PROMOTING EXPLORATORY TEACHING IN MATHEMATICS

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Abstract In this paper we present a design experiment on a continued professional development (CPD) course for mathematics teachers. It consisted of three teaching cycles. Between them we analyzed the collected data in order to discover the factors affecting the course's effectiveness and improve the next implementation. The general themes of the course, *Introduction to Exploratory Learning in Mathematics*, are teaching methods that promote active learning and exploratory learning environments. The course consists of one-day, on-site training and is aimed at elementary, middle and high school mathematics teachers. It is the first part of a larger CPD unit. We were especially motivated by the recent study of Stylianides & Stylianides (*The Journal of Mathematical Behavior*, *33* (1), 8–29) who proposed that even a very short intervention can impact positively on mathematical problem solving (attitudes) in initial elementary teacher training. Our main research question is thus to replicate and expand on their study: *Can we impact positively on in-service teachers' mathematical thinking over the course of a one-day seminar?* In this article we describe the goals and implementation of our one-day course, some observations made during the implementations and conclusions. We replicate the findings of Stylianides and Stylianides (2014) that their "blond hair problem" makes a great impression on the participants. However, we found that the intervention did not have a substantial effect, at least in the short term, on what were considered good problems to use in an exploratory setting.

1 Introduction

The Finnish teacher education system is generally regarded to be of high quality, and it has been indicated as one of the reasons for Finnish erstwhile highly successful results in the international PISA studies (Ministry of Education, 2009). However, in continued professional development (CPD, i.e. studies after graduating from the university), the situation in Finland is not so good. According to the PISA 2012 report (OECD, 2013) only about 35 % of mathematics teachers had participated in CPD courses "with a focus on mathematics". In the PISA report, Finland is classified as a country where CPD is compulsory. This is true if so-called VESO-days are calculated as CPD. Every teacher has to participate in three VESO-training days per year, each 6 hours in length. However, VESO-"training" can also include events like a gathering of the schools teachers the day before classes start to review school practices. Anecdotal evidence suggests that there is a great disparity between teachers in how actively they seek out CPD opportunities.

In recent years, one of the actors offering CPD courses in STEM subjects has been the national LUMA Centre Finland network. Major funding for these courses has been provided by the National Board of Education (Opetushallitus). The purpose of this article is to describe

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one such course planned and implemented by staff of the University of Oulu during 2013 and 2014.

The funding agency asked for the course to be implemented in a long-term fashion with credits given. In our case two full, one-year implementations were given, the first in the academic year 2013–14 and the second in the calendar year 2014. The modules in the spring of 2014 were attended by participants in both groups.

In order to attract a viable number of participants to the intensive course, we decided to organize the one-day starting event as a stand-alone course which was repeated a total of five times at various locations in Northern and central Finland. This module was called *Introduction to Exploratory Learning in Mathematics* and was organized for the first time in October 2013. The course is situated in the framework of mathematical problem solving research, especially following Schoenfeld (1985, 1992, 2007).

In December 2014 we decided to develop this one-day course within a *design research* framework. The aim of a design experiment is both to develop theory and to study a design in practice. This research method includes testing, analyzing and revising the design. With this iterative method it is possible to collect versatile information about the functionality of the design and to develop its effectiveness. (Cobb, Confey, diSessa, Lehrer & Schauble, 2003)

In our case, the iterative process consisted of three teaching cycles, where an analysis period follows after each implementation of the course. In the analysis period the collected data is analyzed in order to discover the factors affecting the course's effectiveness. Then the course design is revised and the changes made in order to enhance the effectiveness of the course are discussed with the research team.

Studies aimed at impacting mathematical problem solving beliefs have been long-term interventions (Chapman, 1999; Perrenet & Taconis, 2009; Swars, Smith, Smith, & Hart, 2009). Thus we were especially intrigued by the recent study of Stylianides & Stylianides (2014) who proposed that even a very short (75 minutes) intervention can have a positive impact on attitudes during initial elementary teacher training. We included in our design as one component the *blond hair problem* of these authors, as described below. Our main research question is the following:

> Can we replicate the findings of Stylianides & Stylianides (2014) in a Finnish, CPD context? That is to say, can we impact positively on in-service teachers' mathematical thinking over the course of a one-day seminar?

In this article we describe the goals and implementation of our one-day course, some observations made during the implementations and conclusions. In addition, one of the instructors, Vuokko Kangas, has written a reflection on how participating as a teacher in the unit affected her own teaching views.

