Appendix 1

Coding scheme and hierarchy of strategies

Core strategy	Observable solution strategy	References in earlier research
No explanation	No solution explanation provided	Several researchers have reported unclear and unambiguous approaches or students not able to explain thinking.
Not possible to determine the strategy	Erroneous: Calculations with random numbers (addition, subtraction, multiplication and division)	
	Correct: Not visible in this sample	
1 Intuitive strategies without ability to demonstrate thinking with mathematically valid explanations	Erroneous: Intuitive approach based on visual representations: drawing, measuring, comparing, observations on pictures. Failing to take the relative nature of the task into consideration.	Karplus, 1983; Lamon, 1993; Langrall & Swafford, 2000
	Correct: Demonstrating understanding of relative thinking, indications on pre-proportional reasoning. Approaching the problem by intuitive methods, for example drawing or visual comparison.	
2 Build-up or build-down/scale-down	Erroneous: Building up or scaling down by skip-counting until the anticipated quantity is reached, errors due to failing to understand the relative nature of the task	Hart, 1984; Tourniaire and Pulos, 1985; Lesh et al., 1988; Lamon, 1993; Kaput & West, 1994; Langrall & Swafford, 2000; Christou & Philippou, 2002
	Correct: Building up or scaling down by skip-counting until the anticipated quantity is reached, demonstrating understanding of the nature of the task	
3 Additive reasoning	Erroneous: Basing decisions on addition or subtraction, but failing to understand the relative nature of the task	Hart, 1984; Tourniaire and Pulos, 1985; Lesh et al., 1988; Baxter & Junker, 2001; Fujimura, 2001; Misailidou & Williams, 2003; Van Dooren et al., 2010
	Correct: Basing decisions on addition or subtraction, demonstrating some understanding of relative nature of the	

	task, but not necessarily able to justify thinking	
4 Multiplicative reasoning	Erroneous: Basing decisions on multiplication or division, but failing to demonstrate understanding of the relative nature of the task or failing to expand the knowledge to cover the whole concept	Hart, 1984; Tourniaire and Pulos, 1985; Lesh et al., 1988; Baxter & Junker, 2001; Van Dooren et al., 2010
	Correct: Basing decisions on multiplication or division, demonstrating some understanding of the relative nature of the task, but failing to provide a mathematically justified explanation for reasoning	
5 Ratio or unit factor approach	Erroneous: Demonstrating understanding of relative nature of task, but failing to proceed into correct end result	Hart, 1984; Lamon, 1993; Kaput & West, 1994; Langrall & Swafford, 2000, Baxter & Junker, 2001; Christou & Philippou, 2001; Fujimura, 2001
	Correct: Demonstrating relative thinking between quantities by using ratio or unit factor approach in solving the unknown quantity	
6 Formal operations with ability to provide mathematically valid explanations	Erroneous: Not visible in this sample	Lamon, 1993; Langrall & Swafford, 2000; Baxter & Junker, 2001
	Correct: Formal operations, demonstrating ability to create and use generalisable formulas, expressing problem-solving process by using algebraic symbols and “mathematical language”	

Appendix 2

Students' ability to solve tasks correctly

		Pre-test in the beginning of fifth grade (*N*=25 or 24*)	Post-test at the end of fifth grade (*N*=25)
Task 1	Correct answer	84% (*N=21)*	80% (*N=20)*
	Erroneous answer	16% (*N=4)*	12% (*N=3)*
	No answer	0%	0%
	Unclear answer	0%	8% (*N=2)*
Task 2	Correct answer	84% (*N=21)*	92% (*N=23)*
	Erroneous answer	16% (*N=4)*	4% (*N=1)*
	No answer	0%	0%
	Unclear answer	0%	4% (*N=1)*
Task 3	Completely correct answer, chose both options	4% (*N=1)*	4% (*N=1)*
	Partially correct answer, chose one option	92% (*N=23)*	88% (*N=22)*
	Erroneous answer	0%	4% (*N=1)*
	No answer	4% (*N=1)*	0%
	Unclear answer	0%	4% (*N=1)*
Task 4	Correct answer	88% (*N=22)*	84% (*N=21)*
	Erroneous answer	12% (*N=3)*	12% (*N=3)*
	No answer	0%	0%
	Unclear answer	0%	4% (*N=1)*
Task 5	Correct answer	80% (*N=20)*	88% (*N=22)*

