{S_{XY}}(1 - \mathrm{S_{XY}})} \tag{C.2}$$

where $\mathrm{S_{XY}}$ is the similarity of sub-lexicons of thematic blocks X and Y. The similarity appearing here is the cos-similarity of networks (i.e. here lexicons) defined through centrality values of nodes of the network (Newman, 2018), now in form $\mathrm{S_{XY}} = (\bar{\gamma}_\mathrm{X} \cdot \bar{\gamma}_\mathrm{Y})/(|\bar{\gamma}_\mathrm{X}||\bar{\gamma}_\mathrm{X}|)$, where $\bar{\gamma}_\mathrm{X} = (\gamma_1, \gamma_2, \dots, \gamma_N)_\mathrm{X}$ and $\bar{\gamma}_\mathrm{Y} = (\gamma_1, \gamma_2, \dots, \gamma_N)_\mathrm{Y}$ are vectors containing the communicability centralities of words appearing in blocks X and Y. If sub-lexicons X and Y are identical and similarity is S=1, there is no new information that can be passed on from X to Y. On the other hand, if the sub-lexicons of X and Y are completely different, Y cannot easily (or at all) accommodate information from X. In addition, the flow of information from X to Y must depend on the information content of X, related to its total communicability Γ_X.

The complete DAG is now specified by thematic block and with weights $\mathrm{W_{XY}}$ between given blocks X and Y. Examples, all with the same skeletal structure, are shown in Figures 3 and 4. In these figures, only the sizes of the nodes are different. The sizes of the nodes correspond to in- and out-going total communicabilities, interpreted as in- and out-going information flows $\mathrm{I_{IN}}(\mathrm{X})$ and $\mathrm{I_{OUT}}(\mathrm{X})$ for each node X. These are obtained directly from weight matrix **W** with elements $[\mathbf{W}]_\mathrm{XY} = \mathrm{W_{XY}}$, where $\mathrm{W_{XY}}$ is defined by Eq. C.2., as off-diagonal sums (Benzi et al., 2013; Estrada, 2012; Estrada et al., 2012)

$$\mathrm{I_{IN}}(\mathrm{X}) = Z^{-1} \sum_{\mathrm{Y} \neq \mathrm{X}} \exp[\beta \mathbf{W}]_\mathrm{YX} \tag{C.3}$$

$$\mathrm{I_{OUT}}(\mathrm{X}) = Z^{-1} \sum_{\mathrm{Y} \neq \mathrm{X}} \exp[\beta \mathbf{W}]_\mathrm{XY}$$

with a weight factor $\beta > 1$ and normalization $Z = \mathrm{Tr}\exp[\beta \mathbf{W}]$, where Tr[.] is a matrix trace. All computations with $\beta \approx 3$ lead to stabilized values of $\mathrm{I_{IN}}$ and $\mathrm{I_{OUT}}$. Usually, either $\mathrm{I_{IN}}$ or $\mathrm{I_{OUT}}$ dominates, as is seen in Figs 3 and 4, allowing us to call a node either a receiving or sending block (node) in DAG. Therefore, sending (sources of information) and receiving blocks can be discerned by the analysis. This completes our formalization of argument structures in pre-service teachers' texts.