2 Course philosophy and goals

Early in the course development we were looking at intensive interventions like Chapman's (1999). However, for practical reasons we had to adopt a more modular approach including the stand-alone one-day introductory course under investigation here. In the course we approached exploratory learning through two constituent aspects: mathematical problems solving and co-operative learning.

By mathematical problems solving we understand a teaching philosophy in which pupils are expected to do more than imitate a procedure presented by the teacher. This aspect of the course concerns the pupils' relation to mathematics. In the problem solving lecture in the course (see section 4.3) we used Ryve's (2007) version of Schoenfeld's definition:

A (mathematical) problem is a task that one does not have a solution method (algorithm) for, but that one can solve with the knowledge and skills possessed and that one is motivated to solve.

Like Chapman (1999), we believe that mathematical problem-solving instruction is not mainly about teaching skills and processes, but rather about enable pupils to think for themselves. Also, unless teachers share this view, their instructional practices are not likely to satisfy our conception of a problem solving based lesson (Stein & Kaufman 2010). Ryve, Hemmi & Börjesson (2013) found evidence that this is the case also for Finnish teachers. Ryve (2007) pointed out that problem solving is also a way of developing other mathematical competencies.

Teachers' attitudes and beliefs on mathematics impact their perceptions about how mathematics should be taught (Hannula, Kaasila, Laine & Pehkonen, 2005). Laine and Hannula (2010) found that pre-service elementary teachers' views on mathematics are often limited to seeing it as calculations and following procedures. Some of the students even state they are afraid of teaching mathematics. In particular attitudes affect how teachers employ problems solving activities in the classroom (Ryve, 2007).

Such beliefs and attitudes are formed through one's own experience over a long period of time. Changing such deeply held attitudes is very challenging. According to Wilcox, Schram, Lappan & Lanier (1991) such change requires that one is confronted with one's counterproductive beliefs. Then through acknowledging and re-evaluating the beliefs it is possible to change them. Stylianides & Stylianides (2014) state that the memories that are effecting the students' attitudes should be replaced with new, positive ones in order to be able to influence their attitudes. The intervention has to be powerful and even dramatic for this to be possible.

The design experiment entailed developing the course with an iterative method and specific goals. The effectiveness of our course was evaluated against the following goals:

1. To impact positively on the participants' attitudes towards and beliefs about problem solving and its role in mathematics teaching.
2. To provide examples of concrete ways to implement problem solving and co-operative learning in mathematics teaching.
3. To promote the participants' self-confidence in using exploratory teaching.

3 Research methods and data analysis

There were 18 participants in the first implementation organized in Oulu, 12 in the second implementation organized in Rovaniemi and 12 in the third organized in Kajaani. The majority or participants in each case were mathematics teachers (grades 7–12), with a minority (2–4) of elementary teachers and a few pre-service teachers.

The instructors of the course were Peter Hästö, Marko Leinonen and Vuokko Kangas of the University of Oulu. In the course organized in Rovaniemi, Anna-Maija Partanen of the University of Lapland participated as a guest lecturer and instructor in the course.

Data was collected by videotaping whole-class discussions, audio recording group conversations, with questionnaires at the beginning and at the end of the course, by saving the written solutions of the participants to various tasks. The data gathering methods were developed over the course of the design experiment; consequently, the data gathered from different iterations are not fully comparable.

The discussion below is based primarily on the post-questionnaires (see Appendix 2) and the video recordings. In the analysis the participants' evaluations of the effectiveness of each course unit was compared to the activity seen in the video, and where necessary the audio recordings were consulted to get insight into the work done in the group. Since the video and audio was used only as corroboratory material, it was not deemed necessary to transcribe it.

4 Course

Introduction to exploratory learning in mathematics is a one-day, on-site training for elementary, middle and high school mathematics teachers. The general themes of the course are teaching methods that promote active learning and the use of exploratory learning in mathematics, especially problem solving and co-operative learning. The meaning of understanding in emphasized and the answer-oriented nature of school mathematics is questioned. Participants were encouraged to think about what is relevant in mathematics: the process or the product.

The course consists of five units, with indicative lengths:

- Non-standard solutions and flexible equation solving, 60 minutes
- The blond hair problem, 60–75 minutes
- Lecture on problem solving, 30 minutes
- A practical example, 30 minutes
- Co-operative learning and problem-centered lesson –workshop, 75–90 minutes

The aim of the course is to have positive effects on the attitudes and beliefs of mathematics teachers. It is clear that accomplishing that in one day is a challenge. Hence one of the purposes of this course is to recruit the participants also to the other parts of the larger CPD unit. On the other hand introduction to exploratory learning is an independent part of the education and it is our aim to make it as effective as possible.