	Erroneous answer	20% (*N=5)*	8% (*N=2)*
	No answer	0%	0%
	Unclear answer	0%	4% (*N=1)*
Task 6A	Correct answer	24% (*N=6)*	68% (*N=17)*
	Erroneous answer	44% (*N=11)*	24% (*N=6)*
	No answer	32% (*N=8)*	8% (*N=2)*
Task 6B	Correct answer	8% (*N=2)*	48% (*N=12)*
	Erroneous answer	56% (*N=14)*	28% (*N=7)*
	No answer	36% (*N=9)*	24% (*N=6)*
Task 7	Correct answer	67% (*N=16)*	80% (*N=20)*
	Erroneous answer	29% (*N=7)*	16% (*N=4)*
	No answer	4% (*N=1)*	4% (*N=1)*
Task 8	Correct answer	33% (*N=8)*	60% (*N=15)*
	Erroneous answer	50% (*N=12)*	32% (*N=8)*
	No answer	17% (*N=4)*	8% (*N=2)*
Task 9	Correct answer	17% (*N=4)*	52% (*N=13)*
	Erroneous answer	54% (*N=13)*	40% (*N=10)*
	No answer	29% (*N=7)*	8% (*N=2)*

Note: One student was absent in pre-test tasks 7-9.

Appendix 3

Range of different strategies observed in tasks 6A and 6B

Task 6A	Observed solution	Pre-test in the beginning of fifth grade (*N*=25)	Post-test at the end of fifth grade (*N*=25)
No answer or explanation	Did not answer the question	8 students (32%)	2 students (8%)
	No explanation	8 students (32%)	3 students (12%)
Erroneous approaches	Erroneous random calculations	0 students (0%)	1 student (4%)
	Erroneous intuitive approaches	1 student (4%)	3 students (12%)
	Erroneous additive reasoning	3 students (12%)	1 student (4%)
	Erroneous multiplicative reasoning	1 student (4%)	5 students (20%)
Correct approaches	Correct ratio or unit factor approach	4 students (16%)	10 students (40%)

Task 6B	Observed solution	Pre-test in the beginning of fifth grade (*N*=25)	Post-test at the end of fifth grade (*N*=25)
No answer or explanation	Did not answer the question	9 students (36%)	6 students (24%)
	No explanation	11 students (44%)	8 students (32%)
Erroneous approaches	Erroneous random calculations	2 students (8%)	1 student (4%)
	Erroneous additive reasoning	2 students (8%)	3 students (12%)
Correct approaches	Correct intuitive strategies	1 student (4%)	0 students (0%)
	Correct additive reasoning	1 student (4%)	2 students (8%)
	Correct multiplicative reasoning	0 students (0%)	2 students (8%)
	Correct formal operations with generalizable formulas	0 students (0%)	2 students (8%)

Appendix 4

Range of different strategies observed in tasks 7 and 9

Task 7	Observed solution	Pre-test in the beginning of fifth grade (*N*=24)	Post-test at the end of fifth grade (*N*=25)
No answer or explanation	Did not answer the question	1 student (4%)	1 student (4%)
	No explanation	8 students (33%)	0 students (0%)
Erroneous approaches	Erroneous building up or scaling down	1 student (4%)	1 student (4%)
	Erroneous multiplicative reasoning	3 students (13%)	3 students (12%)
Correct approaches	Correct building up or scaling down	2 students (8%)	2 students (8%)
	Correct additive reasoning	1 student (4%)	0 students (0%)
	Correct multiplicative reasoning	8 students (33%)	18 students (72%)

Note: One student was absent in pre-test tasks 7-9.

Task 9	Observed solution	Pre-test in the beginning of fifth grade (*N*=24)	Post-test at the end of fifth grade (*N*=25)
No answer or explanation	Did not answer the question		
	No explanation	12 students (50%)	6 students (24%)
Erroneous approaches	Erroneous random calculations	0 students (0%)	2 students (8%)
	Erroneous building up or scaling down	0 students (0%)	1 student (4%)
	Erroneous additive reasoning	0 students (0%)	5 students (20%)
	Erroneous multiplicative reasoning	4 students (17%)	1 student (4%)
Correct approaches	Correct building up or scaling down	1 student (4%)	2 students (8%)
	Correct ratio or unit factor approach	0 students (0%)	5 students (20%)
	Correct formal operations with generalizable formulas	0 students (0%)	1 student (4%)

Note: One student was absent in pre-test tasks 7-9.