4.1 Non-standard solutions and flexible equation solving

The first one-hour unit of the course deals with non-standard solutions and flexible equation solving. The aim of this unit is to expose the answer-orientated nature of school mathematics and to explicate the difference between focusing on process or product. For example the importance of the grading scale is emphasized: does it encourage students to depend on their own reasoning or to memorize algorithms?

First the participants are given a group assignment to grade incorrect solutions, in both standard and non-standard approaches. These solutions are presented in Appendix 1. The groups' results are aggregated in a whole class discussion. The participants explain their decisions and they are encouraged to general conversation about the solutions, the meaning of the correct answer and the written reasoning for the answer in the grading of a mathematic task. In this unit we also recorded the conversations in some of the groups to get more information on their reasoning and possible differences in opinions.

Another point of this unit was to question routine-oriented teaching methods, e.g. students solving as many mathematics tasks as possible in order to learn to apply a given procedure. Both parts of this unit aimed to give the participants practical ways to direct their teaching away from the answer- and routine-orientated mathematics and towards understanding.

The next part of the first unit is flexible equation solving (cf. solution 2 b in Appendix 1). This unit is based on the papers of Star & Seifert (2006) and Rittle-Johnson & Star (2007). The participants are shown multiple linear, one-variable equations and asked to formalize the standard procedure and to describe the possible flexible solving methods that can be used to solve them.

The purpose of this unit is to make the participants see the many possible solutions to a "routine task" and to see the possible advantages of flexible methods in equation solving. Star & Seifert (2006) state that the exclusive use of standard algorithms leads to memorization rather than understanding. They suggest that by teaching flexible equation solving the students' understanding can be enhanced. The main points of the articles were presented: by teaching flexible equation solving methods the students also learned the standard procedure; and the benefits of this teaching method were not restricted to above average students.

4.2 The blond hair problem

The implementation of the second unit, the blond hair problem, follows closely the instructional intervention described in detail by Stylianides & Stylianides (2014). They have developed the intervention as a design experiment to study "whether a positive impact on four specific student problem solving beliefs, which are common and counterproductive, can be achieved with an intervention of short duration." (p. 8) The goal of this unit is to make a positive effect on the participants' problem solving beliefs and attitudes.

4.3 Problem solving lecture

The third unit of the course is a half-hour lecture on problem solving. The aim of this short lecture is to make sure that the participants have a shared understanding of what is meant by "problem solving" in this course, in preparation for the lesson planning/analyzing workshop. Participants were also engaged in a discussion on the pros and cons of this kind of approach; the fact that a problem solving approach also means giving up some element of a traditional classroom (teachers' level of control; amount of material and procedures memorized, etc.) was made explicit.

4.4 A practical example

In the fourth units we aimed to concentrate in offering the participants a practical aspect to exploratory learning and problem solving. This unit was a "changing part" in the course. In the implementations organized outside Oulu we benefitted the local expertise and hence the instructor team of the course varied. In the first and third implementations of this unit Vuokko Kangas gave a lecture on how to make time for exploratory teaching methods. In the second implementation Anna-Maija Partanen presented practical examples of exploratory teaching methods that she has used in her own teaching.

4.5 Co-operative learning and problem-centered lesson –workshop

The final unit of the course proved the most challenging. Therefore, it was substantially changed between each of the iterations.

4.5.1 The first implementation

The title of the last unit is co-operative learning and problem-centered lesson -workshop. It is 75–90 minutes long. This practice-oriented unit aims to offer a concrete example of a problem-centered lesson that the participants can put to use in their own classrooms. Additionally, we sought to acquaint the participants with planning problem-centered lessons.

First there is a short lecture about what co-operative learning is and how it can be implemented in elementary school. The participants are then divided into small groups on the basis of the grade they teach. In the first implementation the participants were given an assignment to plan a problem-centered lesson on a given topic. The topics for elementary school, middle school and high school were respectively dividing a fraction by an integer, dividing a polynomial by a monomial and dividing a polynomial by a polynomial. All three instructors circulated the groups and provided support.

4.5.2 The second implementation

The observations made in the first implementation suggested that this unit did not reach its' goals. Instead of making changes to the assignment itself, the level of support was increased. Examples of problem-centered approaches to each three topics were written up in advance so that they could be presented to the participants if needed. The assumption was that with concrete materials the participants would be able to come up with an idea for a problem-centered lesson.

4.5.3 *The third implementation*

Due to the limited success of this unit, it was revised completely. The idea of the participants planning a lesson was abandoned. We decided to revise the design so that the assignment was not to plan a lesson, but to analyze one. The participants were given short videos of exploratory teaching methods and the assignment to evaluate these methods. The aim was that the participants see a practical example and analyze it from the perspective of problem-centered teaching.

The video analysis was guided by a few given questions. All of the videos were selected from the site *teachingchannel.com*.

5 Observations

We present detailed data from the first and last parts of the one-day course.

5.1 Non-standard solutions

The unit non-standard solutions provoked good conversations about the meaning of the grading scale and the demanding of written reasoning as a part of the solution in mathematics tasks. In the first implementation in one of the groups the differences of opinions resulted in a rather intense but illustrative conversation:

(Talking about task 2 of Appendix 1. Participant 1 is an experienced mathematics teacher and participant 2 is a pre-service mathematics teacher.)

P1 The solution 2a shows that the solver knows what he/she is doing, the solution is logical. The solution 2b is not.

P2 In my opinion the solution 2a shows that the solver has learned the algorithm and the solution 2b shows that the solver has a deeper understanding.

P1 I thought immediately that the solver in 2b does not know what he/she is doing.

P2 How did you come to that conclusion?

P1 Well, my practical experience tells me that. These things have to be taught through an algorithm.

P2 But will the students ever learn to apply, if they are only taught to follow the standard algorithm?

P1 This really isn't an application task, this is a routine task.

This conversation and other comments illustrate the common opinion that some topics have to be taught by presenting the students the standard procedure and giving some examples of using it. Participant 1 is convinced that the solver in 2b does not understand what he is doing because he is not following the standard procedure, and would therefore grade the solution 2a higher than 2b. Clearly this kind of attitude and grading does not support pupil creativity.

In the third implementation there was conversation about the participants' views on what each solver has been thinking while writing the solutions presented in Appendix 1. The conversation revealed many possible interpretations of the level of the solvers' understanding. Some participants (for example participant 1 in the presented conversation) see the use of the standard algorithm as a sign of understanding. Furthermore, participant 1 interprets the use of a non-standard method as a sign of a poor understanding in the absence of any other information about the solver.

However, in all implementations the majority of the participants evaluated the non-standard solutions to be at least as valuable as the standard solutions (cf. Appendix 1). Some participants stated that none, or only the very advanced ones, of their students would use the non-standard methods.

The versatile conversations that arose in this unit in the first implementation can be seen as a sign of its effectiveness. By hearing multiple points of view that the topic evoked, participants have the possibility to re-evaluate their own opinions. Most conversations occurred in the small groups and therefore all the opinions were not presented to the whole class. Even though a good conversation is one of the most effective ways to make people re-evaluate their attitudes, it depends on the participants' activeness and even personalities to a large extent. Hence there was variation in the effect of the conversations between the implementations.

Setting up the conversations is very much dependent on the personalities of the participants and the amount of participants. There was less plenary conversation in the second implementation than in the first implementation. We inferred that one of the elements effecting the conversation might the seating arrangement of the participants.

5.2 Flexible equation solving

Many comments revealed the opinion that below average students cannot benefit from these kinds of teaching methods. It was made clear that in the research of Star & Seifert (2006) the teaching of flexible equation solving had positive outcomes regardless of their mathematical abilities.

However, a common perspective about the teaching of flexible equation solving methods appeared to be that it could be used in ability grouping for the advanced students. Many participants stated that they could show the flexible equation solving methods to above average students as additional information after teaching the standard procedure. Some participants expressed that especially below average students will benefit from flexible solution methods. The common opinion appeared to be that the below average students have to be taught by presenting the routine procedure. Due to this observation one of the aims of the unit flexible equation solving became to question the idea that the below average students cannot benefit from learning flexible methods.

In the second implementation one of the participants, a special education teacher, stated that in her experience below average students would benefit from flexible teaching methods. She stated that these kinds of methods leave more space for the diverse ways of thinking than traditional methods.

This unit appeared to serve its goals relatively well and only few changes were made during the development process. Traditional attitudes towards non-standard solutions emerged in both parts of this unit, as well as voices challenging these views.

5.3 Co-operative learning and problem-centered lesson –workshop

The first implementation had many shortcomings, especially concerning the assignment to plan a problem-centered lesson. Most of the participants stated that the given topics were too difficult to be taught by exploratory teaching methods. Most of the groups had trouble getting started even with guidance. The aim of this unit, generating a workable example, was not reached in the first implementation.

Especially the high school teachers' topic, dividing a polynomial by a polynomial, was experienced to be very difficult. The following comment demonstrates the conceptions and attitudes that some of the participants had on problem-centered teaching methods: "Some topics [dividing a polynomial by a polynomial] have to be taught by traditional methods, through routine procedure." This group abandoned the given topic straight away and wanted to change topic.

The preparation of the participants over the course of the day was insufficient for them to undertake the proposed planning task, even with considerable support. Indeed, most participants in the first two implementations gave up in their effort to come up with ideas of lessons on the given topics. Therefore the aim of generating concrete examples of lesson plans was not reached.

The observations of this implementation indicate that the goals were better achieved than in previous iterations. The results from the post-questionnaire suggest that the participants saw this unit to be the most useful for them. The unit was experienced to be the best one in providing practical tools, which was its main. However, our in-class observations indicate that the analyses of the lessons were not as profound as they could have been.

Since the post-questionnaire was revised between iterations, it cannot be used to evaluate the success of the changes made. However, the observations from the video indicate that in the third implementation the timing and completion of the assignments improved. In contrast to the first two iterations, there was now proper time for both group work and a plenary summary.

This unit revealed both counterproductive attitudes towards problem-centered teaching methods and insufficient ideas about what problems in mathematics can be. The concept of teaching a new topic in mathematics by using problem solving appeared to be unfamiliar to many participants. It was surprising how difficult the assignment was experienced to be. This raises questions about the effectiveness and usefulness of the other units of the course which were supposed to provide the tools for planning.

The observations made in this unit reveal the pervasiveness of the idea that problem-centered mathematics has to be related to the real world. All groups seemed to be focused on coming up with an idea that would associate the topic to the real world. After stating the topic to be difficult, a group of high school teachers said: "We decided to abandon the idea of planning a problem-centered lesson on this topic [dividing a polynomial by a polynomial] and change the topic to something easier, like geometry." The participants in this case found geometry to be an easier topic, because with it they could come up with a real-world problem. It is worth noticing that they nevertheless did not seem too concerned about the authenticity of the real-world problems.

Questioning the compulsory real-world connection of problems became one of the central aims of the problem solving lecture in the second and third implementation. We emphasized further that a problem in mathematics can be purely mathematical. In the third implementation one discussion suggested that the obstacle is not a lack of understanding of this idea, but of the willingness to adopt it. The participants seemed to accept the idea when an example is presented, but adopting it so that it would be possible for them to apply the idea in their own teaching seems to be difficult.

6 Conclusions

The data confirms our expectation that participants find using exploratory methods in their mathematics teaching quite difficult, despite a willingness to use them. Participation in the course during their free time suggests that the teachers are willing to make an effort to learn exploratory methods; this was also evident from the pre-questionnaire. Nevertheless, the habits and attitudes of traditional teaching methods were found resilient to change in most of the participants.

The opinion that exploratory teaching methods are useful mainly as a differentiating method for above-average students was found to be surprisingly common among the participants. While such a thought was also observed in a comparable CPD course on flexible solution methods in the US, it seemed much less prominent there (Yakes & Star, 2011). This may be a reflection of a certain degree of traditionalism which has been observed in Finnish teachers (Andrews, Ryve, Hemmi & Sayer, 2014; Partanen & Kilhamn, 2013).

According to the participants, lack of time, equipment and ready-to-use materials are the biggest obstacles to using exploratory teaching methods. The role of the teacher communities was also raised: the possible resistance from colleagues affects the teachers' willingness to change their teaching methods. Wilcox et al. (1991) state that especially for beginning teachers who are willing to deviate from the traditional teaching methods in mathematics, the support of colleagues who share their vision is essential. These are all external reasons, although elements such as lack of self-confidence in changing teaching methods or fear of failure as a teacher can be seen as internal reasons.

Unsurprisingly, no-one offered internal reasons hindering their use of exploratory methods. However, we found that most participants' rather narrow view on the type of problems they would consider using in exploratory teaching was a severe impediment to their ability to plan an exploratory lesson. It stands to reason that it would thus also be an impediment for adopting and successfully implementing exploratory lessons. The aim to affect the participants' attitudes towards exploratory teaching methods can be seen as an attempt to affect these internal barriers.

Our data from the blond hair problem support the findings of Stylianides and Stylianides (2014) that the intervention is very effective in achieving its goals. Most of the participants experienced both frustration and the feeling of success in the solving process; the intervention was very effective in challenging and questioning the participants' views on problem solving. Since we studied the intervention in a broader context, we can evaluate its effectiveness also on the participants' thinking in terms of planning the use of an exploratory lesson. We found that it did not have a substantial effect, at least in the short term, on what participants considered good problems to use in an exploratory setting. It may of course be that this memorable problem will exert its influence over a longer time scale.

Accepting the idea that a problem-centered perspective can be purely mathematical seems difficult. We observed a strong tendency to associate the term “problem” with real-world problems. One aim of the course, especially in the problem solving lecture, was to provide an alternate, broader conception of “problem”. We found, however, that this idea was only superficially accepted by most participants: when planning their own interventions they reverted to the old idea of looking for real-world problems. Although changing core (teaching) beliefs is known to be very challenging (e.g., Chapman, 1999), we were surprised at the resistance of renegotiating the meaning of even such a rather cognitive level term. We are not aware of descriptions in these terms in the literature of this challenge of adopting problems solving based approaches.

Our data show that many of the participants' views have been questioned and challenged during the course, which can be seen as the first step in broadening their perspective.

In every implementation of the course the participants stated in the post-questionnaires that they would have wanted more practical examples. Naturally, the course also offered tools to overcome the external reasons such as lack of materials. However, it seems difficult to balance the participants' wishes for concrete tips on the use of exploratory methods with the instructors' view that a realignment of teachers' attitudes and beliefs is important for the successful use of these methods.

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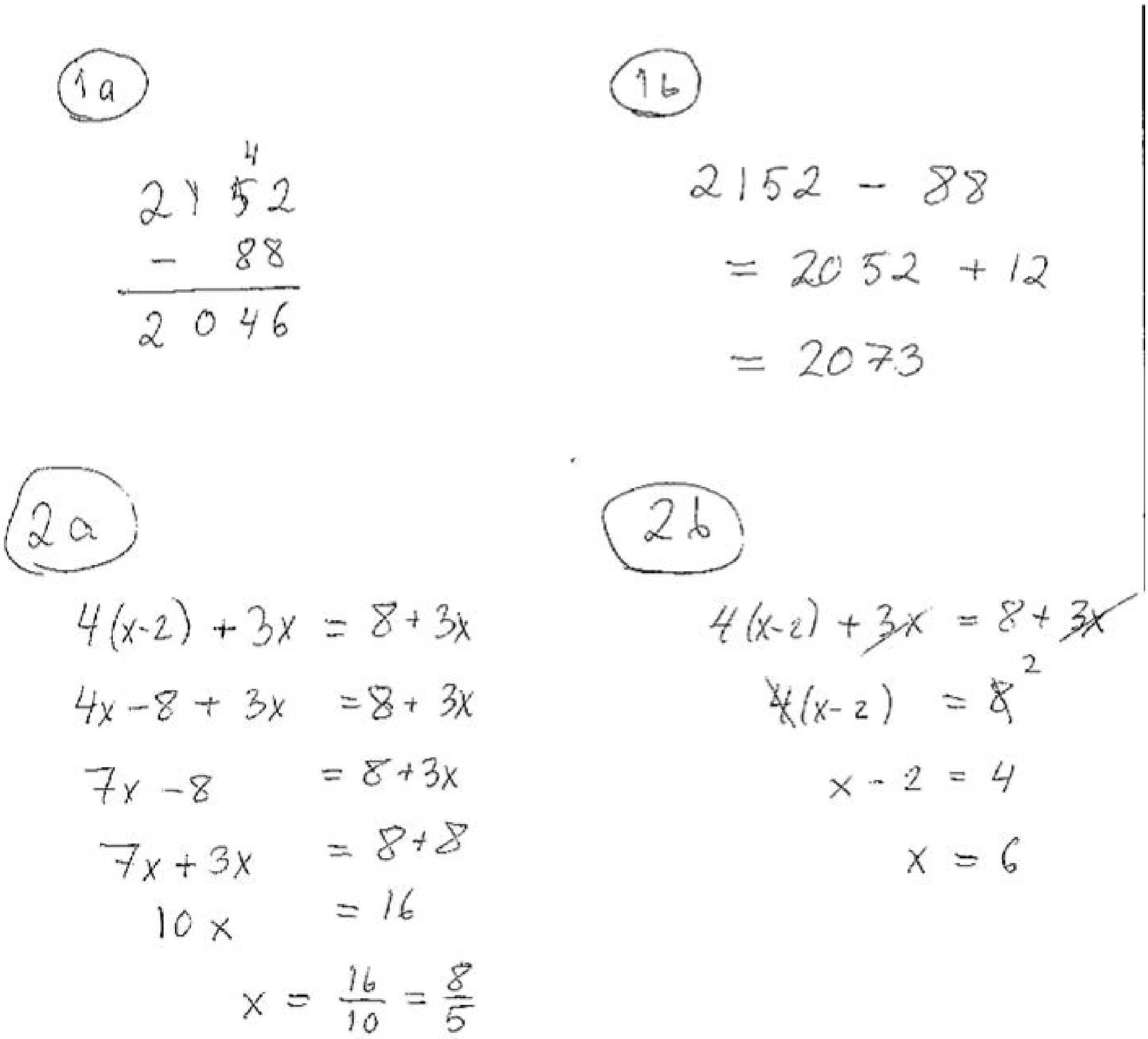
Appendix 1


1a
4
2152
- 88
2046
1b
2152 - 88
= 2052 + 12
= 2073
2a
4(x-2) + 3x = 8 + 3x
4x - 8 + 3x = 8 + 3x
7x - 8 = 8 + 3x
7x + 3x = 8 + 8
10x = 16
x = 16/10 = 8/5
2b
4(x-2) + 3x = 8 + 3x
2
4(x-2) = 8
x - 2 = 4
x = 6

Appendix 2: Post-questionnaire

The assessment of the course

Which of the sections/ topics of this course did you find to be the most useful for you? Why?

Do you think that this course will affect your teaching methods in some way in the future? How?

Did this course offer you concrete ways that you can implement in your mathematics teaching? What ways?

Section	This section provided practical ways that can be used in teaching	This section provoked thoughts and questioned/ challenged my own view	I think that this section will have an effect on my teaching methods in the future	I found this section to be useful
Non-standard solutions and flexible equation solving (9.45–10.45)				
The blond hair problem (10.45->)				
Problem solving lecture (12.45–13.15)				
A practical example (13.15–13.45)				

Co-operative learning and problem-centered lesson -workshop (13.45->)				

Asses the actualization of the claims presented in the top part of the chart. Use the scale from 1 to five by writing the number that describes your experience. Assess the actualization of the claims separately for every section of the course. Aim to remember the thoughts that were raised in each section.

1 – I disagree
2 – I disagree to some extent
3 – I don't know
4 – I agree to some extent
5 – I agree

INSTRUCTOR’S PERSONAL DEVELOPMENT DURING THE DESIGN EXPERIMENT

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1 Background

I work as a senior class teacher and pre-service tutor in Oulu Normal School. I graduated as a teacher of mathematics and later also qualified as a class teacher. I have been teaching mathematics at many grade levels, including middle school.

I have always been interested in exploratory learning and cooperative problem solving. A few years ago I worked on a ”philosophy in mathematics” pilot project, which investigated the impact of dialogical meta-discussion on children’s attitudes toward mathematics. Experts from academic subject faculties participated in the project. This cooperation was clearly beneficial for our student teachers and made me appreciate flexible contacts between the academy and the training school.

At the beginning of the current project, I expected to obtain many new ideas for tutoring student teachers in problem-solving mathematics instruction. I mostly expected the project to provide a handy set of problem-solving tasks and ideas that all participants could use in their own classrooms. I also assumed that participation in the project would promote cooperation between the academic faculty and the training school and improve my substance knowledge of mathematics.

2 Planning and preparation of the course

The design experiment course was planned over a long period with active email discussions and numerous meetings.

I found the planning of the course extremely inspiring. I appreciated the company of distinguished mathematics experts, but I also felt I would have things to share from my experience as a training school teacher and a tutor of student teachers. I thought I could share experiences of enthusiastic children who were able to excel themselves.

Having read the article by (Stylianides & Stylianides 2014), I expected their frame of reference to be applicable to our course without much modification. The planning meetings chaired by Peter Hästö, however, made me re-consider things. I realized that we should aim to involve each participant in a more profound process, to make them reflect on how pupils really learn and how we could make our classroom instruction more problem-based. As it turned out, I was also personally drawn into this learning process.

3 Implementation of the course

Since I use many cooperative learning methods with my pupils, my primary role in the project was to share my teaching experiences. Secondly, I was to serve as a tutor in the practical situations where methods of exploratory learning were applied. This challenged me to test cooperative problem solving even more actively in my classroom. I particularly aimed to find problem-solving tasks suitable for different grade levels and projects of differing length and depth.

For many of the participants the main motivation to attend the course had been a desire to acquire new materials and ideas as well as a set of new and relevant problem-solving tasks. The tasks that I presented inspired such lively discussion that we fell behind the schedule. Many participants began to solve the problems right away, sharing their experiences of having had their own pupils work on similar tasks. They also discussed the age levels and types of pupils that would best benefit from each task.

I was surprised to see how well teachers of different grade levels were able to discuss the challenges of problem-based learning. As the course progressed, however, we considered it best to divide the groups by grade level. Such groups shared enough background and general interest to be able to apply to practice what they had learnt during the course.

While discussing with the pairs or small groups working on ”flexible problem-solving” tasks and test grading, I realized that teachers may hold highly divergent views. Surprisingly many of them focus on the outcome rather than the process. However, it is important to understand how pupils think. It would therefore be good to perceive even the smallest signs of logic in their work. If we only look at the final solution, which may be wrong because of a minor oversight at some stage of the problem-solving process, we may miss the potentially ingenious shortcuts or insights used by the pupil while working to reach the solution.

The approaches to teaching problem solving can be divided into three groups (Schroeder & Lester, 1989):

1. teaching for problem solving (traditional),
2. teaching the process of problem solving (heuristics), and
3. teaching through problem solving (PBL)

We asked all course participants to work out their preferential mutual weighting of these three approaches.

4 Personal development between course sessions

After the first course session it was easy to point out development goals for the timing of course modules, practical arrangements, working practices and other formal matters. In addition to these, I realized that I had mostly concentrated on sharing useful tips and interesting ideas instead of aiming at a profound change in the core beliefs, which is what we should aim at while tutoring teachers and developing our own competence. Maybe I had only contributed my old standard idea of teaching the problem-solving process, though I had applied methods of exploratory and cooperative learning? I also received valuable feedback from my colleagues, and that together with my own insights helped me to re-formulate my course module. I chose to focus my message even more clearly on Schroeder & Lester’s third item, teaching through problem solving.

Teaching through problem solving is relatively easy in elementary instruction. But how about more advanced mathematics? Could this method be applied to dividing by fractions? Or even dividing by polynomials? This topic inspired lively discussion during the early course sessions already.

Based on feedback on the first course session, we asked the participants of the next session to list some of the topics they would be teaching soon after the course. We thereby wanted to customize the workshops on relevant topics. We also asked them to design grade-specific problem-solving methods for learning division by fractions or division by polynomials.

I found this assignment especially inspiring and implemented it in my own classroom. By using concrete tools, most pupils learnt easily to reduce and expand fractions and willingly shared their insights with their classmates.

Although my pupils liked this way of working, they were a bit worried about not using their textbooks at all, and the student teachers working with the class shared this concern. This inspired us to discuss in greater depth what is really important in learning, how learning takes place, and what things best help pupils to learn. These discussion encouraged the pupils to set learning goals for themselves and to learn more self-control.

I also feel that this concern of both the pupils and the student teachers about missing out much of the textbook material is amenable to a practical solution. There are online mathematics games and teaching programs that children like to use during their free time. Also, if they learn to use reasoning and problem solving, they need not burden their memory with so many algorithms. It remains for the teacher to find a suitable balance between learning standard calculations and learning to solve problems by reasoning.

5 My key insights during the course

I am sure that, after this course, I will never be able to teach mathematics with a good conscience without doing my best to help the pupils explore topics through problem solving. That will make them want to learn mathematics and will maintain their interest and motivation. The new Finnish curriculum contains many references to phenomenon-based (multidisciplinary) learning. I think this bodes well for mathematics, naturally provided that the multidisciplinary modules are designed to highlight the importance of mathematics for understanding, comparing, and analyzing phenomena as well as for exploring their background and causal relations.

The articles and videos covered during the course helped me find even more interesting articles and videos on the teaching of mathematics. I thus collected a lot of useful material that I can use in my efforts to support and encourage student teachers to apply problem-based methods.

When I joined this design experiment, I expected to be able to teach others and share my problem-based teaching style. Now that the course is over, I realize that I ended up having a fantastic learning experience myself. Moreover, the colleagues interested in mathematics instruction that I met during the course will make up a valuable network for developing and sharing ideas and creating and implementing the new curriculum.

I learnt something new during each course session. Having worked as a teacher for a long time, I especially appreciated the close contacts with the academic faculty. I gained new insights and more profound mathematical competence, which will be reflected in both my classroom work and my role as a tutor of student teachers. I now feel more confident about the key aspects of the didactics of mathematics and more inspired to teach.

6 Main learning points

1. Pupils like to discuss topics together, and different learners help the group to find a synergistic solution. Problem solving is perfectly compatible with cooperative learning.
2. Problem solving can be used to teach children mathematics. "A (mathematical) problem is a task that one does not have a solution method (algorithm) for, but that one can solve with the knowledge and skills possessed and that one is motivated to solve." (Schoenfeld, 1992)
3. It is important to develop interactive skills. We should encourage children to verbalize their thoughts.
4. Teaching problem-solving skills is consistent with the essence of mathematics: it highlights reasoning rather than calculation, the process rather than the outcome.

References

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